Geometrical Drawing

for

Art Students

Morris

RAW

GEOMETRICAL DRAWING

FOR

ART STUDENTS

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BY

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IN LONGMANS' ELEMENTARY SCIENCE MANUALS
'THE TEACHING OF DRAWING'

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PREFACE TO THE FIRST EDITION

THIS little book has been prepared to meet the wants of those students who only require the Geometry necessary for the Art Student's course. The work necessary for the Second and Third Grade Art Certificates is fully covered, and the student who has thoroughly mastered the contents of the book will find himself well equipped either for examination or for taking up a more advanced course of the subject.

The book contains over seven hundred figures arranged in a convenient form, and a very complete and exhaustive collection of exercises, and covers rather more ground than is absolutely necessary for the South Kensington Examination in Geometrical Drawing. The Chapter on Solid Geometry has been made unusually full, as the Author's experience is that one of the student's chief difficulties is the want of sufficient variety of examples in this important branch of the subject.

The Author is indebted to Bradley's 'Practical Geometry' (Library of Useful Knowledge), and his later work published for the Committee of Council on Education, Winter's 'Geometrical Drawing,' Carroll's 'Practical Geometry for Science and Art Students,' and Meyer's 'Handbook of Ornament,' for much useful information.

I. H. M.

SHEFFIELD: *August* 1890.

PREFACE TO THE FOURTH EDITION

THIS Edition has been thoroughly revised to meet the latest requirements of the Syllabus in Geometrical Drawing.

I. H. M.

August 1896.

PREFACE TO THE EIGHTH EDITION

THIS Edition has been revised and enlarged to meet the present requirements of the Syllabus in Geometrical Drawing.

I. H. M.

August 1901.

CONTENTS

SYLLABUS

OF

ART EXAMINATION

GEOMETRICAL DRAWING (ART)

NOTE.—The portions of this book dealing with the present syllabus are indicated in *italics*.

($1\frac{1}{2}$ *hours are allowed for this Examination*)

This examination is intended to test:—

(A) The students' ability to use compasses, T square, set squares, protractor, and scales, in showing their knowledge of ordinary geometrical constructions, and

(B) Their power of applying these constructions to ornamental and decorative work, which they may do both by freehand drawing and by means of instruments.

There will be first and second class passes. A second class success will be accepted for the Elementary Drawing Certificate, and a first class success for the Art Class Teachers' Certificate. Candidates will be required to qualify in:—

(A) 1. Constructions of triangles (*Chap. V.*), quadrilaterals (*Chap. VI.*), and polygons from given data (*Chap. VIII.* and *Chap. X. probs.* 110–116).

2. Describing circles to satisfy given conditions—passing through given points (*Chap. VII. probs.* 64–70), touching lines and circles (*Chap. XI. probs.* 121–126 and 146–163). Drawing straight lines, touching circles (*Chap. VII. probs.* 71–76).

3. Construction of figures similar to given figures (*Chap. X. probs.* 117–119).

4. Proportional division of lines, including third, fourth, and mean proportional, extreme and mean ratio (*Chap. IV.*). Plain and diagonal scales. Scale of Chords (*Chap. IX.*).

5. Construction of the ellipse, drawing its tangents and normals. Drawing curves defined by simple conditions (*Chap. XIV.*).

6. Inscribing and describing rectilinear figures and circles within and about others (*Chap. XI. probs.* 127–145, 164–177, and *Chap. XII.*).

7. Plans, elevations, and sections of simple geometrical solids, singly or in combination, in simple positions (*Chap. XV.*).

(B) The application of geometrical constructions to setting out schemes of ornamental patterns, construction of units of patterns, spacing of wall and other surfaces for decorative purposes, and construction of arch forms, tracery, and mouldings, etc. (*Chap. XVI.*).

GEOMETRICAL DRAWING

FOR ART STUDENTS

CHAPTER I

INTRODUCTION

In Practical Geometry we apply the principles of Theoretical Geometry to construction by the aid of instruments. The student who has a knowledge of Euclid will find it of considerable service in understanding the principles used, and also in remembering the methods of construction adopted.

It is of the highest importance that the problems should be worked with the greatest possible accuracy and neatness, and in a variety of positions, as problems frequently present fresh difficulties when the position of the points or lines is altered.

The **instruments** used need not be numerous, but should be of the best make and finish that the student can obtain, as inferior instruments frequently cause much trouble and vexation, and render the accuracy so indispensable in a geometrical drawing an impossibility. The following are absolutely essential :—

1. A Drawing-board with a perfectly level surface, and with its corners true right angles. Half-imperial is a very convenient size.

2. A T square.—This is used to draw lines parallel to the edge of the board. It is not advisable to use it for the purpose of drawing *vertical* as well as horizontal lines, as, if the board be not true, inaccuracies will be caused in the drawing. The *vertical* lines are best obtained by using the set square. It is advantageous to have the edge of the T square bound with hard wood, and bevelled. The blade should be screwed on to the head, as this arrangement allows the set squares to be used more freely.

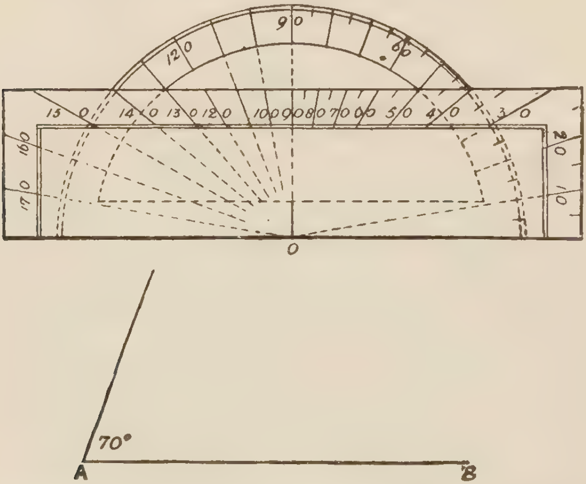
3. Two set squares, having angles of 45° and 60° respectively. These are used to obtain perpendiculars and parallels. Skilful manipulation of these useful instruments will enable the student to construct many simple figures by their aid alone.

4. Pencils.—These should be HH for the construction lines, and H for darkening the lines of the constructed figure. They are best

sharpened like the edge of a *chisel* for geometrical drawing, as the point lasts longer.

5. **Mathematical instruments.**—These should include:—**A compass** with movable pen and pencil legs (those with needle-points are preferable, as they do not make so large a hole in the paper); a pair of **dividers**, for measuring; a **mathematical pen**, for ruling lines in ink. In addition to these, a set of *spring bow compasses*, for describing small arcs and circles, are of great assistance. **Indian ink** must always be used with the instruments, as it does not corrode them; that sold in a liquid form is very convenient. After the pens have been used they should be carefully wiped, to prevent rust.

6. **A protractor.**—This is made either semi-circular or as a flat rule; the latter form is more convenient. The illustration shows



how the flat protractor is made from the round one. The instrument is used as follows:—Suppose an angle of 70° be required at point A in the line A B. Place the mark O, which is always indicated on the protractor, on the point A, keeping the edge of the instrument on the line A B, and make a tick with the pencil at the number 70°; remove the protractor, and join the tick with the point A. An angle of 70° will then have been made with the line A B. The degrees are numbered from each end of the protractor, so that the angle of 70° may be made either to the left or right of the point A.

7. **Paper and pins.**—For ordinary pencil-work cartridge-paper is the most suitable; when the drawings have to be inked in, a better quality is desirable, such as Whatman's smooth papers. Pins with heads soldered on are the most convenient.

8. Parallels and perpendiculars.—These should be obtained by means of the **T** square and set squares. The following instruction is given on the examination papers of the Board of Education:—'Lines parallel or perpendicular to others may be drawn mechanically, without showing any construction. Lines may be bisected by trial.'

CHAPTER II

DEFINITIONS, TERMS USED, &c.

1. A point has no magnitude. It merely indicates *position*, and is marked either by a dot or as in Fig. 1.

LINES.

2. A line has *length* and *position*, but neither breadth nor thickness. It is indicated by the letters placed at its extremities, as A B, Fig. 2. Various methods of drawing lines are used in practice, as thick, thin, dotted, and chain lines. Fig. 2.

3. A straight line is the shortest distance between two given points. It is also called a *right* line.

4. A curved line is nowhere straight. There are endless varieties of curved lines. Fig. 3 is a *simple* curve. Fig. 4 is a *compound* curve.

5. A horizontal line is perfectly level, like the surface of still water. Fig. 5.

6. A vertical line is perfectly upright, like a plumb-line. Fig 6.

7. An oblique line is neither horizontal nor vertical. Fig. 7.

8. Parallel lines are the same distance apart, and cannot meet, however far they may be produced. Fig. 8.

ANGLES.

9. An angle is the inclination to each other of two straight lines which meet together.

10. A right angle.—When one straight line meets another straight line so as to make the adjacent angles (those on each side of the line) equal to one another, each of these angles is a right angle, and the lines are **perpendicular** to each other. In Fig. 9, the angles **A B C** and **A B D** are each right angles.

11. An obtuse angle is greater than a right angle. Fig. 10.

12. An acute angle is less than a right angle. Fig. 11.

13. The vertex is the point where the two lines forming an angle meet, as at A, Fig. 11.

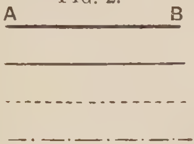
14. Adjacent angles have a common vertex and one common arm. In Fig. 12, the angle D A B is adjacent to the angle D A C.

15. The complement of an angle is the difference between it and a right angle. In Fig. 13, the angle C A B is the complement of the angle C A D, and C A D is the complement of C A B.

16. The supplement of an angle is the difference between it and two right angles. In Fig. 14, the angle C A B is the supplement of the angle C A D, and *vice versâ*.

FIG 1.

FIG. 2.



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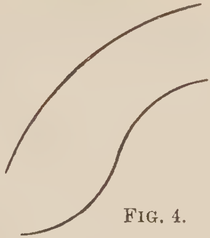


FIG. 4.

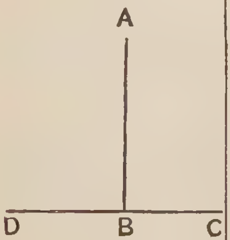
FIG. 5.

FIG. 6.

FIG. 7.

FIG. 8.

FIG. 9.



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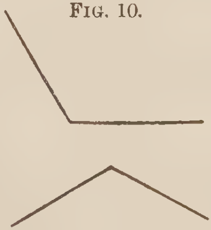

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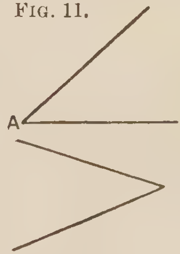

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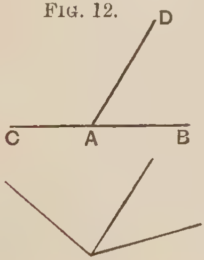

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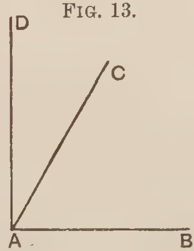
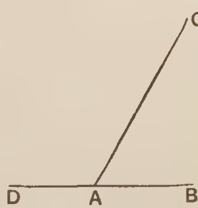


FIG. 14.



TRIANGLES.

17. **A triangle** is a figure contained by three straight lines. If it be supposed to stand upon one of its sides, that side is termed its **base**; the point where the other two sides meet is its **vertex** or **apex**, the angle at the vertex is the **vertical angle**, and the perpendicular from the apex to the base or base produced is the **altitude.** Thus, in Fig. 15, if A B be the base, then C is the vertex or apex, A C B the vertical angle, and C D the altitude.

Triangles are named either from the comparative lengths of their sides, or from the magnitudes of their angles.

1*st. With reference to their sides there are three kinds :—*

18. **An equilateral triangle** has three equal sides. Fig. 16.

19. **An isosceles triangle** has two of its sides equal. Fig. 17.

20. **A scalene triangle** has three unequal sides. Fig. 18.

2*nd. With reference to the angles there are also three kinds :*

21. **A right-angled triangle** has one of its angles a right angle. The side opposite to the right angle is called the **hypotenuse.** In Fig. 19, A B is the hypotenuse.

22. **An obtuse-angled triangle** has one of its angles obtuse. Fig. 20.

23. **An acute-angled triangle** has three acute angles. Fig. 21.

Note.—All the angles of any triangle equal two right angles, or 180°; thus, if two of the angles of a triangle are respectively 60° and 50°, then the remaining angle must be 70°, to make up the 180°.

QUADRILATERALS.

24. **A quadrilateral** figure is bounded by four straight lines. It is also termed a **quadrangle**, from having four angles.

Note.—The four angles of any quadrilateral figure must always equal four right angles, or 360°.

25. **A parallelogram** is a quadrilateral figure in which the opposite sides are parallel. The straight line joining the opposite angles of the parallelogram is called the **diagonal.** In Fig. 22, A B is the diagonal. There are four parallelograms—viz., the square, the rectangle or oblong, the rhombus, and the rhomboid.

26. **A square** has all its sides equal and all its angles right angles. Fig. 22.

27. **A rectangle** or **oblong** has its opposite sides equal and all its angles right angles. Fig. 23.

28. **A rhombus** has all its sides equal, but its angles are not right angles. Fig. 24.

29. **A rhomboid** has its opposite sides equal, but its angles are not right angles. Fig. 25.

30. **A trapezoid** has only two sides parallel. Fig. 26.

31. **A trapezium** has none of its sides parallel, but may have two of its sides equal. Fig. 27. When two of the sides are equal, the figure is sometimes called **a trapezion, or kite.**

Fig. 15.

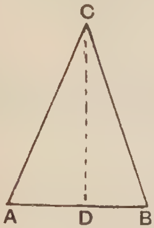


Fig. 16.

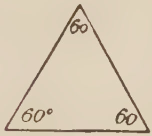


Fig. 17.

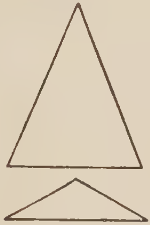


Fig. 18.

Fig. 19.

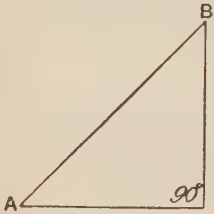
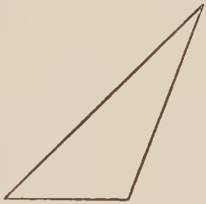


Fig. 20.



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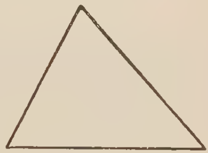


Fig. 22.

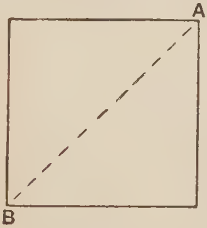


Fig. 23.

Fig. 24.

Fig. 25.

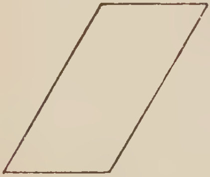
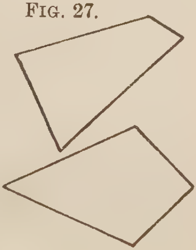


Fig. 26.

Fig. 27.



POLYGONS.

32. **A polygon** is a plane figure bounded by straight lines.

Note.—In some works a polygon is defined as a figure bounded by more than four straight lines.

If the sides of the figure are equal, it is termed a **regular** polygon Fig. 28. If the sides are unequal, it is called an **irregular** polygon. Fig. 29.

Polygons are named according to the number of their sides :—

A pentagon has five sides.
A hexagon has six sides.
A heptagon has seven sides.
An octagon has eight sides.
A nonagon has nine sides.
A decagon has ten sides.
An undecagon has eleven sides.
A duodecagon has twelve sides.

THE CIRCLE.

33. **A circle** is a plane figure contained by one curved line, called the **circumference**, every point of which is equally distant from a point within the circle, called the **centre** (A, Fig. 30).

Note.—The circumference is sometimes spoken of as the circle.

34. **The diameter** of a circle is the straight line passing through its centre, and is terminated at both ends by the circumference, as B C, Fig. 30. It divides the circle into two semicircles. If two diameters be drawn at right angles to each other, the circle is divided into **quadrants**. Fig. 30.

35. **The radius** of a circle is the distance from the centre to the circumference, as A B, Fig. 30.

36. **An arc** of a circle is the portion between any two points in the circumference, as A B, Fig. 31.

37. **A chord** is the straight line joining the ends of an arc, as the straight line C D, Fig. 31.

38. **A segment** is the part of a circle bounded by an arc and its chord. Fig. 32.

39. **A sector** is the part of a circle contained by two radii and the arc between them. Fig. 33.

40. **A tangent** is a straight line which touches a circle, but does not cut it when produced, as A B, Fig. 34.

41. **Concentric circles** have the same centre but different radii. Fig. 35.

FIG. 28.

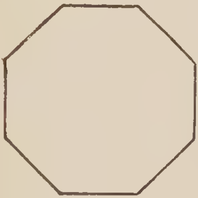


FIG. 29.

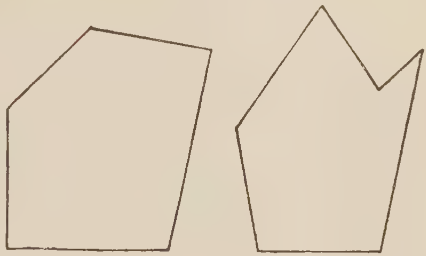


FIG. 30.

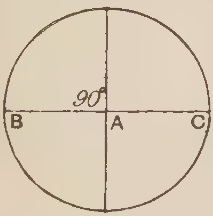


FIG. 31.

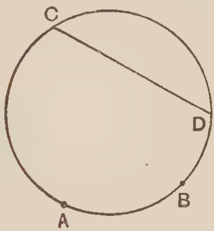


FIG. 32.

FIG. 33.

FIG. 34.

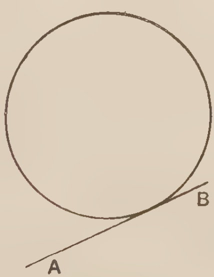
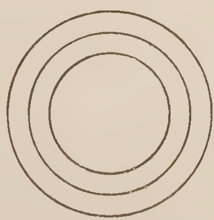


FIG. 35.



CHAPTER III

LINES AND ANGLES

PROBLEM 1.—To bisect a given line.

With centre A, and any distance greater than half the line describe an arc. With centre B, and the same radius, intersect it in C and D. Draw the line C D. Then the line C D bisects the given line at right angles, or is perpendicular to it. (Euc. I. 10.)

The arc of circle E F may be bisected in a similar manner. By this problem a line may be divided into 4 equal parts by again bisecting each half as shown. If those parts be again bisected, the line would be divided into 8 equal parts; and in a similar manner into 16, &c.

PROBLEM 2.—To draw a perpendicular to a given line, from a given point in the line.

Let C be the given point. With C as centre, and any radius, set off equal distances C 1 and C 2. With 1 and 2 as centres, and any radius greater than half 1 2, describe arcs intersecting at D. Draw C D, the required perpendicular.

PROBLEM 3.—To draw a perpendicular when the given point is at or near the end of the line.

Let B be the given point. With centre B describe an arc. With the same radius cut this arc in points 2 and 3. From 2 and 3 describe arcs intersecting at C. Draw B C, the required perpendicular.

Note.—Keep the same radius for describing all the arcs.

PROBLEM 4.—The same. (Another method.)

Take any point C not in the given line A B. Draw C B. With centre C and radius C B describe an arc cutting A B in D. Join C and D, and produce until it cuts the arc in E. Draw B E the required perpendicular.

Note.—All angles in a semicircle are right angles. (Euc. III. 31.)

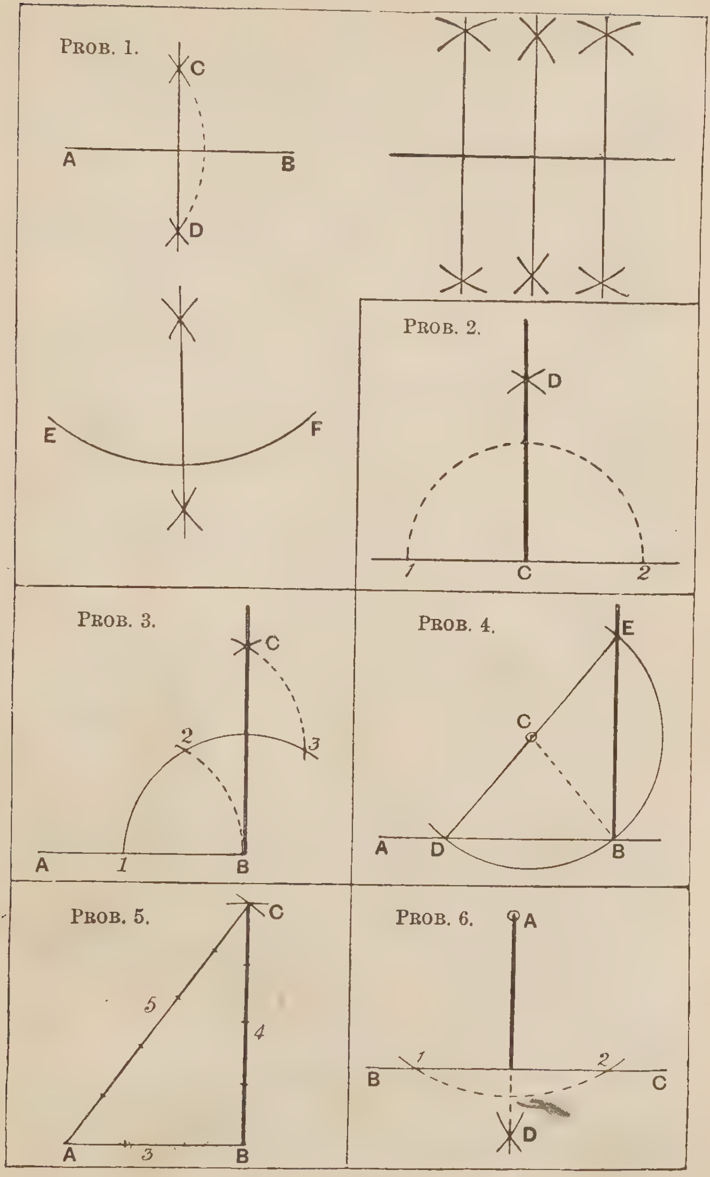
PROBLEM 5.—The same. (For large work when instruments are not available.)

From B set off 3 equal divisions towards A to any convenient unit. With centre B and 4 of the same units describe an arc at C. With centre A and 5 units cut the arc at C. Draw B C the required perpendicular. (Euc. I. 48.)

Note.—Any triangle in which the square on one side equals the sum of the squares on the other two sides is a right-angled triangle. $AC^2 = AB^2 + BC^2$, or $5^2 = 3^2 + 4^2$.

PROBLEM 6.—To draw a perpendicular to a line from a given point without it.

Let A be the given point. With centre A describe an arc cutting the given line B C in points 1 and 2. With centres 1 and 2 describe arcs intersecting at D. Draw A D, the required perpendicular. (Euc. I. 12.)


PROB. 1.
C
A
B
D
E
F
PROB. 2.
D
1
C
2
PROB. 3.
C
2
3
A
1
B
PROB. 4.
E
C
A
D
B
PROB. 5.
C
5
4
A
3
B
PROB. 6.
A
1
2
B
C
D

PROBLEM 7.—To draw a perpendicular when the given point is nearly over the end of the given line.

Let A be the given point. With centre B and radius B A describe an arc. With centre C and radius C A describe an arc cutting the first arc in A and D. Draw the perpendicular A D.

Note.—This method may be adopted for all positions of the point, two of which are shown.

PROBLEM 8.—The same. (Another method.)

Let A be the given point. Take any point D in the line B C. Join A D, and bisect it in E. With centre E describe the semicircle A F D, cutting B C in F. Draw the perpendicular A F. (Euc. III. 31.)

PROBLEM 9.—To draw a line parallel to a given line, at a given distance from it.

Let A B be the given line, and C D the given distance. Take any points 1 and 2, and with radius C D describe two arcs, E and F. Then the line touching the arcs will be the required parallel.

PROBLEM 10.—To draw a line parallel to a given line through a given point.

Let A be the given point, and B C the given line. With any point 1 as centre, and radius 1 A, describe the arc A 2. With A as centre and the same radius describe another arc. Cut off 1 D equal to A 2. Draw A D, the required parallel.

PROBLEM 11.—To bisect a given angle.

With centre A and any radius describe the arc B C. With centres B and C describe arcs intersecting in D. Then the line A D bisects the angle.

By bisecting each half again, the angle will be divided into 4 equal parts, as shown. In a similar manner, it may be divided into 8, 16, &c., equal parts.

PROBLEM 12.—To trisect a right angle.

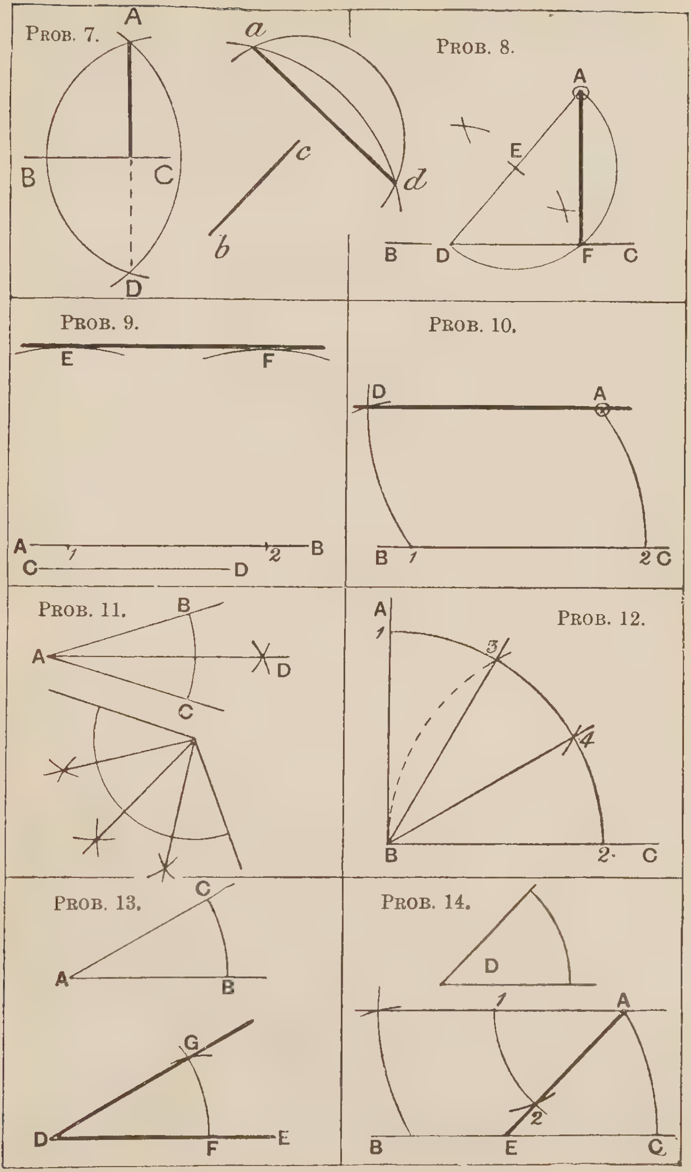
Let A B C be the given right angle. With centre B and any radius describe the arc 1 2. With centres 1 and 2 and the same radius cut the arc in 3 and 4. The lines drawn from B through these points will trisect the right angle.

PROBLEM 13.—To make an angle equal to a given angle.

Let B A C be the given angle. Draw any line, D E. With centres A and D describe arcs B C and F G. Cut off F G equal to B C. (Euc. III. 27.)

PROBLEM 14.—Through a given point to draw a line meeting another line at an angle equal to a given angle.

Let A be the given point, B C the given line, and D the given angle. Through A draw a line parallel to B C. (*Prob.* 10.) At the point A make an angle 1 A 2 equal to the angle D. (*Prob.* 13.) Produce A 2 to E. Then the line A E meets B C, and makes the angle A E C equal to D. (Euc. I. 29.)


PROB. 7.
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PROB. 8.
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PROB. 9.
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PROB. 10.
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PROB. 11.
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PROB. 12.
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PROB. 13.
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PROB. 14.
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PROBLEM 15.—**To bisect the angle made by two converging lines without producing them.**

Let A B and C D be the two lines. Draw a line parallel to A B at any convenient distance, and at the same distance draw another line parallel to C D intersecting the first parallel at E. Bisect the angle thus obtained. Then E F bisects the angle at which A B and C D are inclined.

Notes.—1. The angle of inclination might be found by drawing one parallel only to meet the opposite line. 2. All circles required to touch the lines A B and C D would have their centres in E F.

PROBLEM 16.—**From any given point to draw a line which should meet at the same point as two converging lines would meet if produced.**

Let A be the given point, and B C, D E the two converging lines. Draw any line A 2. Mark a point 1 in the line B C not in the same straight line as A 2. Join A 1 and 2 1, forming the triangle A 2 1. Take any point 3 in D E. From point 3 draw 3 4 parallel to 2 1, and 3 F parallel to 2 A. From 4 draw 4 F parallel to 1 A, forming the triangle 3 4 F. The line A F passing through the corresponding angle of each triangle is the line required.

Notes.—1. Two positions of the point A are given, one between and one outside the lines. 2. Use the set square in obtaining the parallel lines in this and all complicated figures, as the describing of a number of arcs would cause confusion.

PROBLEM 17.—**To find a point in a line equally distant from two given points without it.**

Let A B be the given line, and C and D the given points. Join C and D, and bisect by a perpendicular meeting A B in E. Then E is the required point, and E C equals E D.

Note.—From E a circle could be described passing through C and D.

PROBLEM 18.—**From two given points without a straight line to draw two straight lines to meet the given line and make equal angles with it.**

Let A and B be the two points, and C D the given line. Draw A E perpendicular to C D, by *Prob.* 7. Draw E B, cutting C D in F. Join A and F. Then A F and B F are the required lines.

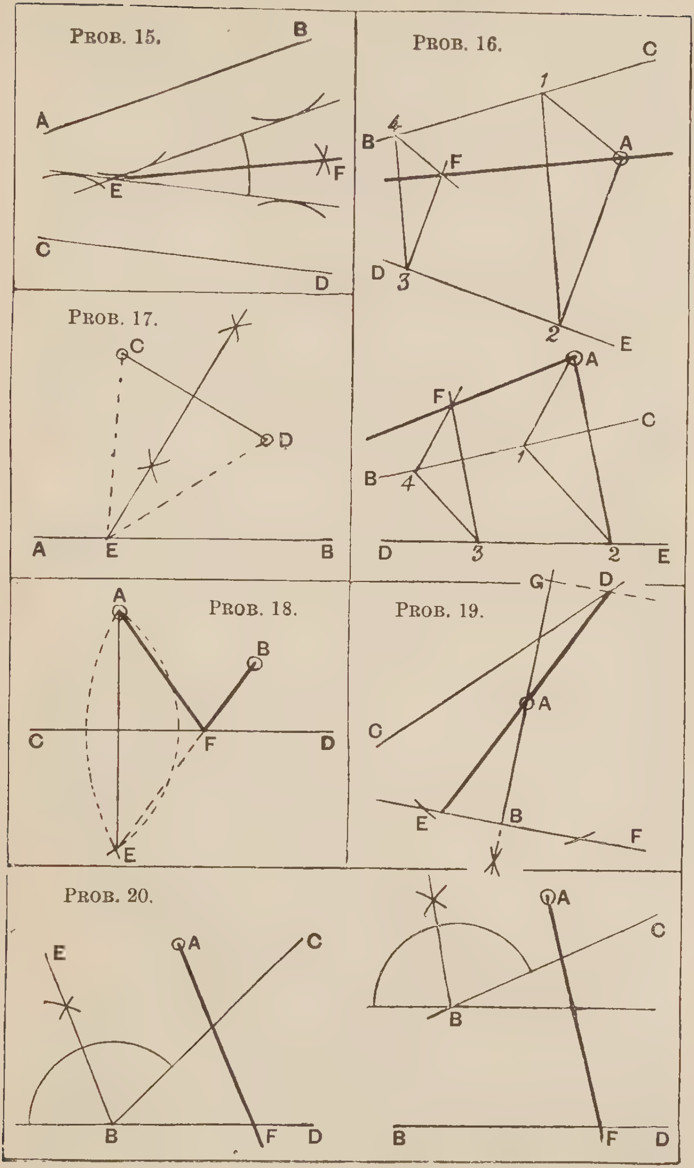
PROBLEM 19.—**Through a given point between two converging lines to draw a straight line which shall be terminated by the given lines and bisected in the given point.**

Let A be the given point, and C D, E F the given lines. Draw A B perpendicular to E F, and produce it, making A G equal to A B. From G draw G D parallel to E F. From D draw D E through the point A. Then D E is bisected in the point A. (Euc. I. 26.)

PROBLEM 20.—**To draw a line from a given point which shall make equal angles with two converging lines.**

Let A be the given point, and B C, B D the converging lines. Produce B D, and bisect the *supplemental* angle thus formed. From A draw A F parallel to the bisecting line, B E.

If the converging lines do not meet, then obtain the *supplemental* angle by drawing a line parallel to B D, as shown in the second figure. Bisect the angle thus formed, and proceed as in the previous figure.


PROB. 15.
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PROB. 16.
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PROB. 17.
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PROB. 18.
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PROB. 19.
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G
PROB. 20.
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C
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F

PROBLEM 21.—From a given point to draw a line so that the part intercepted between two given parallel lines shall be equal to a given distance.

Let A be the given point, B C and D E the two parallel lines, and F the given distance.

Take any point G in B C. With centre G and radius equal to F describe an arc cutting D E in H. Draw G H.

From A draw a line, A K, parallel to G H. Then the distance J K is equal to the given distance F.

The solution when the given point is between the two parallel lines is exactly similar. Both positions are shown.

PROBLEM 22.—To divide a line into any number of equal parts (say 5).

Let A B be the given line. Draw a line at any angle, and set off any convenient distance five times. Draw 5 B, and from the points 4, 3, 2, 1, rule parallels to 5 B with the set square. These parallels will divide the line as required.

PROBLEM 23.—The same. (Without using the set square.)

Draw A 5 at any angle to A B. At B make the angle A B 5 equal to the angle B A 5. *(Prob.* 13.) From A and B, on the lines A 5 and B 5, set off 5 equal distances. Join 5 B, 4 1, 3 2, 2 3, 1 4, and A 5, thus cutting the line A B into 5 equal parts.

PROBLEM 24.—To construct an angle containing a given number of degrees.

The circumference of a circle is supposed to be divided into 360 equal parts, called *degrees*. The radius of a circle may be set off exactly six times round the circumference; hence, if an arc be described, and a portion cut off equal to the radius of the arc, an angle containing 60° will be obtained. With a knowledge of this principle, a variety of angles may be constructed. Constructions for angles of 60°, 120°, 30°, 15°, 45°, and 75° are shown on the opposite page.

For 60°, describe an arc, and cut it with the same radius.

For 120°, describe an arc, and set off twice the radius.

For 30°, obtain an angle of 60°, and bisect it.

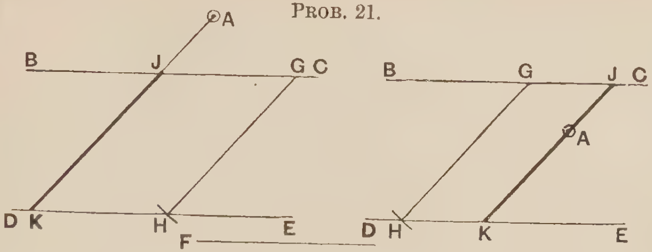
For 15°, obtain an angle of 30° as above, and bisect it.

For 45°, obtain an angle of 30°, and bisect the arc A B. (30° + 15° = 45°.)

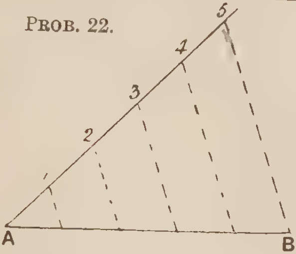
For 75°, draw a right angle, trisect it, and bisect the top division, A B.

Other angles also may be easily made. 150° = 120° + 30°. 105° = 90° + 15°. 135° = 90° + 45°. 22½° = half 45°. 67½° = 45° + 22½°; &c.

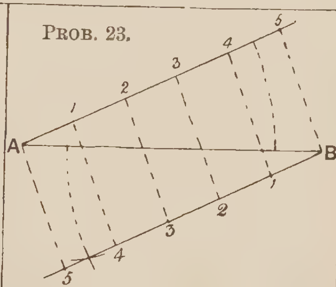
PROB. 21.



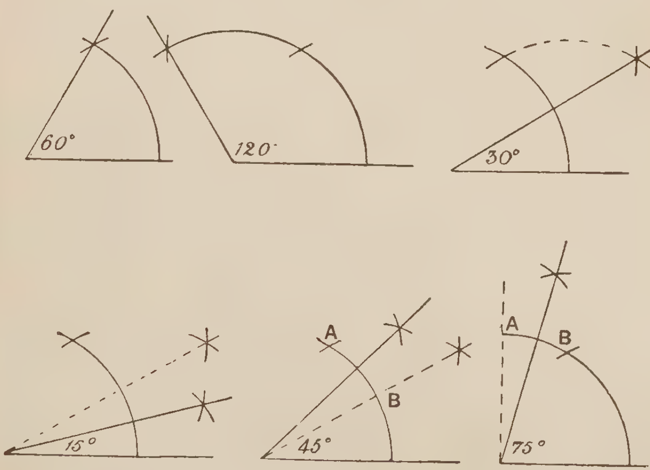
PROB. 22.



PROB. 23.



PROB. 24.



CHAPTER IV

PROPORTIONALS

If we compare two numbers with respect to the number of times one contains the other, a **ratio** is formed. Thus, as $9:3$ is a ratio, and means the same as the fractional expression $\frac{9}{3}$, or that 9 contains 3 three times. If we take two equal ratios, we have a **proportion**; for example, $\frac{9}{3} = \frac{6}{2}$; or $9:3$ as $6:2$.

The first term of a ratio is called the **antecedent**; and the second term, the **consequent.**

If the numbers are in true proportion, the product of the end terms, or **extremes**, equals the product of the middle terms, or **means**.

'*If a straight line be drawn parallel to one side of a triangle, it cuts the other two sides, or those produced, proportionally.*' (Euc. vi. 2.)

PROBLEM 25.—To divide a line in the same manner as another given divided line.

Let A B be the line. It is to be divided similarly to C D.

Draw a line at any angle to A B, and set off C 1, C 2, C 3, and C D on it.

Join D′ and B. Draw parallels to D′ B from 3′, 2′, and 1′. Then **A B** will be divided similarly to **C D**.

PROBLEM 26.—To divide a line into three parts in the same proportion as the numbers 2, 3, and 4.

Let **A B** be the given line. From **A** draw a line at any angle. Set off $2 + 3 + 4$ equal parts. Join 9 and B. From the points 5 and 2 rule parallels cutting **A B**, so that **A C : C D : D B** as $2 : 3 : 4$.

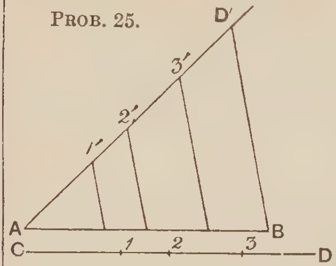
PROBLEM 27.—To find a fourth proportional to three given lines.

Let **A**, **B**, and **C** be the three given lines. Draw **D J** and **D K**, of indefinite length, and at any angle to each other. Set off **D E** equal to **A**, **D F** equal to **B**, and **E G** equal to **C**. Join **E** and **F**. From **G** rule a parallel to **E F**, cutting off **F H**, the fourth proportional required. **D E : D F : : E G : F H.**

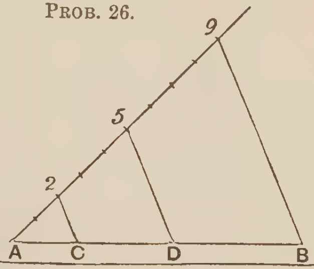
Notes.—1. This construction answers for all cases. It should be worked to scale, and the result verified by the student. If the three lines be 3, $2\frac{1}{2}$, and 2 inches long, then the fourth proportional will be found thus :—$3 : 2\frac{1}{2} :: 2 : F H$. But $3\ F H = 2\frac{1}{2} \times 2$. Therefore, $F H = \frac{2\frac{1}{2} \times 2}{3} = 1\frac{2}{3}$ inches.

2. When the lines are long, it is sometimes more convenient to proceed as in **Problem 27a** :—Draw two lines at any angle as before, and set off D E equal to A, and D F equal to B. Make D G equal to C, and from G draw G H parallel to E F. Then D H will be the fourth proportional **less** than any of the given lines. The same method of construction is applicable when a fourth proportional **greater** than any of the given lines is required, but it is necessary to commence with the shortest line. See **Problem 27b**. Draw two lines at any angle, and set off D E equal to C, D F equal to B, and D G equal to A. Join E F, and from G draw G H parallel to E F. Then D H will be the fourth proportional **greater.**

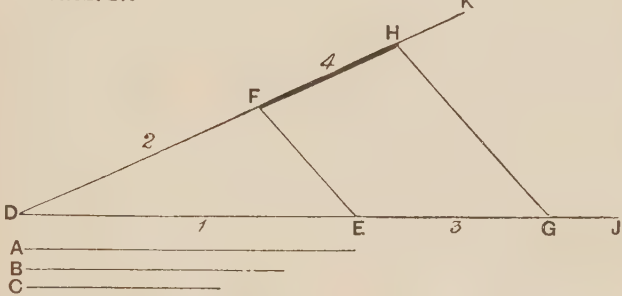
PROB. 25.



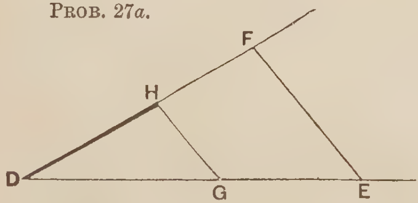
PROB. 26.



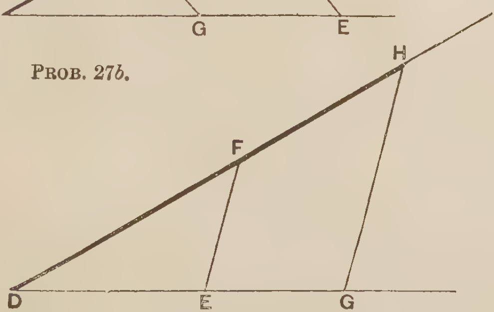
PROB. 27.



PROB. 27*a*.



PROB. 27*b*.



PROBLEM 28.—To find a third proportional to two given lines.
Let A and B be the given lines.

This problem is exactly the same as finding a fourth proportional to three magnitudes, the last two of which are equal. If we take numbers, for example, and require a third proportional to 8 and 4, the statement would read thus—8:4 as 4:2. Here 2 is manifestly the third proportional, because it bears the same relation to 4 that 4 does to 8.

In the case of the given lines, proceed in a similar manner, remembering that the line B is used as both the second and third term.

Draw two lines at any angle. Set off **C D** equal to **A**, and **C E** equal to **B**. Join **D** and **E**. Now set off **D F**, also equal to **B**. Draw **F G** parallel to **D E**. Then **E G** is the required third proportional. **A : B** as **B : E G**.

Note.—The line B may be set off from C as in *Problem* 27*a*: the parallel *f g* cutting off the third proportional C *g*, which will be found to be of exactly the same length as E G.

PROBLEM 29.—To find a mean proportional to two given lines.
Let **A B** and **C D** be the given lines.

Produce **A B**, and set off **B E** equal to **C D**. Bisect **A E**, and describe a semicircle. At **B** draw **B F** perpendicular to **A E**. Then **B F** is the mean proportional required. **A B : B F** as **B F : B E**. (Euc. vi. 13.)

Note.—The square constructed on B F equals the rectangle constructed with sides equal to A B and C D.

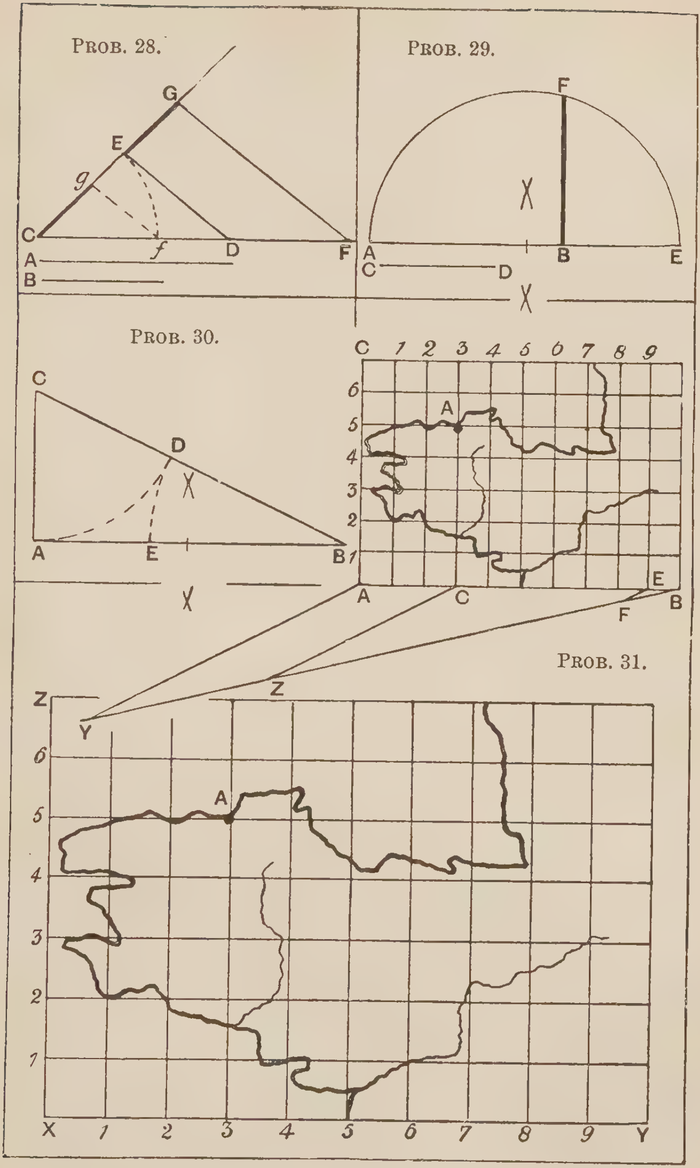
PROBLEM 30.—To divide a line into an extreme and mean ratio - that is, so that one part shall be a mean proportional between the whole line and the other part.

Let **A B** be the given line. Draw **A C** perpendicular to **A B**, and equal to half of it. Join **B** and **C**. With centre **C** and radius **C A** cut off **C D**. With centre **B** and radius **B D** cut off **B E**. Then the line is divided at **E** so that **A E : E B** as **E B : A B**, or so that the rectangle contained by **A E** and **A B** equals the square on **E B**. (Euc. vi. 30 and ii. 11.)

PROBLEM 31.—To make a proportional copy of any irregular figure, such as a plan, map, or picture.

It is required to copy the given map so that the line **A B** shall be enlarged to the line **X Y**. Draw **A C** at right angles to **A B**. Set off equal spaces on **A B** and **A C**, and draw parallels forming a network of squares as shown. We now require the proportionate width of the new drawing—that is, the fourth proportional to the lines **X Y**, **A B**, and **A C**. Set off **X Y** at any angle to **A B**. Join **A Y**. Set off **B C** equal to the width **A C** on **A B**, and draw **C Z** parallel to **A Y**. Then **B Z** is the required width.

The parallel **E F** cuts off **B F** equal to the side of a square in the new drawing; or **X Y** and **X Z** may be divided into the same number of equal parts as **A B** and **A C** are divided into. Draw parallels. With the pencil draw the figure carefully, taking care that each portion of the figure occupies a corresponding square to the original For example, the point **A** is at the junction of vertical 3 and horizontal 5 on the original drawing, therefore it must occupy a similar position on the copy.



Prob. 28.

Prob. 29.

Prob. 30.

Prob. 31.

CHAPTER V

TRIANGLES

PROBLEM 32.—To construct an equilateral triangle on a given straight line.

Let A B be the given line. With centres A and B, and the line A B as radius, describe arcs intersecting at C. Join A C and B C. Then A B C is the required triangle. (Euc. i. 1.)

PROBLEM 33.—To construct an equilateral triangle, the altitude being given.

Let A B be the given altitude. At B draw C D at right angles to A B. With centre A and any radius describe an arc. On each side of A B construct an angle of 30° by cutting the arc in points 2 and 3 from centre 1, and bisecting 1 2 and 1 3 by the lines A C and A D. A D C is the required triangle.

PROBLEM 34.—To construct an isosceles triangle, the base and altitude being given.

Let A B be the base, and C the altitude. Bisect the base by the perpendicular, D E. Make D E equal to C. Draw A E, B E, forming the required triangle, A E B.

PROBLEM 35.—To construct an isosceles triangle, the base and one side being given.

Let A B be the given base, and C the given side. With centres A and B, and radius equal to C, describe arcs intersecting at D. Join D with A and B. A B D is the required triangle.

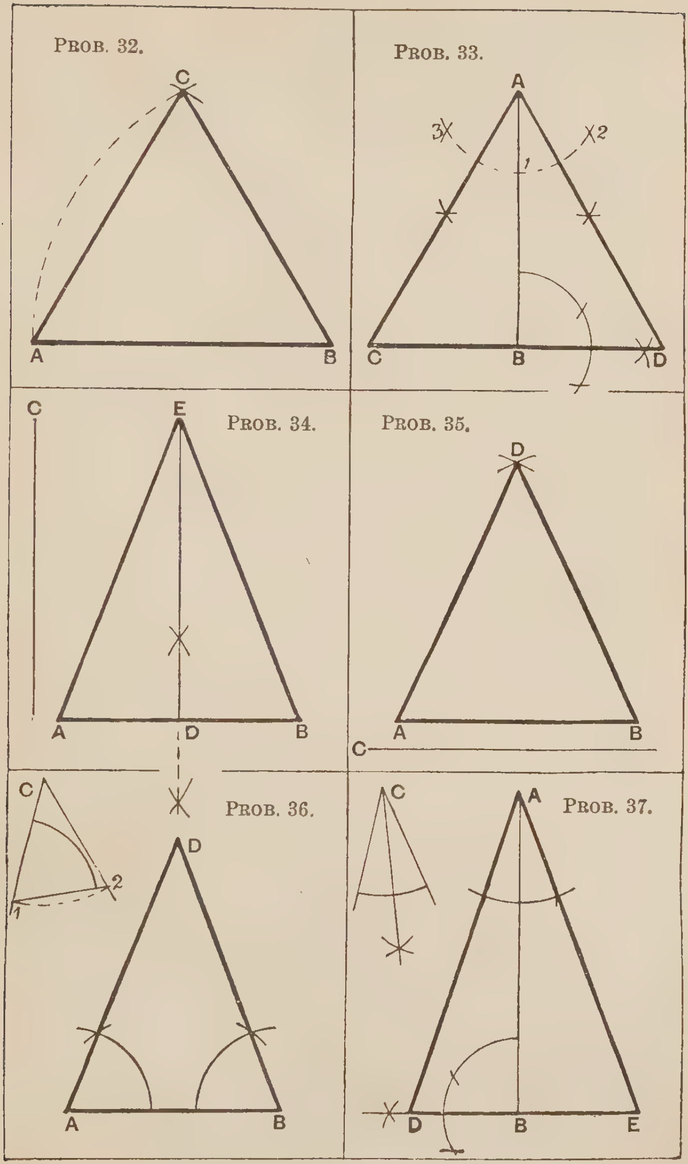
PROBLEM 36.—To construct an isosceles triangle, the base and vertical angle being given.

Let A B be the base, and C the given vertical angle. With centre C describe an arc cutting off equal distances C 1 and C 2. Join 1 and 2, forming an isosceles triangle, C 1 2. At A and B construct angles equal to the angle at 1. (*Prob.* 13.) A B D will be the required triangle.

Note.—A B D and 1 2 C are similar triangles. If the two base angles of the one equal the two base angles of the other, then the vertical angle D must equal the vertical angle C, because the three angles of any triangle must equal 180°. (Euc. i. 32.)

PROBLEM 37.—To construct an isosceles triangle, the altitude and the vertical angle being given.

Let A B be the altitude, and C the vertical angle. Draw D E perpendicular to A B. Bisect the angle C. At A construct angles B A E and B A D, each equal to half the angle C. D E A is the required triangle.


PROB. 32.
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PROB. 33.
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PROB. 34.
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PROB. 35.
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PROB. 36.
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PROB. 37.
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E

PROBLEM 38.—To construct an isosceles triangle, one of the equal sides and an angle at the base being given.

Let **A B** be the side, and **C** the given angle.

Draw any line **D E**. At **D** make an angle equal to **C**, and cut off **D F** equal to **A B**. With centre **F** and radius **F D** describe an arc cutting **D E** in **G**. Draw **F G**. Then **F D G** is the required triangle.

PROBLEM 39.—To construct a right-angled triangle, the base and hypotenuse being given.

Let **A B** be the base, and **C** the hypotenuse.

At **A** erect a perpendicular. With centre **B** and radius equal to **C** cut the perpendicular in **D**. Join **B** and **D**.

PROBLEM 40.—To construct a right-angled triangle, the hypotenuse and an acute angle being given.

Let **A B** be the hypotenuse, and **C** one of the acute angles.

Bisect **A B** in **D**. With centre **D** describe a semicircle on **A B**. At **A** construct an angle, **B A E**, equal to **C**. Draw **B E**. Then **B A E** is the required triangle, the angle in a semicircle being a right angle. (Euc. III. 31.)

PROBLEM 41.—To construct a triangle, the three sides being given.

Let A B, C, and D be the lengths of the three given sides.

With centre A and radius equal to C describe an arc. With centre B and radius equal to D intersect the arc in E. Draw E A and E B. Then A B E is the required triangle. (Euc. I. 22.)

PROBLEM 42.—To construct a triangle, the base and the two base angles being given.

Let A B be the base, and C and D the given angles.

At A and B make angles equal to the given angles C and D. Then A B E is the required triangle.

Note.—Take the same radius for all the arcs.

PROBLEM 43.—On a given base to construct a triangle similar to a given triangle.

Let A B be the given base, and C D E the given triangle.

Make the angles at A and B equal to those at C and D, as in *Prob.* 42.

PROBLEM 44.—To construct a triangle, the altitude and two sides being given.

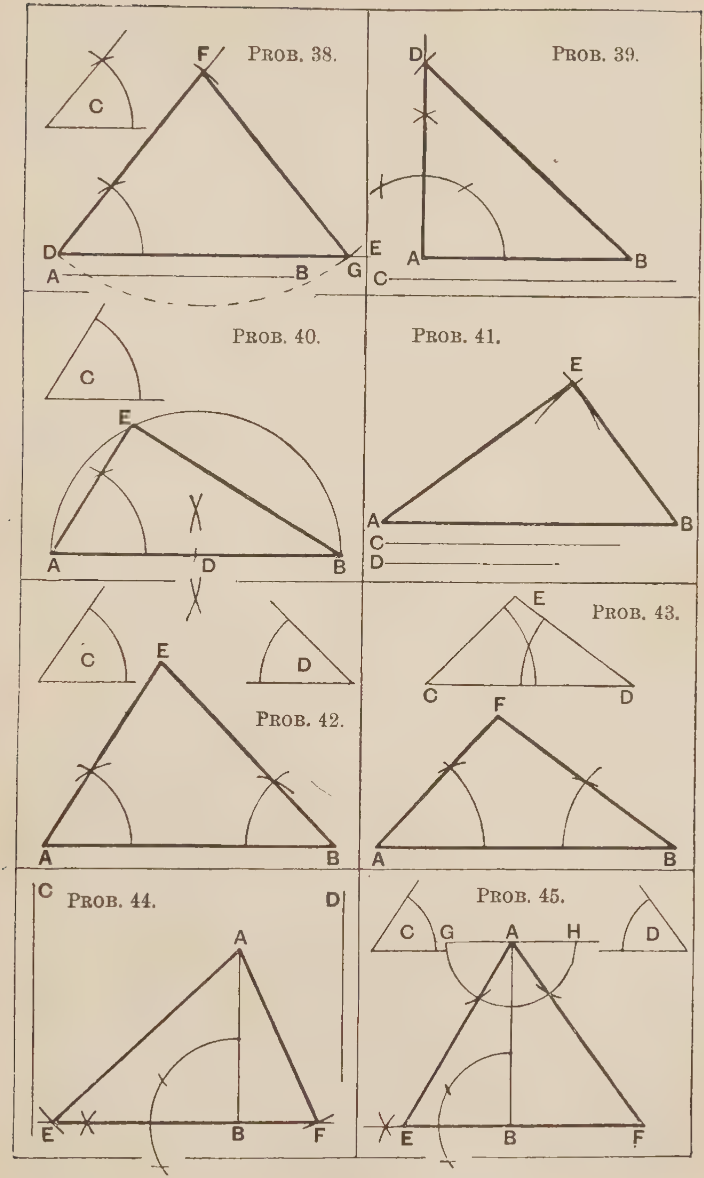
Let A B be the altitude, and C and D the lengths of the sides.

At B draw a base line at right angles to A B. With centre A and radius C cut the base in E. With centre A and radius D cut the base in F. Draw A E, A F.

PROBLEM 45.—To construct a triangle, the altitude and base angles being given.

Let A B be the altitude, and C and D the base angles.

At A and B draw lines G H and E F perpendicular to A B. At A make the angle G A E, equal to C, and the angle F A H, equal to D. A E F is the required triangle.


Prob. 38.
Prob. 39.
Prob. 40.
Prob. 41.
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Prob. 43.
Prob. 44.
Prob. 45.

PROBLEM 46.—To construct a triangle, the base, altitude, and one side being given.

Let A B be the base, C the altitude, D one of the sides.

Draw E F parallel to A B, at a distance equal to C.

With centre A and radius D cut E F in G. A B G is the required triangle.

PROBLEM 47.—To construct a triangle, the base, altitude, and vertical angle being given.

Let A B be the base, C the altitude, and D the vertical angle.

At A construct an angle, B A F, equal to D. Draw A G at right angles to A F. Bisect A B by the perpendicular E G. With centre G and radius G A describe the segment of a circle on A B.

Draw H J parallel to A B, at a distance equal to C. Draw H A and H B. Then A H B is the required triangle.

Notes.—1. Any triangle on A B whose vertex lies in the arc A H J B will have its vertical angle equal to D. (EUC. III. 33.) Hence there will be two triangles satisfying the conditions, A H B and A J B.

2. This problem offers another solution to Problem 36. If an isosceles triangle were required on base A B, with its vertical angle equal to D, it would only be necessary to join A and B with K. (See Prob. 47a.)

PROBLEM 48.—To construct a triangle, the base, one side, and the angle opposite the base being given.

Let A B be the given base, C the side, and D the angle.

At A construct an angle, B A F, equal to the angle D.

Draw A G perpendicular to A F. Bisect A B by the perpendicular E G. With centre G and radius G A describe the segment A H B. With centre A and radius equal to C cut the segment in H. Then A H B is the required triangle.

PROBLEM 49.—To construct a triangle, the base, the sum of the other two sides, and one of the base angles being given.

Let A B be the base, C the sum of the other two sides, and D one of the base angles. At A make an angle equal to D.

Cut off A E equal to C. Draw B E, and bisect it by the perpendicular F G. Join G and B. Then A G B is the required triangle.

Note.—The triangle B G E is isosceles, therefore G E = G B, and A G, G B = A E.

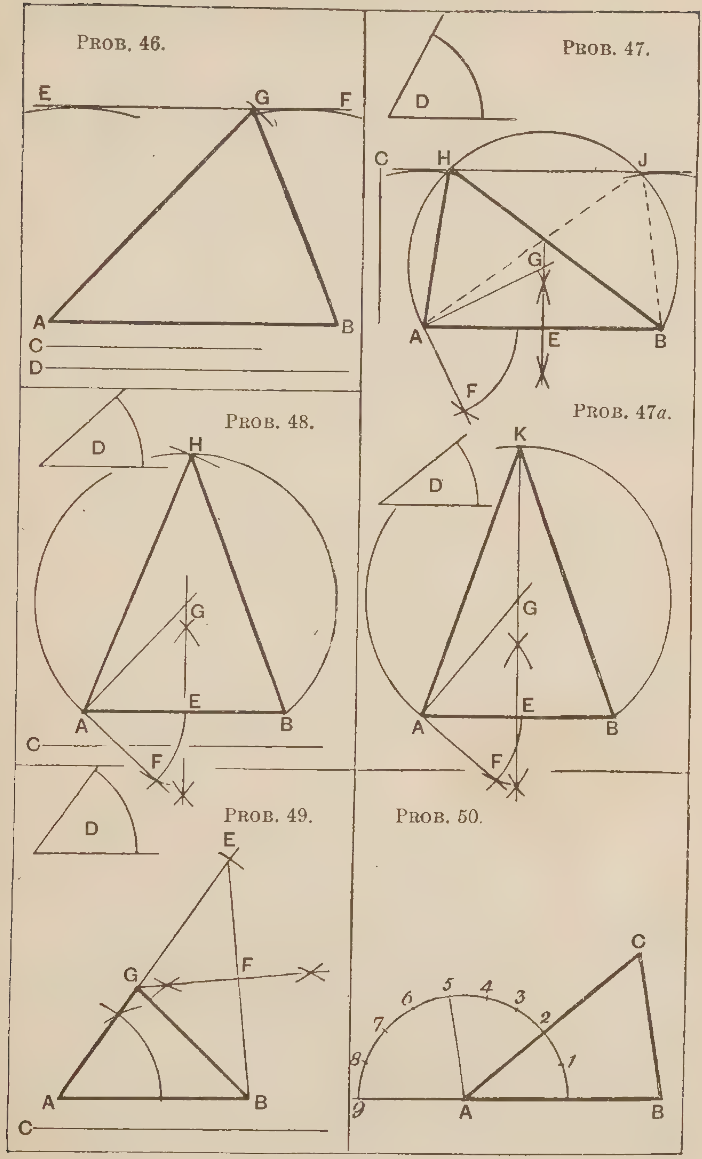
PROBLEM 50.—To construct a triangle, the base and the ratio of the angles being given.

Let A B be the base, and let the angles be as 2 : 3 : 4.

Produce A B, and describe a semicircle. Divide this semicircle in 9 parts (2 + 3 + 4). Draw A 2 and A 5, giving the three angles of the triangle. At B construct an angle, A B C, equal to the angle 9 A 5, and produce A 2 to C, forming the required triangle, A B C.

Notes.—1. B C may be drawn parallel to A 5 to form the angle A B C.

2. The three angles, 9 A 5, 5 A 2, and 2 A B, are equal to two right angles, therefore they must equal the angles of the triangle A B C.


PROB. 46.
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A B
C
D
PROB. 47.
D
C H J
G
A E B
F
PROB. 47a.
K
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A E B
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PROB. 48.
D
H
G
A E B
C
F
PROB. 49.
D
E
G F
A B
C
PROB. 50.
C
5 4 3
6 2
7 1
8
9 A B

PROBLEM 51.—To construct a triangle, the perimeter and two angles being given. (The *perimeter* of any plane figure equals the sum of its sides.)

Let A B be the perimeter, and C and D the two angles.

On A B construct a triangle with its base angles equal to the angles C and D. Bisect the angles at A and B by lines meeting at F. From F draw F G parallel to A E, and F H parallel to E B, giving F G H, the required triangle.

Note.—A G F and B H F are isosceles triangles, therefore A G = G F, and B H = H F. The angle F G H = the angle E A B, and the angle F H G = the angle E B A. (Euc. I. 29.)

PROBLEM 52.—To construct a triangle, the perimeter and the proportion of the sides being given.

Let A B be the perimeter, and let the sides be as 3 : 4 : 5. Draw a line at any angle to A B, and set off 3 + 4 + 5 equal parts. Join B and 12. Divide A B so that A C, C D, and D B shall equal 3, 4, and 5 parts respectively. With centre C and radius C A describe an arc, and with centre D and radius D B intersect the first arc in E. Draw D E and C E, forming the required triangle, C D E.

PROBLEM 53.—To construct a triangle, the base, the ratio of the other two sides, and the angle opposite to the base being given.

Let A B be the given base, C the angle opposite to the base, and let the two sides be as 5 : 6.

Draw any line, D E. At D make an angle equal to C. On D E set off five, and on D F, six, equal parts. Join 5 and 6. On line 5 6 cut off 6 G equal to A B. Draw G H parallel to D 5. Then G H 6 is the required triangle, and is similar to the triangle 6 5 D.

Note.—If A B be longer than 5 6, then both 5 6 and 6 D must be produced as in Problem 53a.

CHAPTER VI

QUADRILATERALS

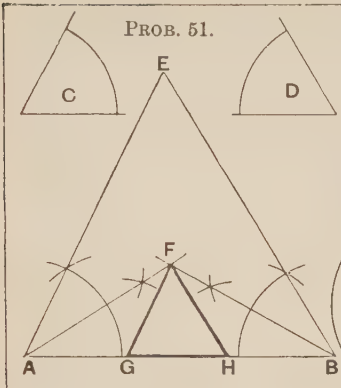
PROBLEM 54.—To construct a square, the side being given.

Let A B be the given side. At A erect a perpendicular, A C, and make it equal to A B. With centres C and B and radius A B describe arcs intersecting at D. Draw C D, B D.

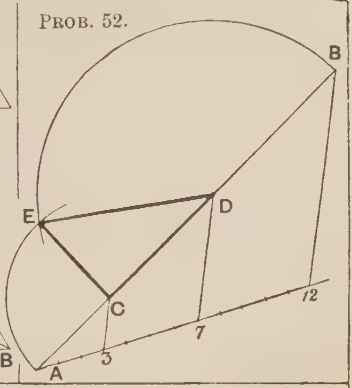
PROBLEM 55.—To construct a square, the diagonal being given.

Let A B be the diagonal. Bisect A B by the perpendicular C D. With centre F and radius F A describe arcs cutting the perpendicular in C and D. Draw A D, D B, B C, and C A.

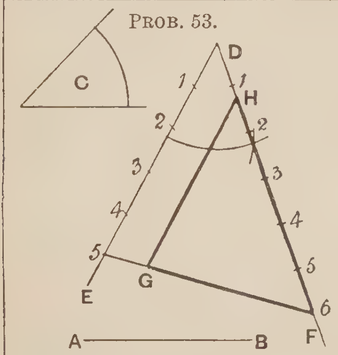
PROB. 51.



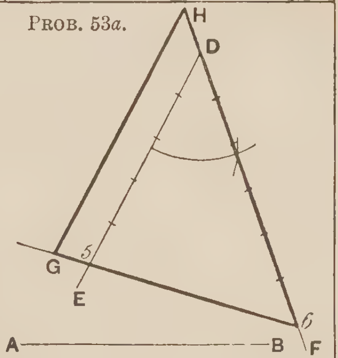
PROB. 52.



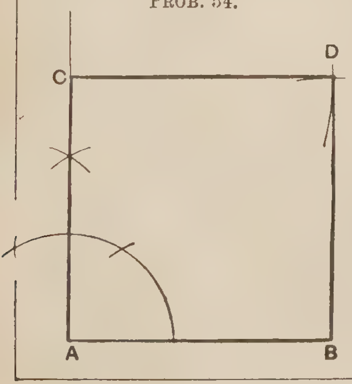
PROB. 53.



PROB. 53*a*.



PROB. 54.



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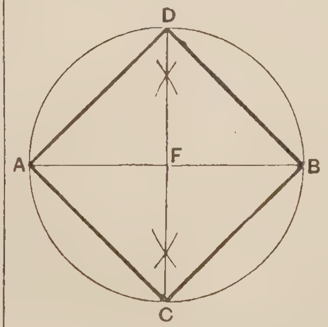


PROBLEM 56.—To construct a rectangle, the two sides being given.

Let A B and C D be the two sides. At A erect a perpendicular. A E, and make it equal to C D. With centre E and radius A B describe an arc, and with centre B and radius C D intersect the arc in F. Draw E F, F B.

PROBLEM 57.—To construct a rectangle, the diagonal and one side being given.

Let A B be the diagonal, and C D the side. Bisect A B in F.

With centre F and radius F B describe a circle. With radius C D and centres A and B cut the circle in G and H. Draw A G, G B, B H, H A.

Note.—The angle in a semicircle is a right angle. (EUC. III. 31.)

PROBLEM 58.—To construct a rhombus, the side and one of the angles being given.

Let A B be the given side, and C the given angle.

At A construct an angle equal to C, and make A D equal to A B.

With centres D and B and radius A B describe arcs intersecting at E. Draw D E, B E.

PROBLEM 59.—To construct a rhombus, the diagonal and side being given.

Let A B be the diagonal, and C the side. With radius C and centres A and B describe arcs intersecting at D and E. Draw A D, D B, B E, E A.

PROBLEM 60.—To construct a rhomboid, the two sides and one of the angles being given.

Let A B and C be the two sides, and D the given angle.

At A construct an angle equal to D, and make A E equal to C.

With centre E and radius A B describe an arc, and with centre B and radius C intersect the arc at F. Draw E F, B F.

PROBLEM 61.—To construct a rhomboid, the diagonal and the two sides being given.

Let A B be the diagonal, and C and D the two sides.

With radius C and centres A and B describe two arcs.

With radius D and the same centres intersect the arcs in E and F. Draw A E, E B, B F, F A.

PROBLEM 62.—To construct a trapezium, the diagonal and two pairs of equal sides being given.

Let A B be the diagonal, and C and D the sides.

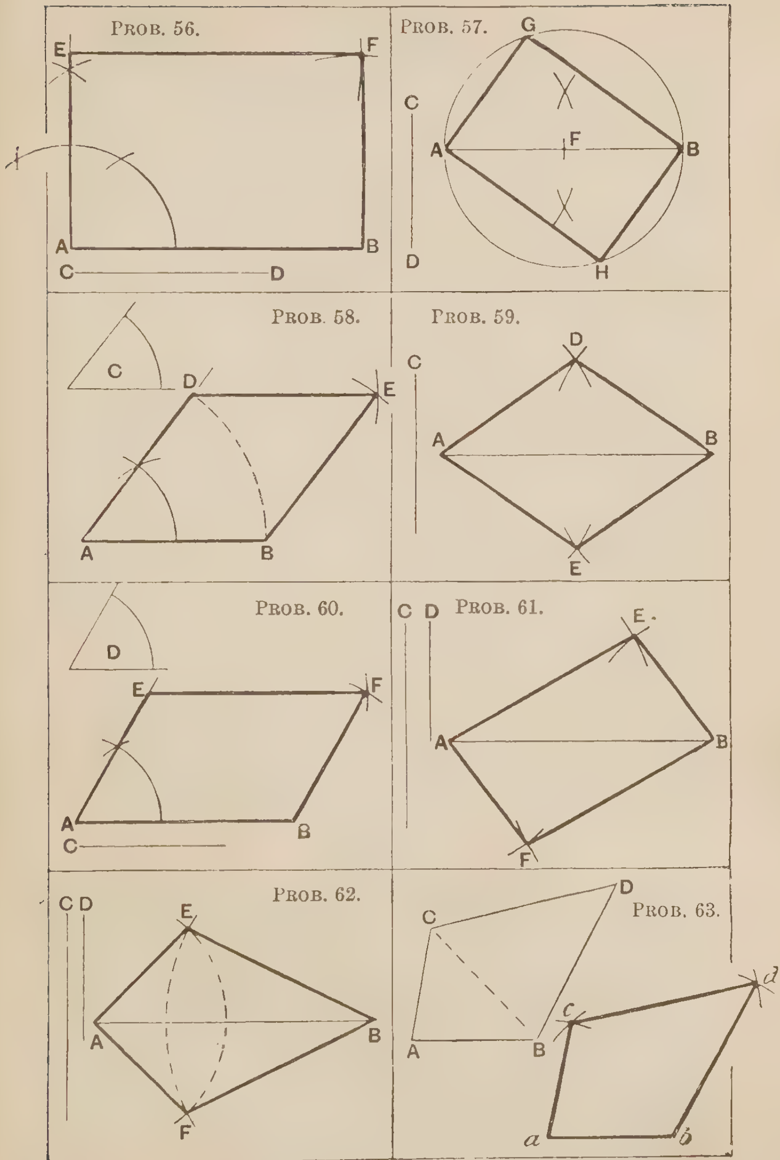
With centre B and radius C describe an arc. With centre A and radius D intersect this arc in E and F. Draw A F, F B, B E, and E A.

PROBLEM 63.—To construct a trapezium equal to another given trapezium.

Let A B C D be the given trapezium. Draw $a\,b$ equal to A B.

With centres a and b, and radii equal to A C and B C, describe arcs intersecting in c. With centres b and c, and radii equal to C D and B D, describe arcs intersecting in d. Draw $a\,c$, $c\,d$, and $b\,d$.

Note.—The same principle of cutting the figure into triangles may be applied to figures with any number of sides.


PROB. 56.
E
F
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C
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PROB. 57.
G
C
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PROB. 58.
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PROB. 59.
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PROB. 60.
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PROB. 61.
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C D
PROB. 62.
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PROB. 63.
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b

EXERCISES

Note.—The exercises taken from the examination papers of the Science and Art Department in Subject I. are marked with the letters *Sc.*, and the year when set. Those from the various Art examinations are marked *Art.* They should be worked in connection with the chapter to which they relate.

CHAPTER III

1. Draw a line 3″ long; mark a point $1\frac{7}{8}$″ below it, and from this point draw a perpendicular to the line.

2. Draw a line, **A B**, 2·5″ long. From a point **C** in it, ·5″ from **A**, draw a line making 45° with **C B**; and at **B** draw a line at 75° with **B C** to meet it.

3. Draw a line 3·5″ long. Divide it into 7 equal parts, and on $\frac{6}{7}$ of the line describe a semicircle.

4. Draw a line 3″ long, and erect a perpendicular from a point $\frac{1}{2}$″ from one end, and without the line, without producing it.

5. From a point **A** draw two lines, **A B**, **A C**, making with each other an angle of 75°; bisect this angle.

6. Set off with your protractor an angle of 57°, and divide it into 4 equal parts.

7. At the extremities of a line 3″ long erect perpendiculars 2″ and $2\frac{1}{2}$″ long. From the upper ends of these perpendiculars draw two equal lines meeting each other in the first line. (*Prob.* 17.)

8. From **A** and **B**, two points 2″ apart, draw two right lines to meet at an angle of 70°. (*Art.*)

9. Through a point **C**, $1\frac{1}{2}$″ from a line **A B**, draw two right lines, one parallel and the other perpendicular to **A B**. (*Art.*)

10. Draw two lines intersecting at an angle of 52° and between them place a line 2·25″ long, making 58° with one of them. (*Sc.* 1884.)

Note.—Draw any line 2·25″ long, and making 58° with one line, and through its extremity draw a parallel to the one line until it meets the other. A line from the point of intersection will be the line required.

11. Draw a line 3·5″ long, and at one extremity erect a perpendicular 1·75″ long. From the top of this perpendicular draw a line to make an angle of 30° with the given line. (*Sc.* 1871.) (*Prob.* 14.)

12. Draw any two parallel lines, **A** and **B**, $1\frac{1}{2}$″ apart. Mark a point **P**, $\frac{3}{4}$″ above **A**. Through **P** draw a line cutting the given lines in points $1\frac{7}{8}$″ apart. (*Sc.* 1888.) (*Prob.* 21.)

13. **A B C** is an obtuse angle, and **X** is a point within it. Determine **X**'s position when **A B** = 2·33″, **B C** = 3·15″, **A B C** = 160°, **A B X** = 65°, **B C X** = 37°. State the lengths of **A X** and **B X**. (*Sc.* 1872.)

CHAPTER IV

1. Find a fourth proportional to three lines whose lengths are $2\frac{1}{4}$″, $3\frac{3}{8}$″, $1\frac{3}{16}$″ respectively.

2. Find a mean proportional between two lines, 2″ and 1″ long, and figure its length on the line.

3. Draw a line 3·75″ long, and divide it in the proportion of the numbers 2, 7, 3, 6. Figure the parts.

4. Find a third proportional to two lines, $1\frac{1}{2}$″ and $1\frac{3}{4}$″ long.

5. Find a line which shall have the same ratio to a line $1\frac{3}{4}$″ long that $2\frac{1}{2}$″ has to $1\frac{1}{2}$″.

Note.—$1\frac{1}{2}$″ : $2\frac{1}{2}$″ :: $1\frac{3}{4}$″ : required line.

6. Draw a line 2″ long, and divide it into 4 unequal parts. Draw another line two-thirds of the length of the first line, and divide it proportionally to it. (*Prob.* 25.)

7. From the extremity **A**, of a line **A B**, obtain the $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$ of the line.

8. Divide a line 3·25″ long into 4 parts, **A**, **B**, **C**, **D**, so that **B** is double of **A**, **C** three times **A**, and **D** four times **A**.

9. Produce a line, **A B**, 3″ long, to a point **P**, so that **B P : A B** as 3 : 5. (*Sc*, 1872.)

Note.—Draw a line at any angle, and set off 5 + 3 parts. Join 5 B, and from 3 draw 3 P parallel to 5 B.

10. Find a point, **P**, in a line **A B** (2″ long), produced, so that **A P : A B** as 7 : 4. (*Sc.*, 1870.)

Note.—Draw a line at any angle to A B, and set off 7 parts. Join 4 B From 7 draw 7 P parallel to 4 B.

11. Divide a line, **A B**, 3·5″ long, in a point **P**, so that the rectangle contained by **A B**, **A P** may be equal to the square on **B P**. (*Sc.*, 1870.) (*Prob.* 30.)

12. Find a mean proportional to two lines, 2·5″ and 1·5″ long respectively. State any Problem that you know of which this is the solution. (*Sc.*, 1871.) (Euc. II. 14.)

CHAPTER V

1. Construct a triangle with sides **AB** = 3″, **A C** = $2\frac{5}{8}$″, **B C** = $1\frac{9}{10}$″, and construct an angle equal to **B A C.**

2. Two lines, **A** and **B**, 4″ long, contain an angle of 45°. From a point **C** in one line draw a line to make equal angles with the two converging lines.

3. Upon a base of $2\frac{1}{2}$″ construct a triangle having two of its angles 75° and 45° respectively, and then construct a similar triangle on a base of 2″. (*Art.*)

4. An isosceles triangle has a base of $1\frac{1}{2}$″ and a vertical angle of 42°. Construct it by using *Problem* 47*a*.

5. Describe a segment of a circle which shall contain an angle equal to the angle of an equilateral triangle. (*Prob.* 47.)

6. Construct an isosceles triangle with its equal sides $2\frac{1}{2}$″ long and the included angle 30°. On the same base describe another isosceles triangle with its vertical angle double that of the first triangle. (*Art.*)

Note.—Bisect the sides, and find the centre for the circumscribing circle. The angle at the centre of a circle is double the angle at the circumference, standing upon the same base. (Euc. III. 20.)

7. Draw a triangle, two of whose sides are 2·5″ and 3″ respectively, the angle opposite the shorter side being 40°. (*Sc.*, 1876.) (*Prob.* 38.)

8. Draw a triangle having its vertical angle 30°, the base 1·7″, and the sides as 4 : 5. (*Prob.* 53.)

9. Construct a triangle of which the sides are as 1, 1·5, 2, the perimeter being 4″. (*Prob.* 52.)

10. Construct a triangle on a base of 2″, altitude 1·75″, and the angle opposite the base 42°. (*Sc.*, 1871.) (*Prob.* 47.)

11. Construct a triangle whose sides, $a\,b$, $b\,c$, $c\,a$, are $3\frac{1}{4}$″, $2\frac{3}{8}$″, and 2″ respectively. On $a\,c$ construct a second triangle, $a\,d\,c$, whose vertical angle, $a\,d\,c$, is equal to the angle $a\,b\,c$, and the side $a\,d$ $1\frac{5}{8}$″. (N.B.—The angles upon the same base and in the same segment of a circle are equal.) (*Sc.*, 1887.)

Note.—At a draw a line making with $a\,c$ an angle equal to $a\,b\,c$, and use *Problem* 48.

CHAPTER VI

1. Construct a square of 4″ sides, bisect the sides, and join the adjacent points of bisection, thus obtaining a second square; bisect the sides of this square, and obtain a third square. Continue the process until five squares have been drawn. (*Sc.*, 1879.)

2. Construct a rhomboid, one diagonal being 2″, and the adjacent sides $1\frac{1}{2}$″ and 1″ respectively. (*Prob.* 61.)

3. Construct a rhombus having an angle of 65° and a base of 3″. Measure its two diagonals accurately, and write down their lengths.

4. The adjacent sides of a trapezium are 2·3″ and 1·8″ long respectively. The included angle is 60°. The other sides are $2\frac{3}{4}$″ and 3″. Construct the figure, and give the length of the longest diagonal.

5. Upon a line $1\frac{7}{8}$″ long describe a square, and divide it by parallel lines, alternately thin and dotted, into 5 equal rectangles.

6. The diagonals of a parallelogram 2·4″ and 4·2″ long contain an angle of 61°. Construct the parallelogram. (*Sc.*, 1883.)

CHAPTER VII

THE CIRCLE AND TANGENTS

PROBLEM 64.—To find the centre of a circle.

Draw any two chords, A B, B C. Bisect them by lines at right angles. The point D, where the bisecting lines intersect, is the centre. (Euc. III. 1, Cor.)

PROBLEM 65.—To describe a circle passing through three given points not in the same straight line.

Let A, B, and C be the three points.

Join A B, B C. Bisect both lines as above. From D, the point where the bisecting lines meet, describe the circle.

PROBLEM 66.—To describe a circle about a triangle.

Bisect two sides, and proceed as in Problem 64.

PROBLEM 67.—To describe an arc equal to a given arc, and having the same radius.

Draw any two chords, A B and C D, in the given arc. Bisect them, and find the centre E, from which the arc was described. With centre O and radius E F describe the arc G H, and make it equal in length to the given arc.

PROBLEM 68.—To draw a tangent to a circle through a given point in the circumference.

Let A be the given point. Find the centre, B. Draw **B A** and produce, making A C equal to A B. With centres B and C describe arcs intersecting at D. Draw D A, the required tangent.

Notes.—1. This method of drawing the perpendicular is preferable to the other methods when there is room to produce the radius, as it is likely to be more accurate.

2. A tangent is always at right angles to the radius.

PROBLEM 69.—To draw a tangent to a circle from a given point without it.

Let A be the given point. Find the centre, B, and draw **B A**. Bisect B A, and describe a semicircle cutting the given circle in C. Draw A C, the required tangent.

Note.—If the whole circle on A B were described, another tangent might be obtained.

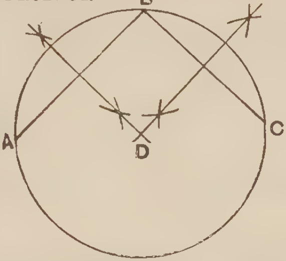
PROBLEM 70.—To draw a tangent to an arc from a point in it, when the centre is not accessible.

Let A be the given point.

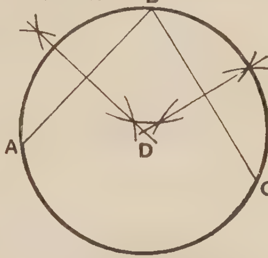
With centre A describe a circle. With centres B and C describe arcs intersecting in E and F. Draw E F. At A draw the tangent at right angles to E F, using H and G as centres for the intersecting arcs.

Another solution is shown in **Problem 70*a*.** Draw a chord, A B, from the given point A. Bisect this chord. Draw C A. Make the angle C A D equal to C A B. Then D E is the required tangent, the angle D A C being equal to the angle A B C. (Euc. III. 32.)

PROB. 64.



PROB. 65.



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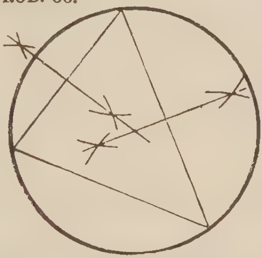

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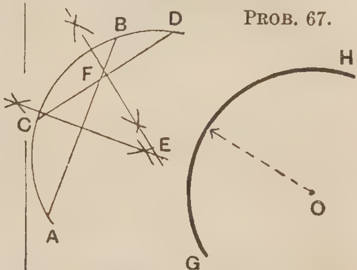

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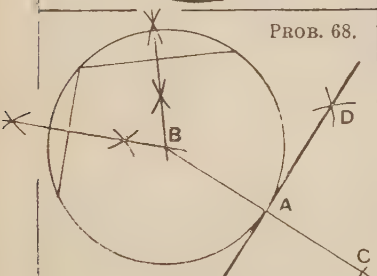

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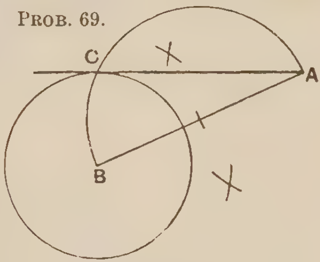
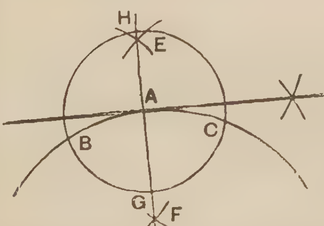
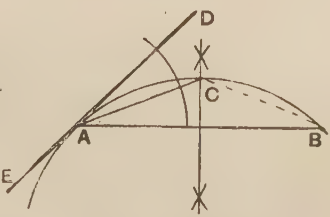


PROB. 70.



PROB. 70*a*.



PROBLEM 71.—To draw a tangent to a circle which shall be parallel to a given straight line.

Let A B be the given straight line. Find the centre, C, of the circle. From C draw a perpendicular to A B. Through D, the point where the perpendicular cuts the circumference, draw E F at right angles to C D.

Note.—In all problems where the centre of the circle is not given, it must be found as shown in Problem 64.

PROBLEM 72.—To draw two tangents to a circle to meet at a given angle.

Let the given angle in this case equal 60°.

From the centre, B, draw any line, B C. Take any point, C, and on each side of the line B C construct an angle equal to *half* the given angle—in this case 30°. Now apply Problem 71, and draw perpendiculars B E, B D. Through F and G draw parallels to E C and D C.

PROBLEM 73.—To draw a common tangent to two equal circles which shall not cross the line joining their centres. (This is commonly called an **exterior tangent.**)

Join the centres A and B. At A and B draw perpendiculars A C, B D. Through the points C and D draw the exterior tangent.

PROBLEM 74.—To draw a common tangent to two equal circles which shall cross the line joining their centres. (Interior tangent.)

Join the centres A and B. Bisect A B in C, and A C in D. With centre D and radius D A describe a semicircle. Draw A E, and from B draw B F parallel to A E. (Use set square.) Through E and F draw the interior tangent.

PROBLEM 75.—To draw an exterior tangent to two unequal circles.

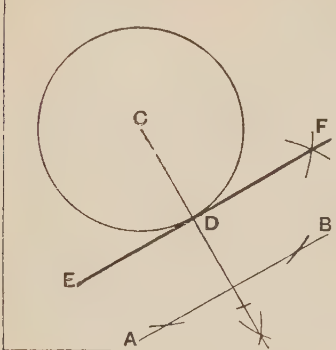
Join the centres A and B. From C set off C D equal to B E, the radius of the smaller circle. With centre A and radius A D (the *difference* of the radii of the given circles) describe a circle. Bisect A B, and describe a semicircle. From B draw B F, a tangent to the small, described circle. Through F draw A G, and from B draw B H parallel to A G. Through the points G and H draw the exterior tangent.

PROBLEM 76.—To draw an interior tangent to two unequal circles.

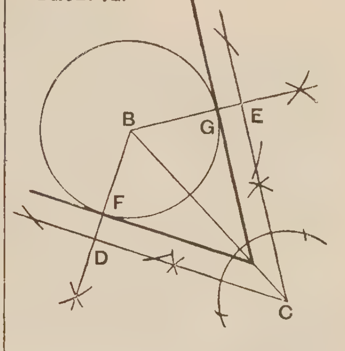
Join the centres A and B. From C set off C D equal to B E. With centre A and radius A D (the *sum* of the radii of the given circles) describe a circle. Bisect A B, and describe a semicircle. From B draw B F, a tangent to the large, described circle. Draw A F, and from B draw B G parallel to A F. Through the points G and H draw the interior tangent.

Note.—In Problems 73, 74, 75, 76, two tangents might be drawn, if required, as shown in dotted line on Problem 74.

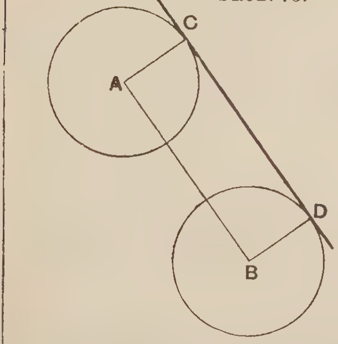
PROB. 71.



PROB. 72.



PROB. 73.



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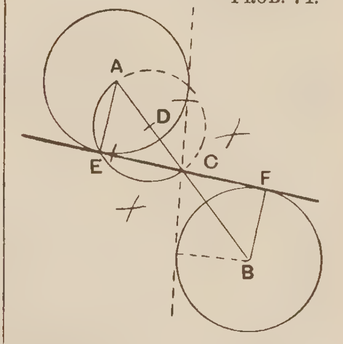
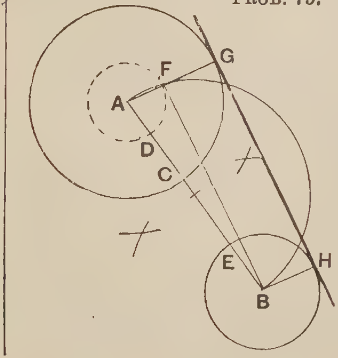
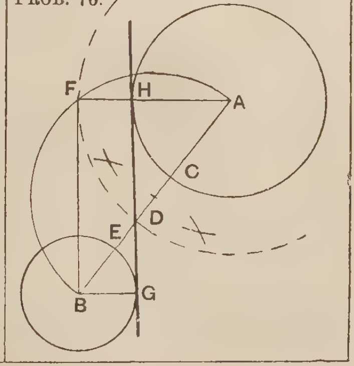


PROB. 75.



PROB. 76.



CHAPTER VIII

REGULAR POLYGONS

PROBLEM 77.—To construct ANY regular polygon on a given line, A B (in this case a pentagon).

Produce A B, and with centre A and radius A B describe a semicircle. Divide the semicircle into as many equal parts as the required figure has sides (in this case five) with the dividers. Join A with 2, giving another side of the polygon. (Always join A with the second division for any polygon.) Bisect the two sides, A B and A 2, by lines meeting at O, giving the centre of the polygon. With centre O and radius O B describe a circle. Mark off B E and E D equal to A B. Draw B E, E D, D 2, forming the required pentagon.

Notes.—1. The accuracy of this construction depends upon the correct division of the semicircle and the care with which the centre is obtained

2. A semicircle may be divided into three equal parts by marking off its radius three times. To get 6 equal parts, bisect each third part; to get 9, trisect each third part. To get 4 equal parts, bisect each half; by bisecting again, 8 will be obtained. If the set squares are perfectly accurate, the semicircle may be divided into 4 by using the 45°, and into 3 by using the 60°, set square.

PROBLEM 78.—To construct a regular hexagon on a given line, A B. (Special method.)

With centres A and B and radius A B describe arcs intersecting in O. With centre O and the same radius describe a circle, and set off A B round it. Join the points, thus forming the required hexagon.

Note.—The side B C may be obtained by using the 60° set square, and the hexagon quickly set up by this means.

PROBLEM 79.—To construct a regular octagon on a given line, A B. (Special method.)

Erect perpendiculars at A and B. Produce A B both ways. With centres A and B and radius A B describe quadrants. Bisect each quadrant, obtaining B E and A F, two more sides of the octagon. From E and F draw parallels to A C and B D, and make them equal to A B. Draw E F and G H. Make K D and L C equal to J B. Join the points H, D, C, G.

Notes.—1. The lines from A and B through M and N will also give the points H and G.

2. B E, A F, H D, G C may be readily obtained by using the 45° set square.

PROBLEM 80.—To construct a regular polygon by using the protractor.

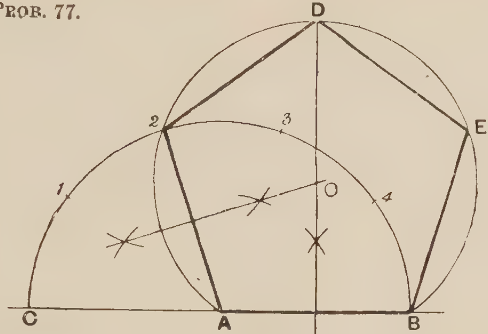
The number of degrees in each angle of a regular polygon may be readily found as follows :—*From twice as many right angles as the figure has sides subtract four right angles, and divide this result by the number of angles the figure contains.* (Euc. I. 32, Cor.)

Suppose the regular polygon to be a pentagon.

The pentagon has *five* sides ; therefore take *ten* right angles. Deduct *four*, leaving *six* right angles. The solution will then be $\frac{6 \times 90^\circ}{5} = 108^\circ$. The angles for other polygons may be found in the same manner. For an octagon the solution would be $\frac{12 \times 90^\circ}{8} = 135^\circ$.

Place the protractor on A B, with its centre on B (see page 2). Mark 108°, reading from the left. Place the centre on A, and mark 108°. Make A D and B C equal to A B. With centres D and C and radius A B describe arcs intersecting in E. Draw D E, E C.

PROB. 77.



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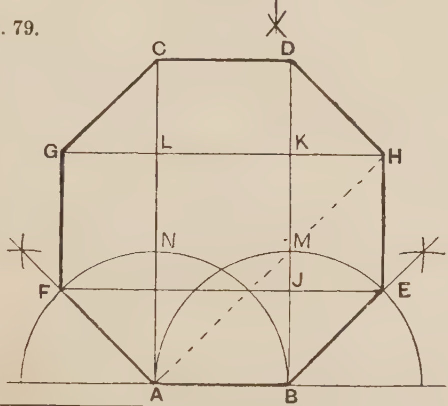

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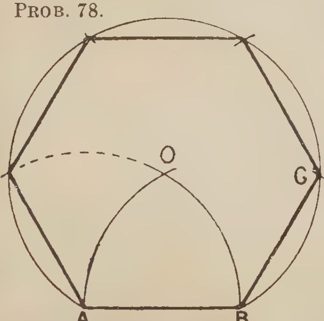
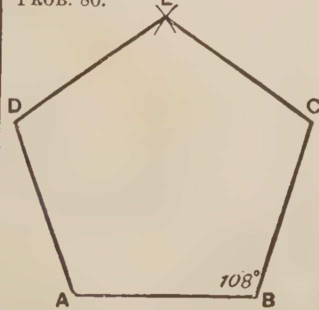


PROB. 80.



PROBLEM 81.—To inscribe ANY regular polygon in a circle (approximate).

Draw the diameter A B. (If the centre of the circle be not given, draw a chord, and bisect it.) Divide A B into 7 equal parts. With centres A and B describe arcs intersecting at C. From C, through the second part, rule C D, cutting off A D, one of the sides of the polygon. Set off A D round the circle, and join the points.

Notes.—1. The greatest care must be exercised in dividing the line and in drawing the line from C exactly through the point 2.

2. The circumference of the circle may be divided into the number of parts required with the dividers, and the points joined to form the figure.

3. The hexagon, octagon, and duodecagon are more easily inscribed in a circle by special methods.

PROBLEM 82.—To inscribe a regular hexagon in a circle. (Special method.)

Draw the diameter A B. With centres A and B set off the radius on each side. Join the points.

PROBLEM 83.—To inscribe a regular octagon in a circle. (Special method.)

Draw the diameter A B. Bisect it at right angles by the diameter C D. Bisect each quadrant thus formed, cutting the circumference into 8 equal portions. Join the points thus obtained.

PROBLEM 84.—To inscribe a regular duodecagon in a circle. (Special method.)

Draw two diameters, at right angles to each other, as before.

With centres A, B, C, D, describe arcs passing through the centre of the circle and cutting the circumference. Join the points thus obtained, forming the required polygon.

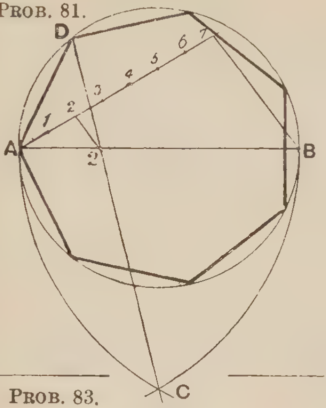
PROBLEM 85.—To complete a regular polygon, two adjacent sides and the included angle being given.

Let A B, B C, be the two sides, and A B C the included angle (in this case 120°). Bisect the two sides. Describe a circle passing through the points A, B, C. Set off the length of one side round the circumference, and join the points thus obtained.

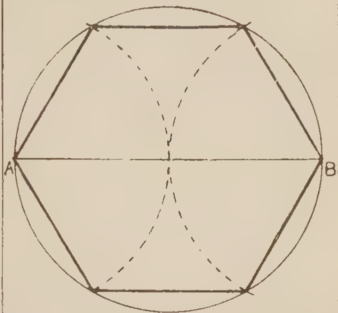
PROBLEM 86.—To inscribe ANY regular polygon in a circle by using the protractor.

If all the angles of a polygon be joined with the centre, there will be formed as many equal isosceles triangles as the polygon has sides, having their vertical angles at the centre equal. All the angles at the centre equal four right angles. (Euc. I. 15, Cor. 2.) Hence, if 360° be divided by the number of sides the regular polygon has, the magnitudes of the central angles will be obtained. For example, it is required to inscribe a regular pentagon in a circle by this method. The central angle in this case will be $\frac{360^\circ}{5} = 72^\circ$. Draw any radius, O A. Make the angle A O B equal to 72°, with the protractor. Set off the distance A B round the circumference, and join the points. A B C D E is the required pentagon.

PROB. 81.



PROB. 82.



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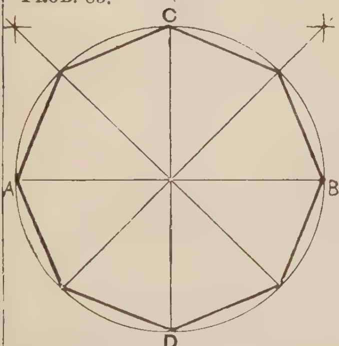

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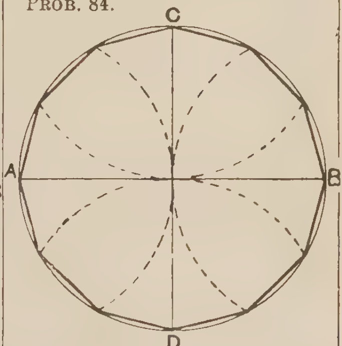

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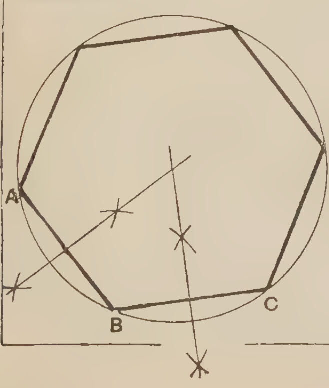
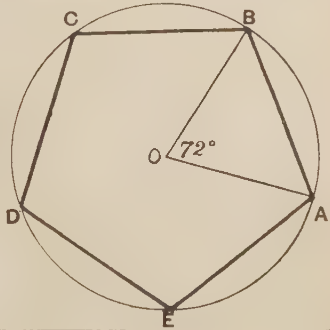


PROB. 86.



PROBLEM 87.—To describe ANY regular polygon about a circle (say a pentagon).

Divide the circumference of the given circle into as many equal parts as the polygon has sides, in the same manner as for the inscribed polygon. (*Prob.* 81, *Note* 2.) From the centre, O, draw lines through each point. Draw A B, one of the sides of the inscribed pentagon. Draw the tangent C D parallel to A B. Make O E, O F, O G, equal to O D. Draw D E, E F, F G, G C, tangential to the circle. Then C D E F G is the required pentagon.

PROBLEM 88.—To construct ANY regular polygon, the length of any diagonal being given.

Let A B be the length of one of the *longer* diagonals of a regular heptagon.

On any base, C *d*, construct a regular heptagon. (*Prob.* 77.) From C draw the diagonals, and produce them, if necessary. Mark off C F equal to A B on one of the longer diagonals. Draw F E, E D, F G, G H, H J, parallel to the corresponding sides of the heptagon.

Notes.—1. If the length of the shorter diagonals be given, proceed in an exactly similar manner.

2. If one of the longer diagonals of a hexagon, octagon, or duodecagon be given, bisect it, describe the circumscribing circle, and proceed as in Problems 82, 83, and 84.

PROBLEM 89.—To construct ANY regular polygon, having the diameter given (say a pentagon).

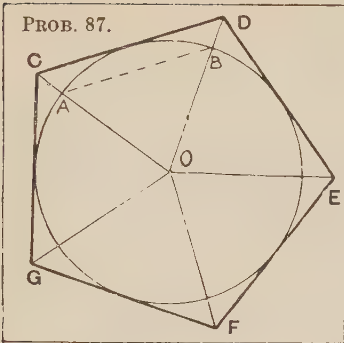
Note.—The **diameter** divides the polygon into two equal parts. It is the line passing through the centre of the polygon, and terminated at the middle points of two opposite sides in a polygon having an *even* number of sides. When the sides are *uneven*, the diameter is drawn through the centre, from one angle to the middle of the opposite side.

Let A B be the given diameter. Through B draw a line at right angles to A B. Set off B *c*, B *d*, any equal distances on each side of B. Describe a pentagon on *c d*. Through *e* and *f* draw lines from B. From A draw A E and A F parallel to *a c* and *a f*, and from E and F draw E D and F C parallel to *e d* and *f c*. Then C D E A F is the required pentagon.

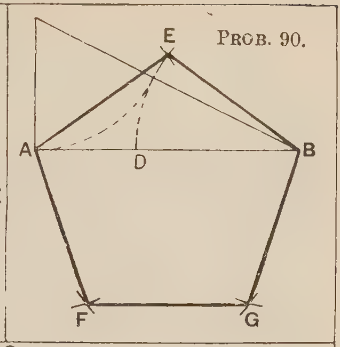
PROBLEM 90.—To construct a regular pentagon, the diagonal being given. (Special method.)

Let A B be the diagonal. An important property of the pentagon is, that if the diagonal be divided into an extreme and mean proportion (*Prob.* 30), *the greater segment* of the diagonal so divided will equal the *side* of the pentagon.

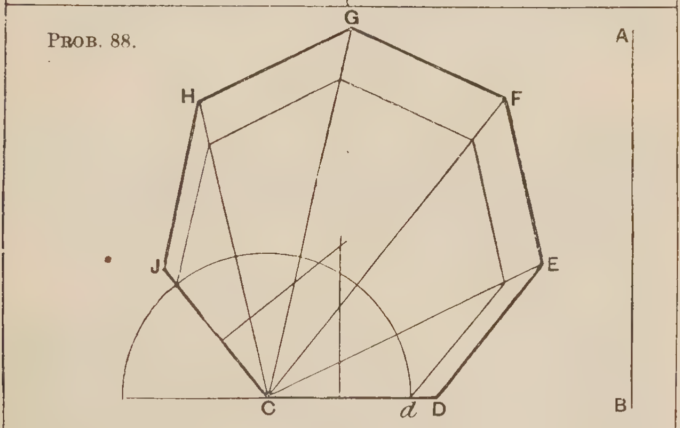
Divide A B into an extreme and mean ratio at D. Then B D, the greater segment, will be the side of the pentagon. With centres A and B and radius B D describe arcs intersecting at E. Draw B E, E A, two sides of the pentagon. With centre B and radius B A describe an arc. With centre A and radius A E intersect the arc in F. With centres F and B and radius F A describe arcs intersecting in G. Draw A F, F G, G B.



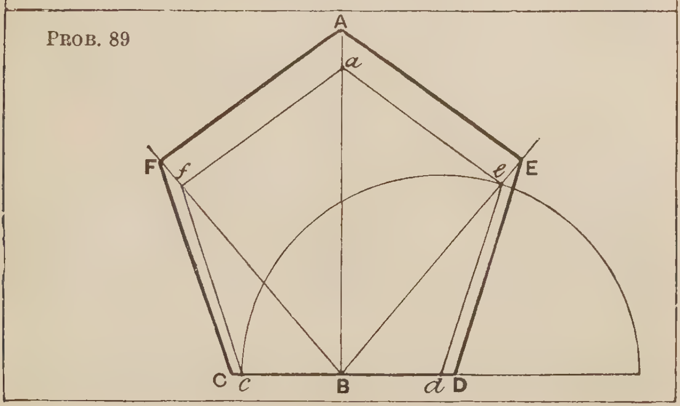
PROB. 87.



PROB. 90.



PROB. 88.



PROB. 89

CHAPTER IX

SCALES

Drawings are not usually made of the same size as the objects which they represent. It would be manifestly inconvenient to draw the plan of a building its proper size, as the drawing would be too large for use; but if a drawing were made on which every yard of the object was represented by an inch, then we should have a diagram showing the relative proportion of the parts, but drawn to a different *scale*. As every inch on the drawing represents a true length of one yard, the drawing would be on a *scale of* 1 *inch to* 1 *yard, or* $\frac{1}{36}$, because each line on the drawing is $\frac{1}{36}$ part of its true length. This fraction is called the **representative fraction**, and shows the ratio each line on the drawing bears to the object delineated.

The scale of a drawing may be stated in *words*, as 1 *in. to* 1 *yd.*; by its *representative fraction*, as $\frac{1}{36}$; or by *drawing a line divided into equal parts*, each representing the unit used.

When the scale shows equal divisions only, it is called a **plain scale.**

Scales, to be of use, should fulfil the following conditions:—1. Divided with great *accuracy*, and carefully numbered. 2. Long enough to measure the principal lines of the drawing. 3. The zero (0) must always be between the unit and its subdivisions. 4. The name of the scale should be written on it, and the representative fraction shown. 5. Numbered decimally in the primary divisions.

PLAIN SCALES

PROBLEM 91.—To construct a scale of $\frac{1}{2}$ in. to 1 ft., or $\frac{1}{24}$, to measure 6 ft.

Rule two parallel lines about $\frac{1}{12}$th of an inch apart. Set off 6 half inches. Divide the first part into 4, showing spaces of 3 in. Figure the scale as shown, placing the *zero* between the feet and the subdivision into inches.

PROBLEM 92.—To construct a scale of $\frac{3}{4}$ in. to 1 mile, showing miles and furlongs, and measuring 4 miles.

Draw two lines as before. Set off 4 spaces of $\frac{3}{4}$ in. each. Divide the first part into 8, showing the furlongs, and number as shown.

Note.—The representative fraction in this scale is $\frac{\frac{3}{4}\text{ in.}}{1\text{ m.}} = \frac{\frac{3}{4}\text{ in.}}{63360\text{ in.}} = \frac{1}{84480}$. Every line of the object drawn to this scale would be 84480 times as long as that of the drawing.

PROBLEM 93.—Draw a scale of $\frac{1}{60}$, to show yards and feet, and measuring 5 yds.

The scale has to be long enough to measure lines 5 yds. long; therefore, its total length will be $\frac{5}{60}$ or $\frac{1}{12}$ of 1 yard, that is, 3 in. Draw two lines, 3 in. long. Divide, and number as shown.

PROBLEM 94.—Construct a scale of ·875 in. to 10 ft., to measure 40 ft.

If 10 ft. are represented by ·875 in., then 40 ft. would be represented by $\cdot875 \times 4 = 3\cdot5$ in. Take a line $3\frac{1}{2}$ in. long, divide it into 4 equal parts, giving distances of 10 ft. Divide the first part into 10 equal parts, showing feet.

Representative fraction $= \frac{\cdot875\text{ in.}}{10\text{ ft.}} = \frac{\cdot875\text{ in.}}{120\text{ in.}} = \frac{\frac{7}{8}\text{ in.}}{120\text{ in.}} = \frac{7}{960}$.

PROBLEM 95.—Draw a scale showing $5\frac{1}{2}$ yds. to 1 in., to measure 20 yds.

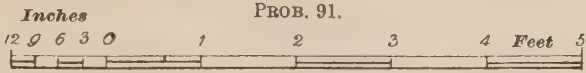
If $5\frac{1}{2}$ yds. be represented by 1 in., then 2 in. will represent 11 yds. Draw a line, and set off A B, 2 in.

Divide this distance into 11 equal parts. Each of these parts will represent 1 yard. Add 9 more parts, to make up the 20, and complete the scale as shown

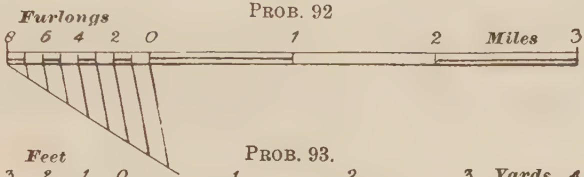
PROBLEM 96.—One inch represents 13 ft. 4 in. Draw the corresponding scale, divided generally into 10-ft. lengths, with one such length subdivided into single feet. (*Sc.*, 1885.)

The representative fraction $= \frac{1\text{ in.}}{13\text{ ft. }4\text{ in.}} = \frac{1}{160}$. The *unit* is a distance of 10 ft. If 160 in. be represented by 1 in., then 10 ft., or 120 in., will be represented by $\frac{120}{160} = \frac{3}{4}$in. Set off distances of $\frac{3}{4}$ in., representing 10 ft. Subdivide the left-hand division into 10 equal parts, showing single feet.

PROB. 91.



PROB. 92



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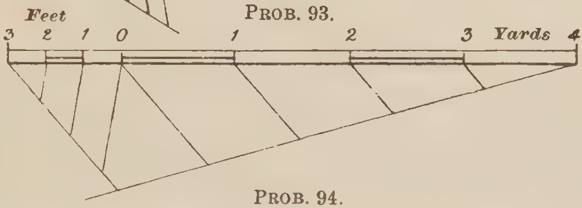

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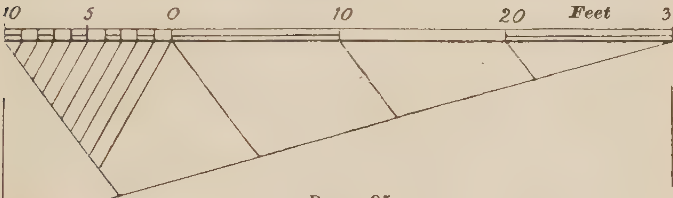

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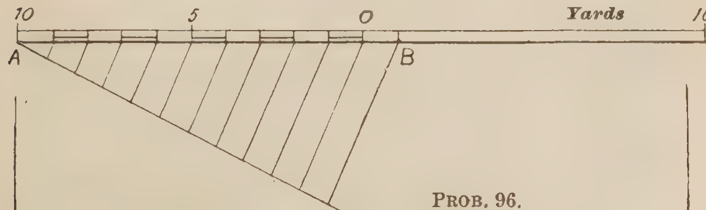
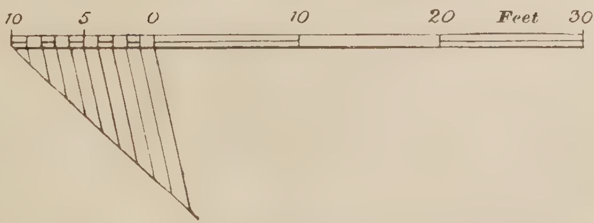


PROB. 96.



PROBLEM 97.—The line A B represents $3\frac{1}{2}$ yards. Construct a scale, showing yards and feet, to measure 10 yards.

Divide **A B** into 7 equal parts, showing $\frac{1}{2}$-yards. Produce the line to the required length. Subdivide the left-hand division into 3 equal parts, showing feet.

PROBLEM 98.—The given line, A B, is 2 ft. 5 in. long by scale. Produce it so as to make it 6 ft.

Draw **A C** at any angle, and set off 2 ft. 5 in. to any convenient scale; $2\frac{5}{12}$ in. will do. Draw **B C**, and rule parallels cutting **A B** into 2 ft. 5 in. Add on the required number of feet.

DIAGONAL SCALES

By means of the diagonal scale very minute distances may be measured with great accuracy. The principle of its construction is as follows:—If the rectangle **A B C D**, on the opposite page, be divided into 8 equal parts by parallels to **A B**, and the diagonal **D B** be drawn, then a number of similar triangles will be formed. (Euc. vi. 2.) If **D** 4 is half **D A**, then 4 a will be half **A B**. In the same manner, **D** 1 is $\frac{7}{8}$ of **D A**; therefore, 1 b will be $\frac{7}{8}$ of **A B**.

From a *plain scale* we get two dimensions, such as yards and feet; from a *diagonal scale* we may obtain three dimensions, as yards, feet, and inches.

PROBLEM 99.—Draw a diagonal scale showing inches and tenths.

Draw a line **A B**, and set off inches. At **A** erect a perpendicular, and set off 10 equal parts to any convenient unit, and from each part draw parallel lines to **A B**. Erect verticals at each primary division. Draw the diagonal from 0 to 10.

Note.—We may obtain from this scale inches and tenths, thus:—Suppose $1\frac{7}{10}$, or 1·7 in., be required. Place the dividers where *vertical* 1 meets *horizontal* 7 (point e), and open until point f is reached. $a\,b = 1\frac{1}{10}$, or 1·1 in.; $c\,d = 1\frac{2}{10}$, or 1·2 in.; $g\,h = 2\frac{8}{10}$, or 2·8 in.

PROBLEM 100.—Draw a diagonal scale showing inches and hundredths of an inch.

Draw the 11 parallels, and set up the verticals as before. Now divide **A** 0 into 10 equal parts, and join the first part on the left with 10 at the end of the top line. Rule parallels from each of the parts as shown.

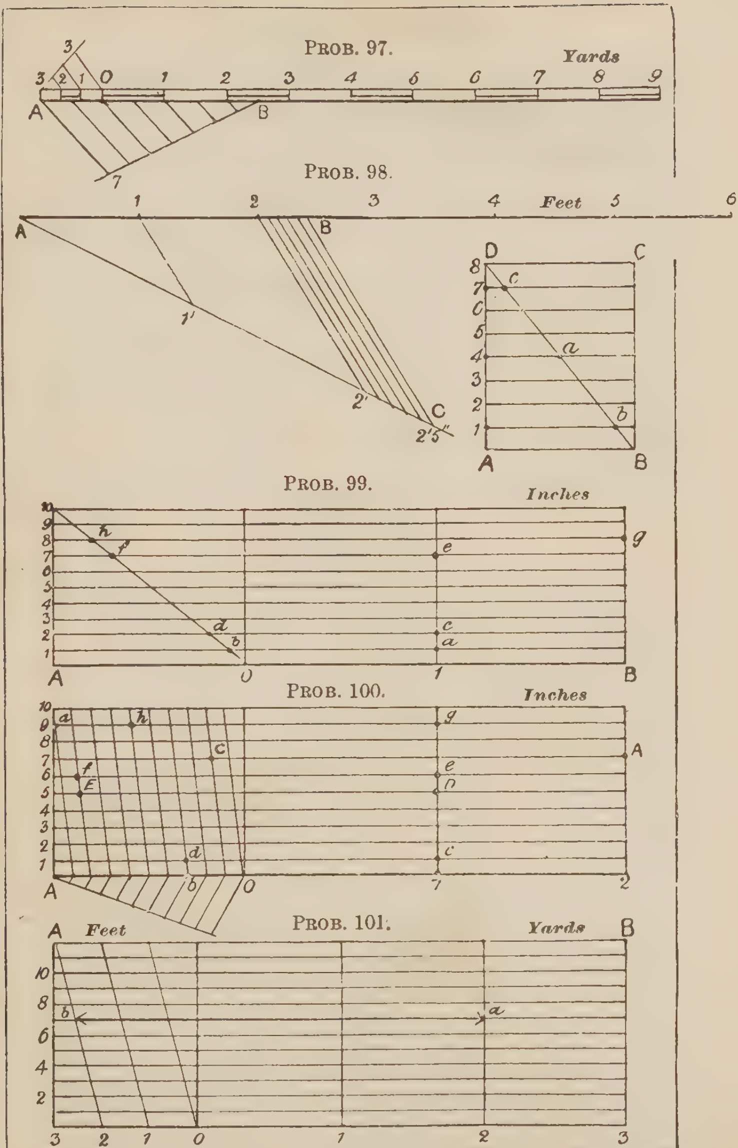
Note.—Each part on the line **A 0** shows tenths of an inch, and the distance 9 a on the second line from the top will be $\frac{1}{10}$ of a tenth—that is, $\frac{1}{100}$ of an inch. The distance 1 $b = 1\frac{3}{10}$, or 1·3; $c\,d = 1\frac{31}{100}$, or 1·31; $e\,f = 1\frac{86}{100}$, or 1·86.

If 1·59 inches were required from the scale, place one point of the dividers on the point g, where the vertical 1 meets the horizontal 9; open the dividers to the point h, where the diagonal 5 meets the horizontal 9.

PROBLEM 101.—Draw a scale of $\frac{1}{48}$th to show yards, feet, and inches, and show a distance of 2 yds. 2 ft. 7 in. on the scale.

A scale of $\frac{1}{48}$ would be $\frac{3}{4}$ in. to the yard. Draw a line, **A B**, and set off distances of $\frac{3}{4}$ in. Divide the firstpart into 3, showing feet. Draw the vertical **A** 3, and set off 12 equal parts. Rule parallels, and complete the scale.

To obtain the required distance, place one point of the dividers on a, where vertical 2 meets the horizontal 7, and extend the other point to b, where the diagonal 2 intersects the horizontal 7.


PROB. 97.
Yards
3 2 1 0 1 2 3 4 5 6 7 8 9
A B 7
PROB. 98.
1 2 3 4 Feet 5 6
A B 1' 2' C 2'5"
D C 8 7 6 5 4 3 2 1 A B c a b
PROB. 99.
Inches
10 9 8 7 6 5 4 3 2 1
h f d b e c a g
A 0 1 B
PROB. 100.
Inches
10 9 8 7 6 5 4 3 2 1
a h C f E d b g e D c A
A 0 1 2
PROB. 101.
A Feet Yards B
10 8 6 4 2
b a
3 2 1 0 1 2 3

THE SCALE OF CHORDS

This scale is used to measure angles, and is usually marked on a scale by the letters 'C' or 'CHO.'

PROBLEM 102.—To construct a scale of chords.

Describe the quadrant A B C. Trisect the arc A C, and subdivide each part again into three, thus dividing the arc into 9 equal parts of 10° each. Make A D equal to A C. From centre A describe arcs from each of the divisions on to the line A D. Complete the scale as shown.

Notes.—1. The distance from A to each division on the scale is the chord of the angle containing that number of degrees. 2. The divisions get smaller as the angle increases towards 90°. 3. The chord of the arc of 60° is always equal to the radius.

PROBLEM 103.—To construct angles of 50° and 100° by means of the scale of chords.

Draw any line, A B. With centre A and *radius* o *to* 60° from the scale, describe an arc. Now take the distance from o to 50°, and intersect the arc in C. Then B A C is the angle of 50°. For 100°, take the distance from o to 50°, and set off from C, giving B A D, the angle of 100° (50° + 50° = 100°). Another way would be to obtain E for 90°, and cut off E D equal to the distance from o to 10° (90° + 10° = 100°).

PROBLEM 104.—To measure the size of an angle by means of the scale of chords.

Let B A C (*Prob.* 103) be the given angle. With radius o to 60° from the scale, describe the arc B C. Take the distance B C, and apply to the scale, giving 50°.

THE SECTOR

This instrument is shown on the opposite page. The most important of the scales marked on it are the line of lines, marked L, and the line of polygons, marked POL. The following problems show some of its uses.

PROBLEM 105.—To bisect a line.

Open the sector until the transverse distance from 10 to 10 on L equals the given line. Then the distance from 5 to 5 will be half the line.

Notes.—1. Measure with the dividers from the inside line, where the dots are marked. 2. The transverse distances at 8 and 4, or 6 and 3, &c., may also be used for the bisection.

PROBLEM 106.—To divide a straight line into 7 equal parts.

Open the sector until the transverse distance from 7 to 7 on L equals the given line. Then the transverse distance from 1 to 1 will be $\frac{1}{7}$th of the given line.

PROBLEM 107.—To find a fourth proportional to three given lines, A, B, C.

From the centre O, on L, set off O D equal to A. Open the sector until the transverse distance at D equals B. Then, if O E be set off equal to C, the transverse distance at E will be the required fourth proportional.

PROBLEM 108.—To inscribe a regular pentagon in a circle.

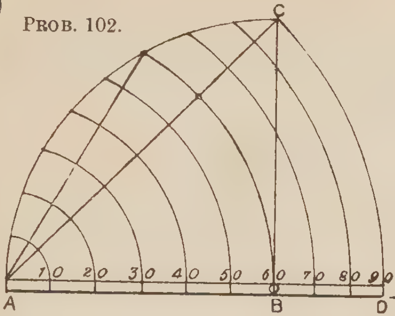
Let O A be the radius of the circle. Open the sector until the transverse distance from 6 to 6 on POL equals O A. Then the transverse distance at 5 will equal the side of the pentagon.

Notes.—1. Always make the radius of the circle equal the distance from 6 to 6. 2. If a heptagon were required, then the distance from 7 to 7 would equal the side.

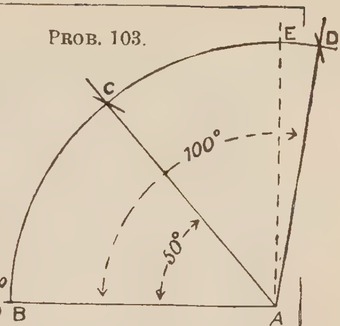
PROBLEM 109.—To construct a regular heptagon on a given line.

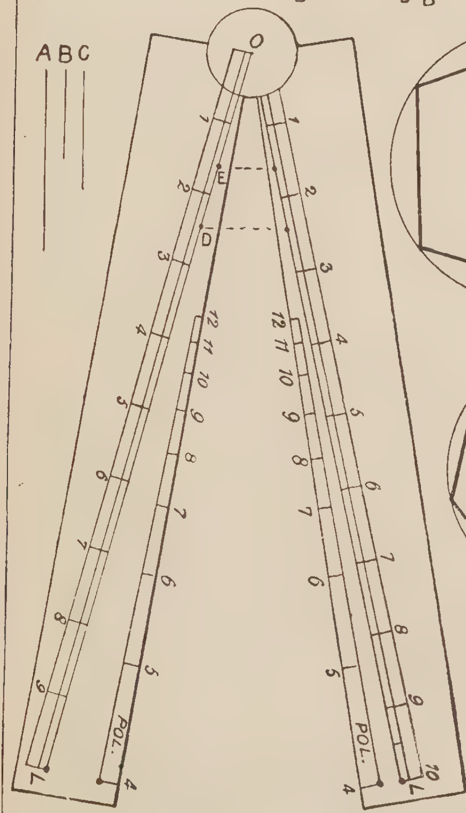
Let A B be the given line. Open the sector until the transverse distance from 7 to 7 equals A B. With A and B as centres, and the distance from 6 to 6 as radius, describe arcs intersecting at O. With centre O and the same radius describe a circle. Set off the distance A B round it.

PROB. 102.



PROB. 103.





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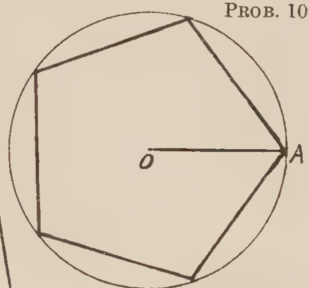

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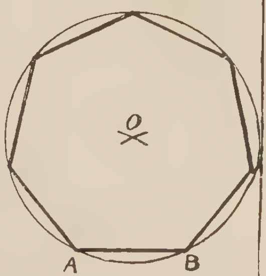


CHAPTER X

THE USE OF SCALES IN THE CONSTRUCTION OF IRREGULAR POLYGONS, AND IN THE REDUCING, ENLARGING, AND COPYING OF PLANE FIGURES.

PROBLEM 110.—To construct an irregular polygon having given the lengths of the sides and the magnitude of the angles.

Sides, A B = 1·3″, B C 1·2″, C D $1\frac{1}{4}$″, D E $1\frac{5}{8}$″, E A 1·4″. Angles, A B C 140°, B A E 100°. First make a rough freehand sketch, lettered and figured, as a guide. Draw A B and make it 1·3″ ($1\frac{3}{10}$) long from the scale. Make the angles A B C and B A E equal to 140° and 100° respectively, either with the protractor or scale of chords. Make B C 1·2″ and A E 1·4″.

With centre C and radius $1\frac{1}{4}$″ describe an arc. With centre E and radius $1\frac{5}{8}$″ intersect the arc in D. Draw E D, C D.

Note.—The angles of figures are usually lettered in order, starting from the first letter.

PROBLEM 111.—To construct an irregular polygon, the lengths of the sides and diagonals being given.

Sides, A B = 1·9″, B C = 1″, C D = 1·85″, D E = 1·75″, E A = $1\frac{3}{8}$″. Diagonals, A C = 2·17″, B E = 2·4″. Make a rough sketch of the figure as before. Draw A B 1·9″ long. With centre B and radius 1″ describe an arc, and with centre A and radius 2·17″ intersect the arc in C. (This distance may be taken from the diagonal scale showing hundredths of an inch; it is marked by the letters A C on the figure for Problem 100.) Draw the triangle A B E in a similar manner. With centres C and E describe arcs of 1·85″ and 1·75″ radii intersecting in D. Draw B C, C D, D E, E A. (The distance C D is shown on the same scale by the letters D E.)

Note.—Problems 111 and 112 are drawn half size.

PROBLEM 112.—To construct an irregular polygon, having given two sides, the lengths of the diagonals drawn from one angle, and the angles between them.

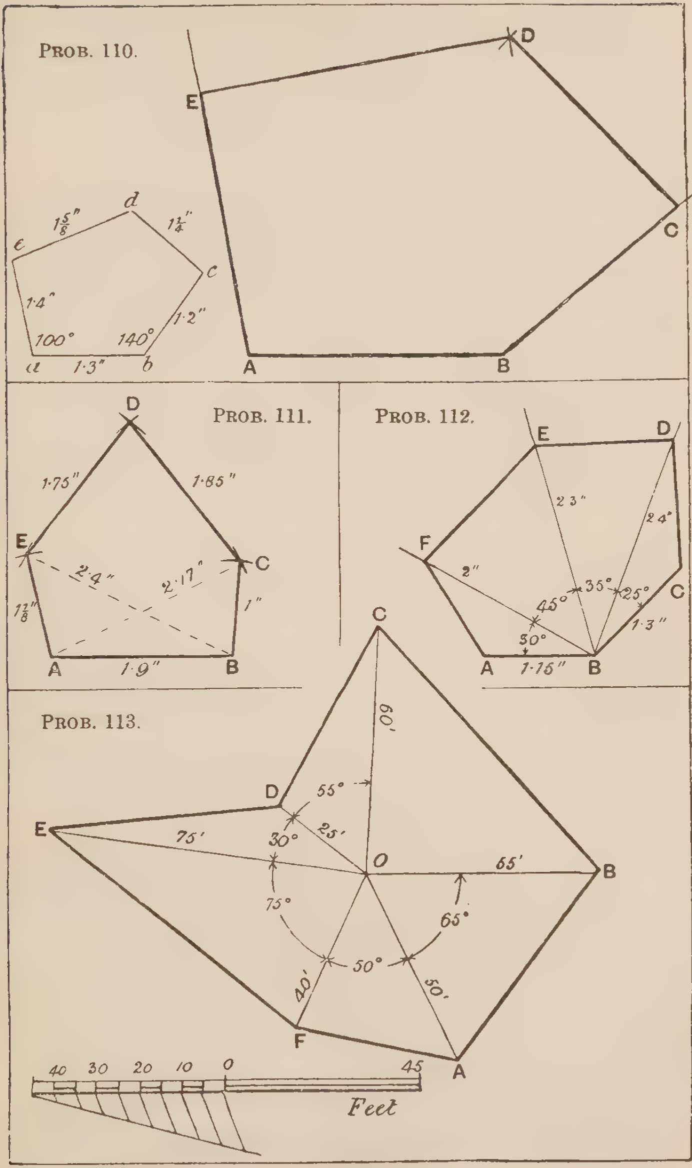
Sides, A B = 1·15″, B C = 1·3″. Diagonals, B D = 2·4″, B E = 2·3″, B F = 2″. Angles, A B F = 30°, F B E = 45°, E B D = 35°, D B C = 25°.

After making a sketch, draw A B = 1·15″. With the protractor set off angles A B F, F B E, E B D, D B C. Make B C, B D, B E, B F equal to the lengths given. Draw C D, D E, E F, F A.

PROBLEM 113.—To construct an irregular polygon having given a point O within the polygon, the distance from the point to each angle, and the angles round the point.

O A = 50′, O B = 55′, O C = 60′, O D = 25′, O E = 75′, O F = 40′. Angles.—A O B = 65°, A O F = 50°, F O E = 75°, E O D = 30°, D O C = 55°. Scale, 45 ft. = 1 inch.

First construct the scale, showing distances of 5 ft. After making a sketch, fix a point O, draw O B, and set out the angles B O A, A O F, F O E, E O D, D O C with the protractor. Take the lengths of the lines given from the scale and join the points, A, B, C, D, E, F, with each other.


PROB. 110.
D
E
C
A
B
d
e
c
a
b
$1\frac{5}{8}''$
$1\frac{1}{4}''$
1·4″
1·2″
100°
140°
1·3″
PROB. 111.
D
E
C
A
B
1·75″
1·85″
2·4″
2·17″
$1\frac{7}{8}''$
1″
1·9″
PROB. 112.
E
D
F
C
A
B
2·3″
2·4″
2″
35°
25°
45°
30°
1·3″
1·15″
PROB. 113.
C
D
E
O
B
F
A
60′
55°
25′
30°
75′
65′
75°
65°
50°
40′
50′
40
30
20
10
0
45
Feet

E 2

PROBLEM 114.—To construct an irregular polygon from a rough diagram, the dimensions on the diagonal and the offsets or ordinates being given.

A E = 3 ch. 50 l., A b = 75 l., A g = 90 l., A c = 2 ch. 20 l., A f = 2 ch. 90 l., A d = 3 ch. Offsets, b B = 1 ch. 10 l., g G = 80 l., c C = 25 l., f F = 1 ch. 30 l., d D = 90 l.

Scale, $\frac{3}{4}$ in. to represent 1 chain.

$$Representative\ Fraction = \frac{\frac{3}{4}\text{ in.}}{66\text{ feet}} = \frac{\frac{3}{4}\text{ in.}}{792\text{ in.}} = \frac{1}{1{,}056}.$$

First construct the scale. As no distances in the figure are required smaller than 5 links it will only be necessary to divide the unit into 20 parts. This will be more accurately performed by means of a diagonal scale. Draw 5 parallel lines. Set off 4 spaces of $\frac{3}{4}$ in. each and draw verticals. Divide the first space into 5 equal parts giving distances of 20 links. Draw the diagonals, thus obtaining distances of 15, 10, and 5 links.

Next draw A E, 3 ch. 50 l. from the scale ($a\,b$ on the scale), and set off A b ($c\,d$ on scale), A g, A c, A f, A d. At the points b, g, c, f, d draw the offsets, taking the distances from the scale. Join the points A, G, F, E, D, C, B.

PROBLEM 115.—To construct an irregular figure from a given figured rough sketch. Scale, 20 ft. = 1 in.

$$Representative\ Fraction = \frac{1\text{ in.}}{20\text{ ft.}} = \frac{1}{240}.$$

First draw the scale. As the longest distance to be measured is under 40 ft. the scale need be only 2 in. long. Divide the first inch into 20 parts.

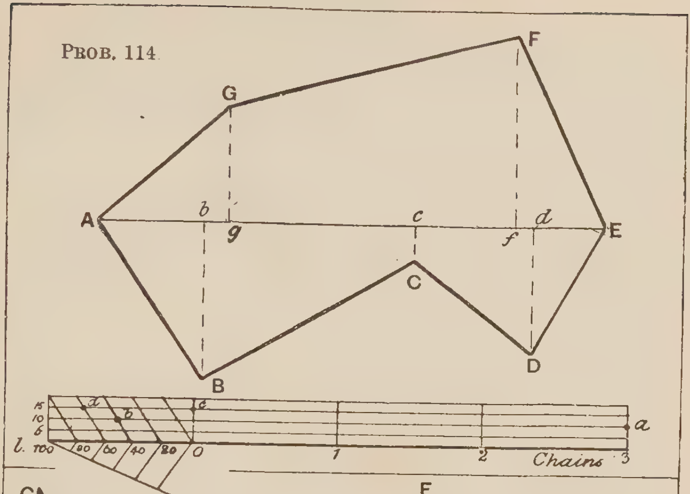
Draw any line A B. At A draw A C at an angle of 90° and make it 27 ft. long by scale. Make an angle of 30° at C and draw C D 15 ft. by scale. At D make an angle of 100° and draw D E 35 ft. long. Proceed in a similar manner for the other lines and angles. From G draw G B at right angles to A B.

PROBLEM 116.—Construct a six-sided polygon $a\,b\,c\,d\,e\,f$, **such that** $e\,f = \frac{a\,e}{2} = \frac{2}{3}\,a\,f$; $d\,e = a\,d$; **and** $a\,b = \frac{2}{3}\,b\,c$. **The rest of the data are given on the figure.** Scale $\frac{1}{2}'' = 10'\,0''$.

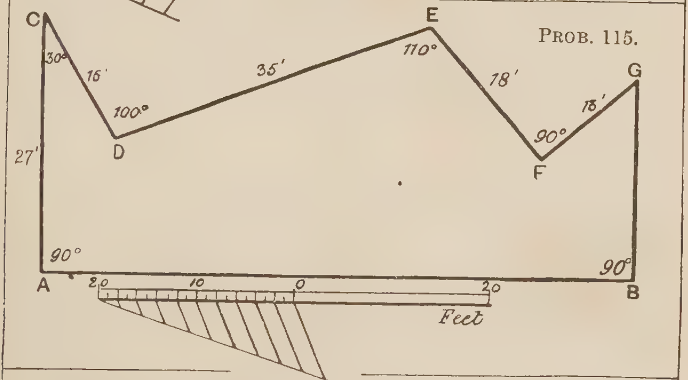
First construct a plain scale showing feet, and long enough to measure $a\,e$. Then draw $a\,e$ 56′ by scale. On $a\,e$ construct the isosceles triangle $a\,d\,e$, making the sides $d\,a$ and $d\,e$ 49′ long. To obtain point c, draw a line parallel to $a\,e$ and 22′ from it, and with centre e and radius 42′ from the scale intersect the parallel. Draw $d\,c$. ($a\,b = \frac{2}{3}\,b\,c = 18'$.) With centre a and radius 18′ describe an arc, and with centre c and radius 27′ intersect the arc in b. Draw $c\,b, a\,b$. ($e\,f = \frac{a\,e}{2} = 28'$, and $\frac{2}{3}\,a\,f = e\,f$, therefore $a\,f = 42'$.) With centre e and radius 28′ describe an arc, and with centre a and radius 42′ intersect the arc in f. Draw $a\,f, f\,e$.

Notes.—1. A figure similar to this was set in the science examination 1889.
2. The figure is drawn half its proper size.

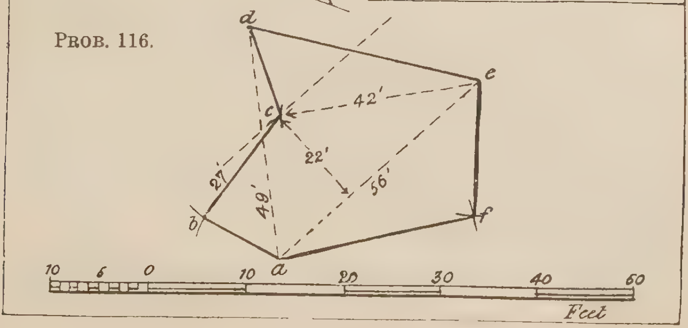
PROB. 114.



PROB. 115.



PROB. 116.



PROBLEM 117.—To make a reduced drawing of a given figure.

Let A B C D E F be the given figure. It is required to reduce it so that each side shall be ⅔ of its length on the given figure.

Method 1.—Draw B F, B E, B D. Divide A B into 3 equal parts. B *a* will be one of the required sides. Draw *a f*, *fe*, *e d*, *d c* parallel to the corresponding sides of the figure.

Note.—The principle is the same as in Prob. 88.

Method 2.—Divide A B into 3 equal parts as before. Take B *a* and place it in the same straight line or parallel to A B. Draw *a f*, *b f* parallel to A F, B F, thus obtaining point *f*. From *f* draw a parallel to F E and from *b* draw a parallel to B E giving point *e*. Proceed in a similar manner for the other sides.

Method 3.—Trisect each side and construct the figure by applying the principle of Prob. 63.

Notes.—1. Two other figures are given showing the application. The sides of both rectangle and triangle are reduced two-thirds.

2. A figure may be reduced to any given size by this method.

PROBLEM 118.—To make an enlarged drawing of a given figure.

Let A B C D E F be the given figure. It is required to enlarge it so that A B may be equal to the given line *a b*.

Draw A C, A E, A D and produce them. Set off *a b* from A. From *b* draw *b c* parallel to B C, and proceed as in the previous problem.

PROBLEM 119.—To enlarge or reduce a drawing by means of a PROPORTIONAL SCALE.

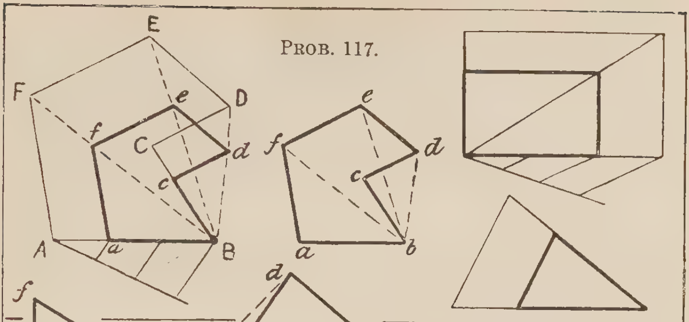
It is required to enlarge the given drawing so that A C shall equal 2½ ins.

Construct a proportional scale by drawing two lines at any angle to each other, set off *a c* equal to A C and *a c′* equal to the required distance, 2½ ins. Join *c c′*. Set off the distances A 1, A 2, A 3, A 4 on *a c*. From each point rule parallels to *c c′* giving the proportionate distances for the new drawing.

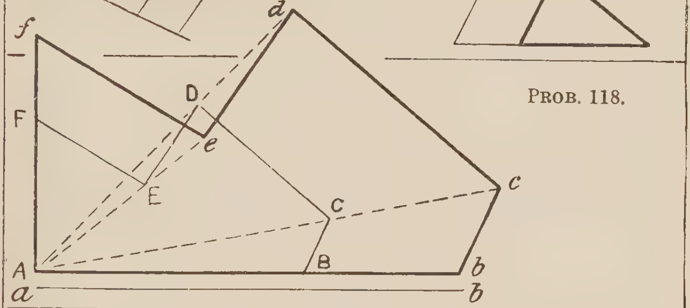
Draw the centre line 2½ ins. long and on it set off the distances *a* 1′, *a* 2′, &c. Through *a′* and *c′* draw perpendiculars. Set off A *x* on *a c* and obtain *a x′* the width of half the door. Set this distance off on each side of *a′* and erect perpendiculars. Through 1′, 2′, 3′, 4′ draw parallels. For the width of the panels set off A 5, A 6 on *a c* and proceed as before.

To reduce a drawing proceed in a similar manner. The only difference will be that the *given* dimensions will be set off on the *longer* line of the scale and the *required* dimensions will be obtained from the *shorter* line.

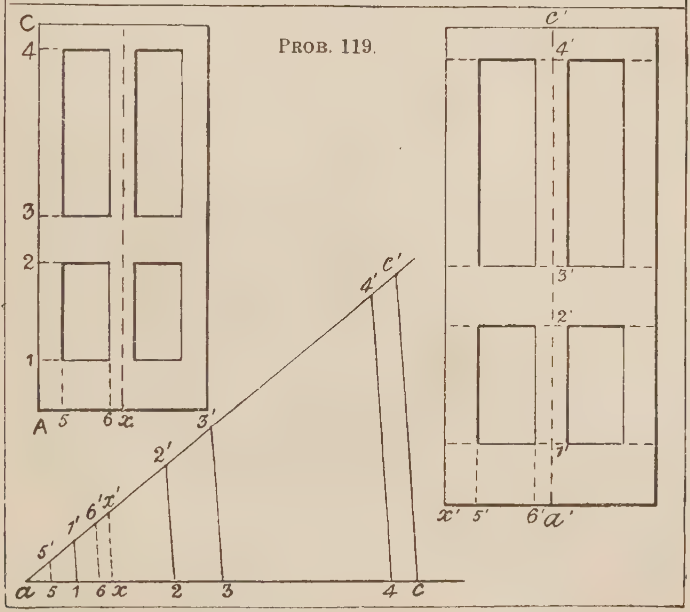
PROBLEM 119a.—To reduce or enlarge a drawing by squaring. See Prob. 31.



PROB. 117.



PROB. 118.



PROB. 119.

PROBLEM 120.—To copy figures.

The copying of given figures is an exercise of considerable importance in helping to form habits of accuracy and quickness in a draughtsman. The proper construction lines must be first obtained, and always in drawing symmetrical figures remember to set out the centre lines first. Several examples are here given, with the proper method of proceeding.

Figure 1 represents a Greek fret. It will be found that the pattern is formed upon ten equi-distant horizontal lines intersected by vertical lines forming squares. Draw the pattern in firmer line, keeping the *shade* line darker.

Note.—The figure is given half size. The same method of construction must be employed when copying any fret.

Figure 2.—Rule the parallels first. Draw the verticals, and with the 60° set square obtain the equilateral triangles as shown. Keep the proper relation between thick and thin lines. Make your copy double the size of the given figure.

Figure 3.—Draw the figure to the dimensions given.

It will be seen that a square of $3''$ sides divided into 9 equal squares will contain the inner lines of the four squares of which the figure is composed. Draw the diagonals, and about each square describe another square at the required distance. In thickening, or inking in the pattern, be careful to notice the portions that overlap.

Figure 4.—Draw the pattern to the dimensions given. As the sides of the small squares and the distances between them are equal, draw a line and set off 5 equal spaces of $\frac{3}{4}''$ each. On this distance construct a square and divide it into 25 equal squares.

To obtain the other lines divide the alternate squares as shown.

Figure 5.—Draw the figure, making the centres of the circles $1\frac{1}{4}$ inch apart. Radius of the large circle $\frac{3}{8}''$, of the smaller one $\frac{1}{8}''$. Draw the centre line, set off the centres and describe the circles. For the centres of the connecting arcs describe equilateral triangles as shown.

PROB. 120.

FIG. 1.

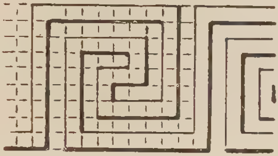


FIG. 2.

FIG. 3.

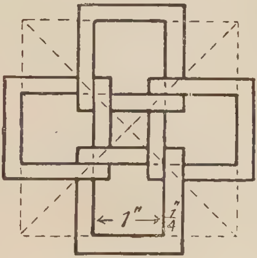


FIG. 4.

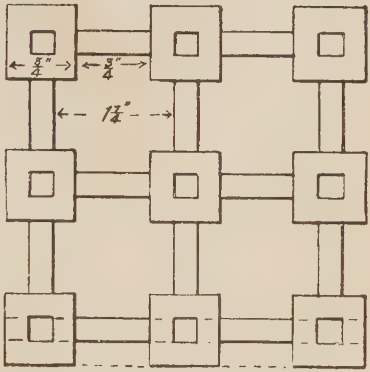


FIG 5.

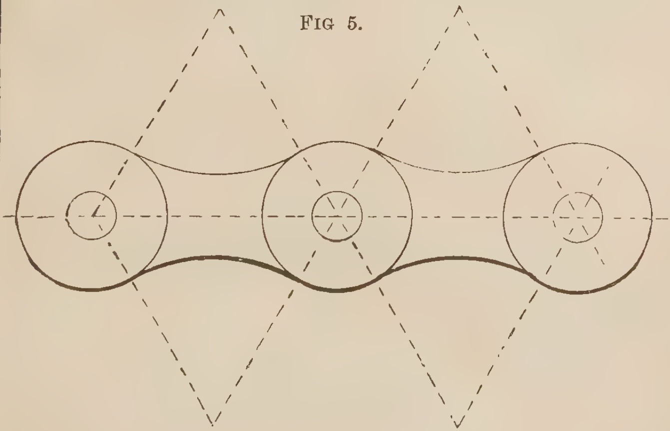


Figure 6.—Copy the double spiral, which is composed of semi-circles $\frac{1}{4}''$ apart; the diameter of the smallest semicircle is $\frac{1}{2}$ in.

Draw a line A B and set off 12 equal spaces of $\frac{1}{4}''$, and with centres 3 and 9 describe three concentric semicircles above the line from 3, and below the line from 9. The centres for the connecting semicircles will be midway between 2, 3, and 9, 10. Describe the arcs which have equal radii before commencing the next.

Figure 7.—Rule the horizontal and vertical lines as shown, and from each centre describe two concentric circles. The finished pattern should be thickened or inked in. Let the centres for each circle be $\frac{5}{8}''$ apart.

Figure 8.—Draw the figure from the given dimensions.

Draw a line A B and set off distances of $\frac{3}{8}''$, $1\frac{1}{4}''$, $\frac{3}{8}''$, $1\frac{1}{4}''$, $\frac{3}{8}''$, $1\frac{1}{4}''$, $\frac{3}{8}''$. Mark the centre for each of the circles and describe them, omitting the portions which are omitted in the copy.

Note.—The given figure is drawn to a smaller scale.

Figure 9.—Draw the figure from the given dimensions.

Begin with the centre lines A B and C D.

Set off on C D distances of $\frac{1}{8}''$, $\frac{1}{4}''$, $\frac{1}{8}''$, $\frac{1}{2}''$, $\frac{1}{8}''$, $\frac{1}{4}''$, $\frac{1}{8}''$. Describe the circles. Set off $1\frac{1}{8}''$ on each side of the centre of A B and draw the vertical lines E and F. Describe the semicircles and complete the figure.

FIG. 6

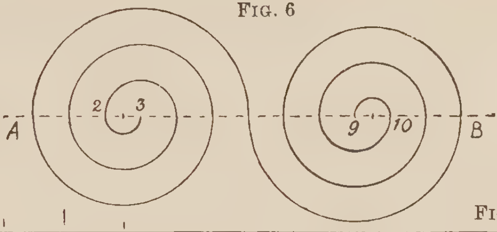


FIG. 7.

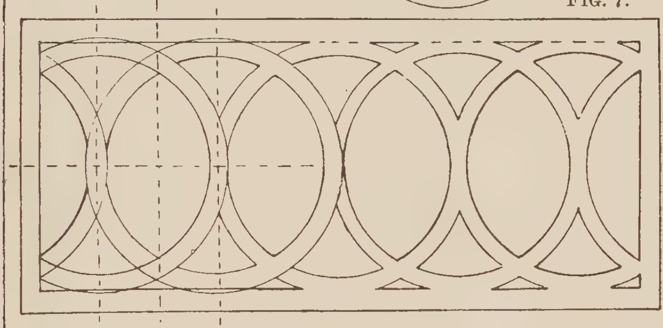


FIG. 8.

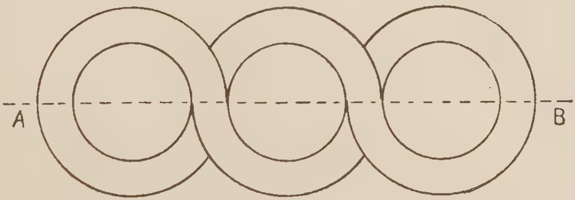
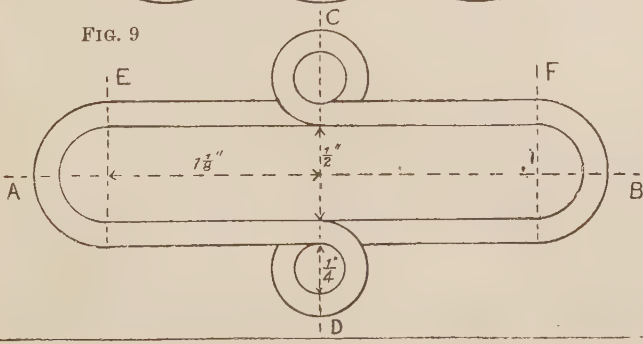


FIG. 9



EXERCISES

Note.—Any figures relating to the following exercises will be found on page 63.

CHAPTER VII

1. Describe a circle $1\frac{1}{4}''$ radius, and from a point **A**, 3″ from its centre, draw a line touching the circle.

2. Describe two circles with radii $\frac{3}{4}''$ and 1″, having their centres $2\frac{1}{2}''$ apart. Draw a right line to touch both these circles.

3. To a circle of 1·25″ radius, draw two tangents which shall contain an angle of 60°. (*Sc.*, 1871.) (*Prob.* 72.)

4. Describe a circle of $1\frac{1}{8}''$ radius, and draw tangents intersecting each other from two points in the circumference 100° apart.

5. Describe a circle 1·5″ radius. Draw a straight line 1·75″ long within the circle. On this line construct an isosceles triangle having its vertical angle in the circumference of the circle.

6. Draw a circular arc of large radius, and draw a tangent to it without finding the centre of the circle.

CHAPTER VIII

1. Construct a regular hexagon having its diagonal 3 inches.

2. On a base of $1\frac{1}{4}''$ construct a regular heptagon.

3. Describe a circle with a diameter of 2·9″. About it describe a regular nonagon.

4. On a side of $\frac{3}{4}''$ set up a regular octagon by means of the set square.

5. In a circle of 3·5″ diameter inscribe a regular pentagon.

6. Construct a regular pentagon whose diagonal shall be 3″. (*Sc.*, 1870.) (*Prob.* 90.)

7. Construct a regular hexagon on a base of ·75″, and on the same base construct a similar hexagon having its side 1·25″ long. (*Prob.* 117.)

8. Copy the hexagons (Fig. 1) which are drawn to a scale of $\frac{3}{4}''$ to 1′ 0″, and make them $\frac{1}{2}''$ to 1′ 0″. (*Art.*)

Note.—The figure is given half its proper size and the construction lines are indicated.

CHAPTER IX

1. Construct a scale 5 ft. long, $1\frac{1}{3}$ in. to 1 ft. to show inches. What is its representative fraction?

2. Construct a scale 2 inches to 1 yard to show $5\frac{1}{2}$ feet. What is its representative fraction?

3. The given line (Fig. 2) is 2 ft. 6 in. long. Produce it to measure 3 ft. 6 in (*Art.*)

4. Convert **A B** 2 ins. long into a diagonal scale showing $\frac{2}{100}$ of the line. (*Art.*)

5. Construct a plain scale $\frac{3}{4}$ in. to 1 ft., and mark upon it a distance of 5 ft. 7 in. by the scale. (*Art.*)

6. Convert the given scale (Fig. 3) into a diagonal scale reading inches. State the representative fraction of this scale. (*Sc.*, 1889.)

Note.—The first division will require dividing into 10 equal parts, and twelve parallels must be ruled to obtain the inches.

7. Construct a scale of chords on a radius of 4·25″ to read to 5°. By means of this scale plot an angle of 75°. (*Sc.*, 1889.)

8. Given a scale of yards (Fig. 4). Deduce from it a scale of feet to read to 1′ 0″ and show 70′ 0″.

Note.—The given scale is $\frac{2}{3}$ its proper size and shows 20 yds. or 60 feet. Divide it into 6 equal parts, showing distances of 10 feet, and add one space on to obtain 70 feet. Subdivide the first part into 10 to obtain one foot.

9. The line $a\,b$ (Fig. 5) represents 3′ 9″. Construct a scale reading inches and showing 10′ 0″. The scale to be correctly figured. (*Sc.*, 1886.)

Note.—Draw a line at any angle from a, and set off $3\frac{3}{4}$ to any unit. Divide $a\,b$ and add the distance required as in Probs. 97 and 98.

10. Construct a triangle with sides 10′ 6″, 14′ 0″, and 16′ 3″. Scale $\frac{1}{8}$″ to 1′ 0″. (*Sc.*, 1884.)

Note.—First construct the scale. As divisions equal to $\frac{1}{4}$ of $\frac{1}{8}$″ will be needed, a diagonal scale should be used.

11. Transfer the given scale (Fig. 6) to your paper and complete it neatly, with figuring, &c. Write down the representative fraction (*Sc.*, 1883.)

Note.—The scale measures 3·75″, therefore its representative fraction will be $\frac{3\cdot75 \text{ in.}}{50 \text{ yds.} \times 3 \times 12}$. Reduce to lowest terms.

12. Make a scale 6″ long to read feet and inches. Fraction $\frac{1}{16}$. (*Sc.*, 1882.)

13. A length of 100 yds. is found to measure 3·6″ on a drawing. What is the fraction of the scale? Construct a scale to read yards, making it not less than 4 ins. long. (*Sc.*, 1881.)

14. Give the representative fraction of a scale on which $3\frac{1}{2}$″ represents 2247′ 0″. Construct a scale of $\frac{1}{20}$, reading inches. (*Sc.*, 1879.)

15. Construct a scale of $\frac{1}{468}$ showing yards. Scale to be properly figured and not less than 7″ long. (*Sc.*, 1878.)

Note.—1 in. stands for 468 ins., that is 13 yards.

16. Construct a scale of $\frac{1}{19}$ to read inches and show 10′ 0″. (*Sc.*, 1877.)

Note.—To find the length of the scale. If 19 feet be represented by 1 foot, what distance will be represented by 10 feet? 19 ft. : 10 ft. as 12 ins. : required length, whence required length $= \frac{12 \times 10}{19} = 6\frac{6}{19}$ ins. Draw a line $6\frac{6}{19}$ ins. long and divide it into 10 equal parts showing feet. Subdivide the first unit into 12 equal parts showing inches.

17. Draw a scale of 12·5 feet to 1 inch and give the fraction. (*Sc.*, 1876.)

18. A line 100 ft. long is represented on a drawing by a line 4″ long. Make a scale of feet for the drawing and give the representative fraction.

19. A line 50 yds. long is measured from **A** to **B** at right angles to a line joining **A** with another point **C**. The angle subtended by the line **A C** is found to be 47°. Find by construction the distance from **A** to **C**. Scale 20 yds. to an inch.

20. Describe a circle of $1\frac{1}{2}$″ radius. Within it inscribe a regular nonagon by means of the sector. (*Prob.* 108.)

CHAPTER X

1. On a base 2″ long construct a figure similar to the given figure (Fig. 7).

2. Construct an irregular pentagon, sides $2\frac{1}{2}''$, $2''$, $1\frac{1}{2}''$, $1''$, and $\frac{1}{2}''$. The angle between the longest and shortest sides to be 100° and the diagonal joining the extremities of the two longest sides $2\frac{1}{2}''$. (First draw a rough sketch.)

3. Draw any irregular four-sided figure, no side less than $1\frac{1}{4}''$. Construct a similar figure whose sides are $1\frac{1}{2}$ times those of the first figure. (*Sc.*, 1879.)

4. The tops of two vertical poles are 85′ 0″ apart. The poles are 40′ 0″ apart. The height of one pole is 12′ 0″. Determine the height of the other pole. Scale 1″ to 10′ 0″. (*Sc.*, 1879.)

5. Draw a circle of 1·25″ radius with centre **O**. The corners of the polygon inscribed in this circle are so that the angles at the centre are as follows: **A O B** = 60°, **B O C** = 70°, **C O D** = 50°, **D O E** = 80°, **E O F** = 50°. Write down the lengths of **A B**, **B C**, **C D**. (*Sc.*, 1871.)

6. Construct an irregular polygon from the following dimensions. Sides **A B** = 2″, **A F** = 1·8″. Diagonals, **A D** = 3·5″, **A E** = 3″. Angles, **A B C** = 33°, **B A C** = 40°, **B A D** = 59°, **B A E** = 118°, **B A F** = 130°.

7. Construct an irregular pentagon having its sides $2''$, $2\frac{1}{4}''$, $2\frac{3}{8}''$, $2\frac{1}{2}''$, $2\frac{3}{4}''$ respectively and with two of its angles right angles.

8. Draw to a scale of 40 yds. to an inch an irregular polygon **A B C D E F**. **A B** = 140 yds., **B C** = 113 yds., angle **A B C** = 130°, **C D** = 82 yds., **A D** = 200 yds., **D E** = 138 yds., **A E** = 140 yds., **E F** = 67 yds., **F A** = 90 yds.

Note.—A diagonal scale will be required, showing the inch divided into 40 parts.

9. The sides of a quadrilateral figure **A B C D** are as follows:—**A B** = 1″, **B C** = 1″, **C D** = $1\frac{1}{4}''$, **A D** = $1\frac{1}{2}''$, and the diagonal **B D** = $1\frac{1}{2}''$. Construct the figure and obtain a similar figure whose perimeter is 4″.

Note.—Find a fourth proportional to the perimeter of the constructed figure ($4\frac{3}{4}''$), the perimeter of the required figure (4″), and the side **A B** of the constructed figure (1″). This will give the side of the required figure. Set off this distance on **B A** and proceed as in Prob. 117.

10. The given figure (Fig. 8) represents a Maltese cross. Two dimensions and an angle are given. Draw the cross from the figured dimensions to a scale of $\frac{1}{8}''$ to 1′ 0″. (*Sc.*, 1888.)

Note.—The chief construction lines are indicated on the figure, which is drawn to a smaller scale. Commence with the inner square 5′ 6″ side. Draw the diagonals and diameters of the square, and produce them. From the centre set off 7′ 3″ on each side and draw parallels. On each side of the diagonal produced set off 20°. From *a* draw a line parallel to the vertical diameter to meet the diagonals. Complete the figure as shown.

11. Copy the given figure (Fig. 9), making the diameter of the circle 4″ and the width between the parallel lines $\frac{1}{4}''$.

Note.—Obtain the points as for the inscribed pentagon by using the protractor.

12. Copy the figure (Fig. 10) to the dimensions given.

Note.—Commence with the dotted lines.

Fig. 1.

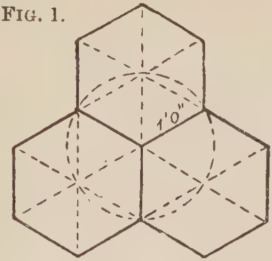


Fig. 7.

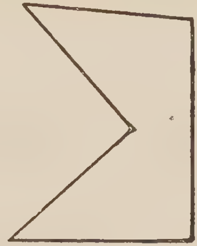


Fig. 2.

Fig. 3.

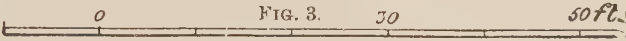


Fig. 4.

Fig. 5.

Fig. 6.

Fig. 8.

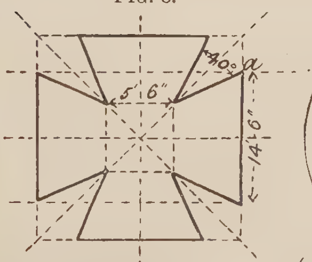
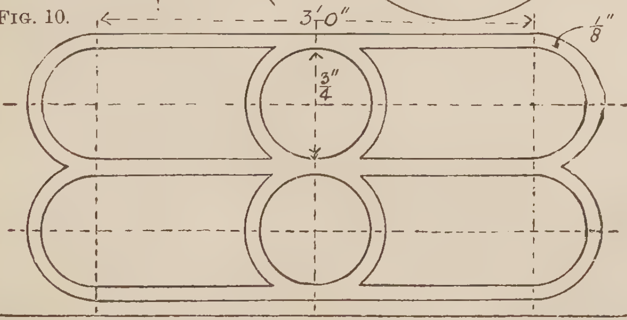


Fig. 9.

Fig. 10.



CHAPTER XI

CIRCLES TOUCHING LINES AND CIRCLES

PROBLEM 121.—To describe a circle passing through a given point and touching a given straight line in a given point.

Let A be the given point, and B the point in the given straight line.

At B draw B O perpendicular to the given line. Join A B.

At A construct the angle O A B equal to the angle O B A. With centre O and radius O B describe the circle.

Note.—The centre must lie in the line B O, because the given line will be a tangent to the circle.

PROBLEM 122.—To describe a circle passing through a given point, touching a given straight line, and having a given radius.

Let A be the given point, B the given line, and C the radius.

Draw a line parallel to B and at a distance equal to C.

With centre A and radius equal to C intersect this line in O.

With centre O and radius equal to C describe the circle.

PROBLEM 123.—To describe a circle of a given radius to touch two converging lines.

Let A B and C D be the two lines and E the radius.

At a distance equal to E draw lines parallel to A B and C D.

From O where the two parallels intersect, with radius equal to E, describe the circle.

PROBLEM 124.—To describe a circle touching three given straight lines which make angles with each other.

Let A B, A C, C D be the given lines.

Bisect the angles at A and C by lines meeting at O. Draw O E perpendicular to C D. With centre O and radius O E describe the circle.

PROBLEM 125.—To describe a circle touching two converging lines and passing through a given point between them.

Let A B, A C be the two converging lines and D the given point.

Bisect the angle B A C by the line A X; the centre of the circle must lie in this line. From any point E draw E F perpendicular to A B and describe a circle touching A B and A C. Join A with the given point D. Draw E G, and from D draw D H parallel to G E. With centre H and radius H D describe the circle.

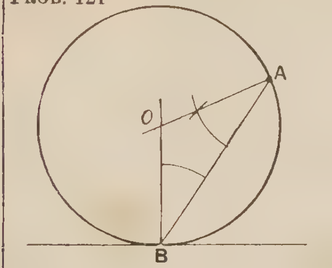
PROBLEM 126.—To describe a succession of circles touching each other and two converging lines.

Let A B, A C be the two lines. Draw A D bisecting the angle B A C.

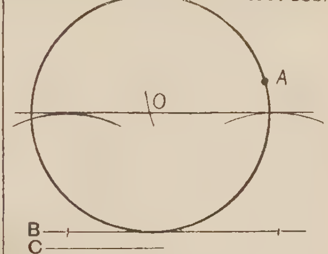
Take any point E, draw E F perpendicular to A C and describe a circle touching the two lines. Draw G H tangential to the circle at J.

Make G K equal to G F. From K draw K L parallel to F E for the centre of the next circle. Proceed in a similar manner for the other circles.

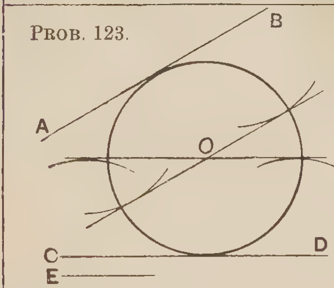
Prob. 121



Prob. 122.



Prob. 123.



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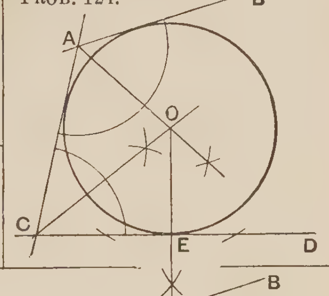
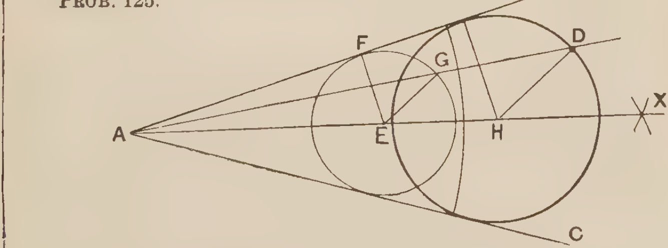
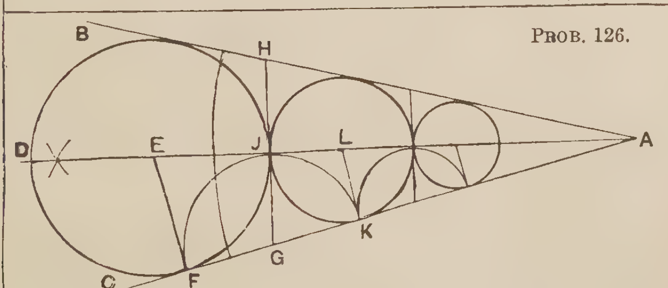


Prob. 125.



Prob. 126.



PROBLEM 127.—To inscribe a circle in a triangle.

Let A B C be the triangle.

Bisect any two angles, and proceed as in *Prob.* 124.

PROBLEM 128.—To inscribe two equal circles in an isosceles triangle, touching each other and two sides of the triangle.

Let A B C be the triangle. Draw the perpendicular A D dividing the triangle into two equal triangles. Find the centre O of the triangle A B D. For the centre of the other triangle join C E, and from O draw a line parallel to B C and meeting C E in P. From centres O and P describe the circles.

PROBLEM 129.—To inscribe three equal circles in an equilateral triangle, each touching one side and two circles.

Let A B C be the triangle. Divide it into three equal triangles by bisecting the sides. Find the centre of the triangle D B C and inscribe a circle. Set off D 2, D 3 each equal to D 1. With centres 2 and 3 describe the circles.

Note.—If parallels to the sides of the triangle be drawn through 1, 2, and 3, the angles 4, 5, and 6 of the triangle thus formed will be the centres for three more equal circles which may be inscribed in the triangle.

PROBLEM 130.—To inscribe four equal circles in a square, each touching one side and two circles.

Let A B C D be the given square.

Draw the diagonals and the diameters of the square.

Find the centre of the triangle A O B as in the preceding problem, and inscribe a circle. Set off the centres as shown, and describe the circles.

PROBLEM 131.—To inscribe in any regular polygon as many equal circles as the figure has sides, each touching one side and two circles.

Let the polygon in this case be a pentagon.

Divide the figure into as many equal triangles as it has sides, and inscribe a circle in each triangle as in the preceding problems.

PROBLEM 132.—To inscribe three circles in a spherical triangle, each touching one side and two circles.

Let A B C be the triangle. Bisect the angles at A and B by lines meeting at O, and draw C D. At D draw a tangent to meet the line through A. Bisect the angle thus formed and inscribe the circles as in *Prob.* 129.

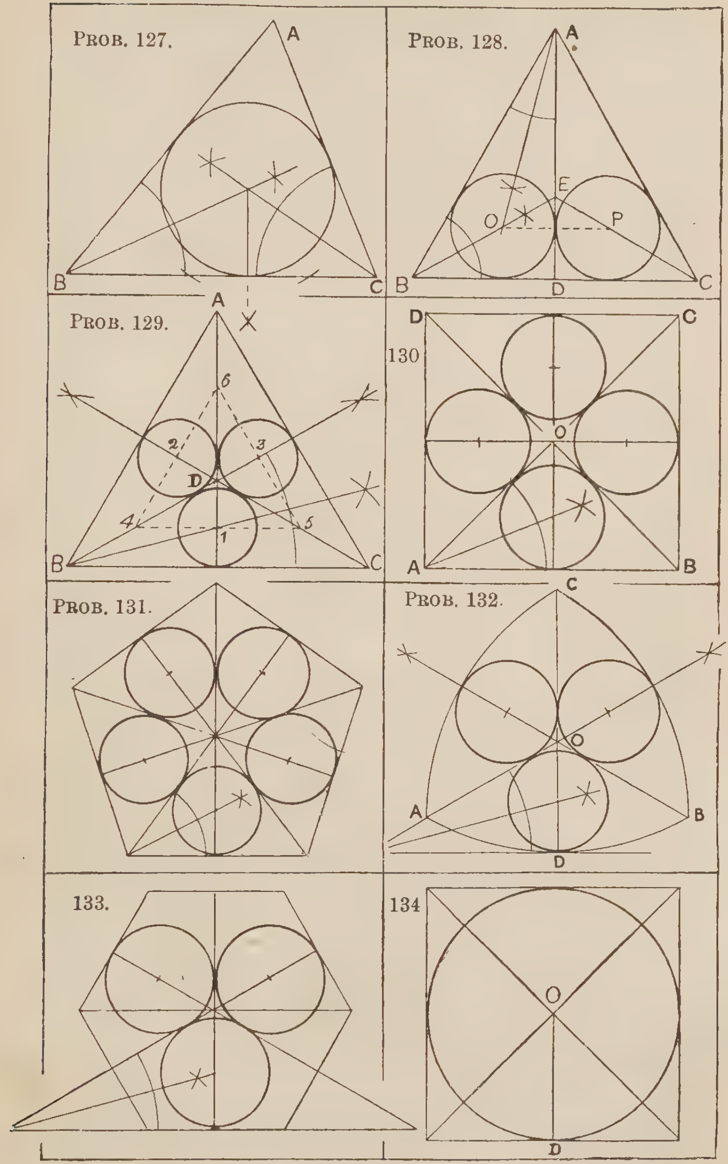
PROBLEM 133.—To inscribe three equal circles in a hexagon, each touching one side and two circles.

Draw the diameters of the figure. Produce two of these, and form a triangle as shown. Inscribe the circles.

Note.—Problems 128 to 134 all depend on Problem 127.

PROBLEM 134.—To inscribe a circle in a square.

Draw the diagonals to find the centre O. For the radius, draw O D perpendicular to the side. With centre O and radius O D describe the circle.



PROB. 127.
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PROB. 128.
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PROB. 129.
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PROB. 131.
PROB. 132.
C
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133.
134
O
D

PROBLEM 135.—To inscribe four equal circles in a square, each touching two sides and two circles.

Draw the diagonals and the diameters of the given square. Draw the diagonals A B, B C, C D, D A, giving the centres of each of the four squares. For the radius of the circles join E and F.

PROBLEM 136.—To inscribe a circle in any regular polygon.

Bisect two of its angles as in the given pentagon. O will be the centre, and O A the radius.

Notes.—1. Bisecting two of the sides by perpendiculars will also give the centre.

2. If the polygon has an even number of sides, the radius must be found by drawing a line from the centre perpendicular to one of the sides.

3. To describe a circle about the figure, O will be the centre and O B the radius.

PROBLEM 137.—To inscribe a circle in a rhombus.

Draw the diagonals to obtain the centre. From O draw O A perpendicular to one of the sides for the radius.

PROBLEM 138.—To inscribe a circle in a trapezion.

Draw the diagonal A B. Bisect one of the other angles. The point O is the centre. For the radius draw a perpendicular to one of the sides.

PROBLEM 139.—To inscribe three equal circles in an equilateral triangle, each touching two sides and two circles.

Bisect each of the angles by the lines A E, B D, and C F, thus dividing the triangle into three equal trapezions. Bisect the angle C F B, obtaining centre 1. Set off centres 2 and 3. To obtain the the radius join 1, 2. With centres 1, 2, 3 inscribe a circle in each trapezion.

Note.—To inscribe in **any regular polygon** as many circles as the figure has sides, each touching two sides and two circles; divide the polygon into as many equal trapezions as the figure has sides, and inscribe a circle in each as above.

PROBLEM 140.—In ANY regular polygon having an even number of sides, to inscribe half as many equal circles as the figure has sides, each touching two sides and two circles.

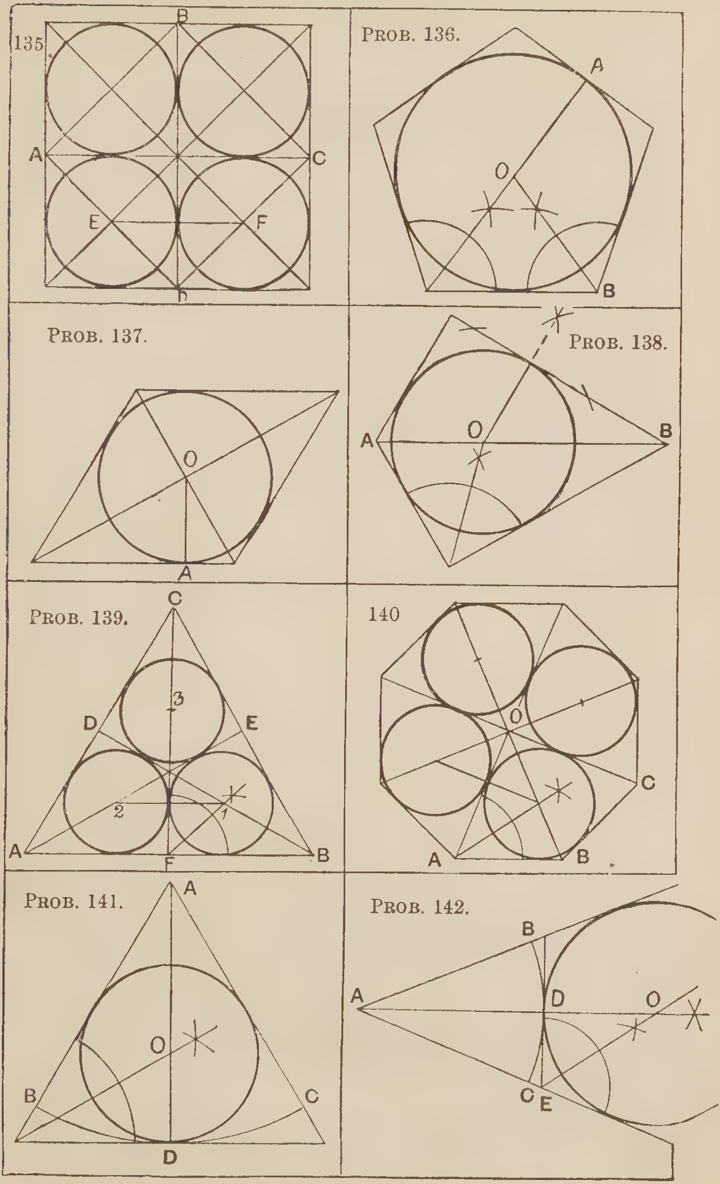
Let the given polygon be an octagon. Draw the diagonals. Inscribe a circle in the trapezion A B C O. Set off the other centres, and proceed as shown.

PROBLEM 141.—To inscribe a circle in a sector.

Let A B C be the sector. Bisect the angle B A C by the line A D. Through D draw a tangent to meet A B, and A C produced. Bisect one of the angles of the triangle thus formed. With centre O and radius O D inscribe the circle.

PROBLEM 142.—To describe a circle to touch the arc of a sector externally and the two radii produced.

Bisect the angle as before, and produce A D. Draw a tangent at D. Bisect the exterior angle at E by the line E O. With centre O and radius O D describe the circle.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PROB. 136.
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PROB. 137.
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PROB. 138.
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PROB. 139.
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PROB. 141.
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PROB. 142.
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E

PROBLEM 143.—To inscribe ANY number of equal circles in a circle.

Let it be required to inscribe four equal circles in the given circle.

Divide the circle into twice as many sectors as circles required. In this case divide the circle into eight equal parts. (Use the 45° set square.) In the sector O A B inscribe a circle (*Prob.* 141). With centre O and radius O C mark off the centres D, E, F, for the other circles.

PROBLEM 144.—To describe ANY number of equal circles about a given circle. Say six.

Divide the given circle into twice as many sectors as there are circles required, and produce the diameters. At A draw a tangent B C.

Bisect the exterior angle at C. D is the centre for one of the circles. Set off the other centres and describe the circles.

PROBLEM 145.—To describe about ANY regular polygon as many circles as the polygon has sides, each touching one side and two circles.

Let the given polygon be a hexagon. From each angle of the hexagon draw lines through the centre, and produce them both ways. Bisect the exterior angle at A by the line A C. C will be the centre for one of the circles. Set off the centres, and proceed as in the preceding problem.

Note.—Circles may be described about a triangle or a quadrilateral by applying the same principles.

PROBLEM 146.—To describe a circle having a given radius, and touching another circle either externally or internally in a given point.

Let A be the given radius, and B the point in the given circle. Find the centre O. Draw O B and produce it. Set off B 1 and B 2 equal to the given radius A. The circle described from 1 will touch externally, that from 2 internally.

PROBLEM 147.—To describe a circle touching a given circle at a given point and passing through another given point either within or without the circle.

1st. Let A be the given point in the circle, and B the given point *within* the circle. Find the centre O of the given circle and join O A. Draw A B and bisect.

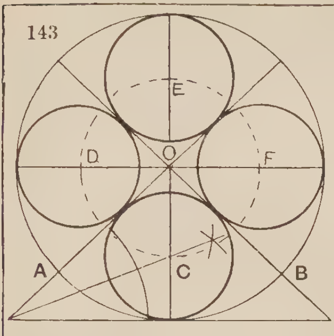
The point C, where the bisecting line intersects the radius O A, is the centre for the required circle.

2nd. Let A be the given point in the circle, and B the given point *without*.

Proceed as in the first case, the bisecting line meeting O A produced.

PROBLEM 148.—To describe a circle touching a given circle and a given straight line, and having a given radius.

Let A be the given radius and B C the given line. Draw a line parallel to B C at a distance equal to A. Find the centre O of the given circle. Draw any radius O 1 and produce it. Set off 1 2 equal to A. With centre O and radius O 2 intersect the parallel line in 3. With centre 3 and radius A describe the circle.



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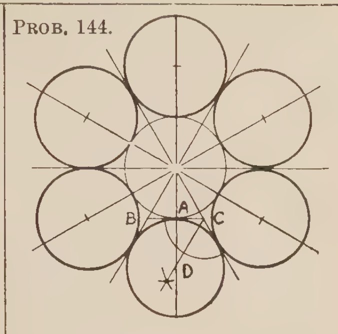
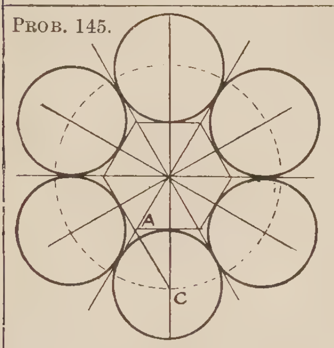


PROB. 144.



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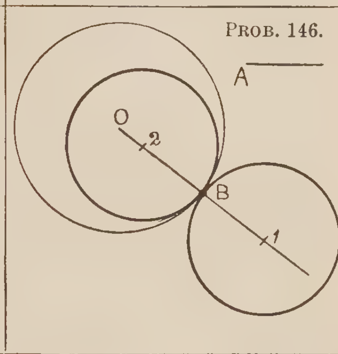
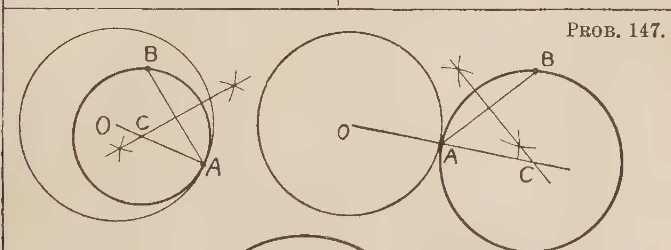


PROB. 146.



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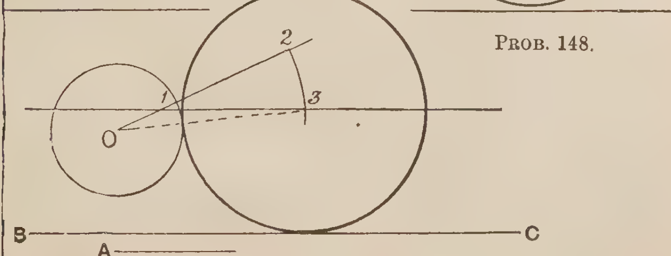


PROB. 148.

PROBLEM 149.—To describe a circle touching a given circle in a given point, and also touching a given straight line.

1st. Externally. Let A be the given point, and B C the given straight line. Find the centre D of the given circle. Join D A and produce it. At A draw a tangent A E to the circle. Set off E F equal to E A. At F draw F O perpendicular to B C. O is the centre for the required circle.

2nd. To include the given circle. Draw the radius D A, and produce it in the opposite direction. Draw the tangent A E. Set off E F equal to E A. Draw F O perpendicular to F E, and from centre O with radius O F describe the circle.

PROBLEM 150.—To describe a circle which shall touch a given circle and a straight line at a given point.

1st. Externally. Let A be the point in the given line.

Find B the centre of the given circle, draw B C perpendicular to the given line, and produce it to D. At A draw a perpendicular. Draw D A cutting the circumference of the circle in E. From B through E draw a line meeting the perpendicular in F. With centre F and radius F A describe the circle.

2nd. To include the given circle. Find the centre B, and draw the perpendiculars from B and A as before. Draw A D, and produce to cut the circumference in E. Draw E B, and produce to meet the perpendicular in F. With centre F and radius F A describe the circle.

PROBLEM 151.—To describe a circle of a given radius, touching two given circles externally.

Let A be the given radius. Find the centres B and C of the given circles, draw radii B D, C E and produce them. Set off D F, E G equal to A. With centre B and radius B F describe an arc. With centre C and radius C G intersect in O. With centre O and radius A describe the circle.

Note.—If the arcs were continued, the centre for another circle would be obtained.

PROBLEM 152.—To describe a circle of a given radius, touching two given circles and including them.

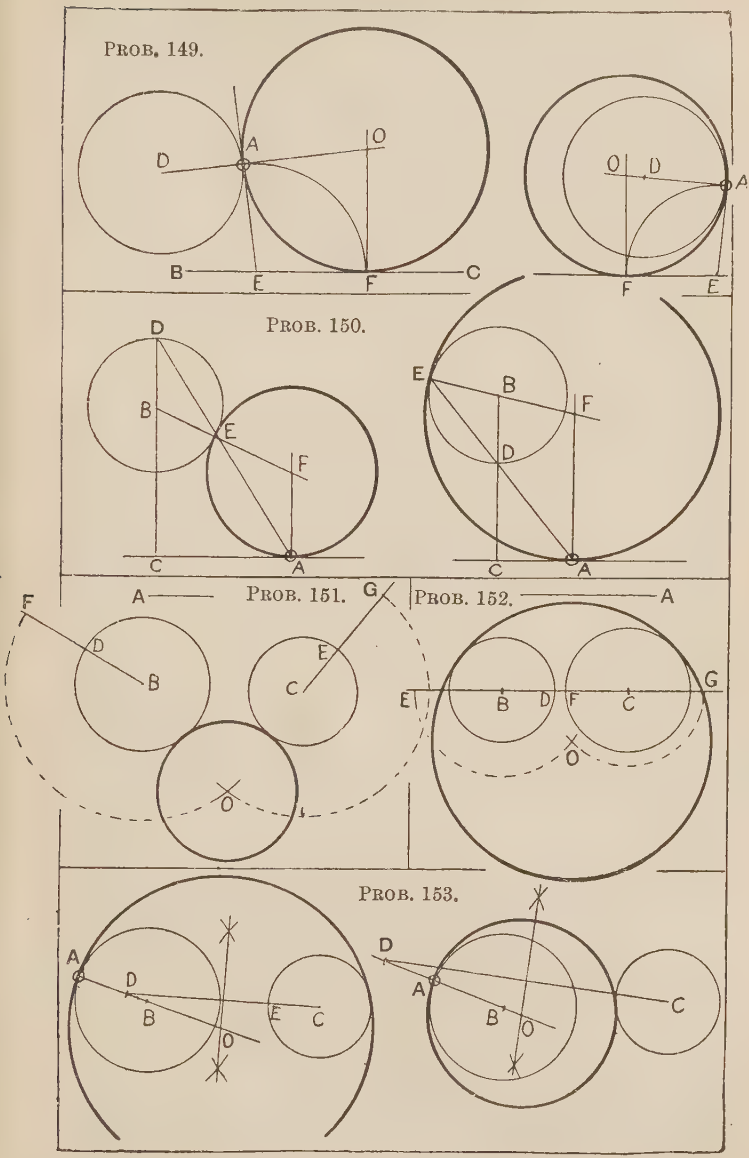
Join the centres of the given circles and produce each way. Set off D E and F G equal to the given radius A. With centres B and C, and radii B E and C G describe arcs intersecting in O. With centre O and radius A describe the circle.

Note.—As in the preceding problem, two circles may be described.

PROBLEM 153.—To describe a circle to touch two given circles, one of them in a given point.

1st. To include both circles. Find the centres B and C. Join A B and produce it. Make A D equal to C E, the radius of the other circle. Draw C D and bisect. O will be the required centre, and O A the radius.

2nd. To include one circle. Find the centres. Draw A B, and produce it both ways. Set off A D equal to the radius of the other circle. Draw D C, and bisect it. With centre O and radius O A describe the circle.



PROB. 149.

PROB. 150.

PROB. 151.

PROB. 152.

PROB. 153.

PROBLEM 154.—To describe three circles touching each other, their radii being given.

Let A, B, C be the radii. Draw any line and from the extremity A set off A C, C B equal to A and B, and describe two of the circles. Set off C E, C D equal to C. From centres A and B with radii A D and B E describe arcs intersecting in O. With centre O and radius C describe the third circle.

PROBLEM 155.—To describe three circles touching each other, the position of their centres being given.

Let A, B, C be the centres of the circles. Join the points, and find the centre of the triangle thus formed. From this centre draw a perpendicular D E to one of the sides.

With centre B and radius B E describe a circle. With centre C and radius C E describe the second circle. With centre A and radius A F describe the third circle.

Note.—The centres must not be in the same straight line.

PROBLEM 156.—To inscribe within a given circle two other circles of given radii, touching each other and the given circle.

Let A and B be the given radii. Draw the diameter, and find the centre F of the given circle. Set off C D equal to A, and describe one of the circles. From points E and J set off E G and J H equal to B. With centre D and radius D G describe an arc. With centre F and radius F H intersect the arc in O. With centre O and radius B describe the other circle. If the arcs be continued on the other side of C J, the centre for another circle will be obtained.

PROBLEM 157.—To describe a circle which shall pass through two given points and touch a given circle.

Let A and B be the two given points. Describe any circle passing through A and B, and cutting the given circle in points C and D. Draw C D to meet A B produced in E. From E draw a tangent E F to the given circle (*Prob.* 69). F will be the point where the described circle will touch the given circle. Describe a circle passing through the points A, B, and F.

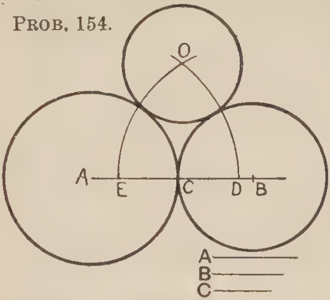
PROBLEM 158.—To describe a circle which shall pass through two given points and touch a given straight line.

1st. When the line joining the two points is parallel to the given line. Join the two given points A and B. Bisect A B by a perpendicular line meeting the given straight line in C. Draw A C and bisect it. With centre O and radius O C describe the circle.

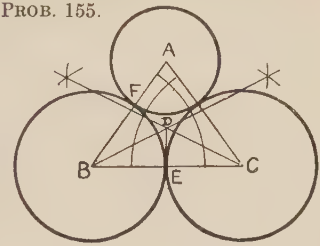
2nd. When the line joining the two points is at an angle to the given line.

Join the points A and B, and produce the line to meet the given line in the point C. Find a mean proportional between A C and C B by bisecting A B, describing a semicircle with D C as radius, and drawing the perpendicular A E. Make C F equal to A E. At F draw the perpendicular F O to meet the perpendicular from D. With centre O and radius O F describe the circle.

PROB. 154.



PROB. 155.



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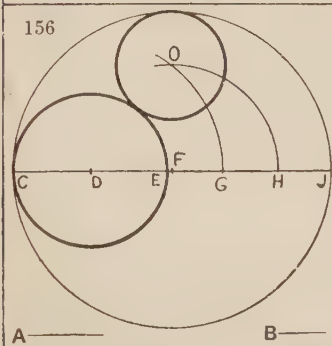

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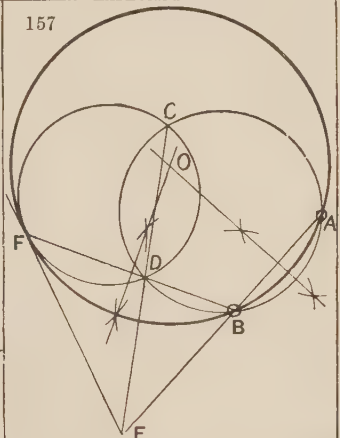
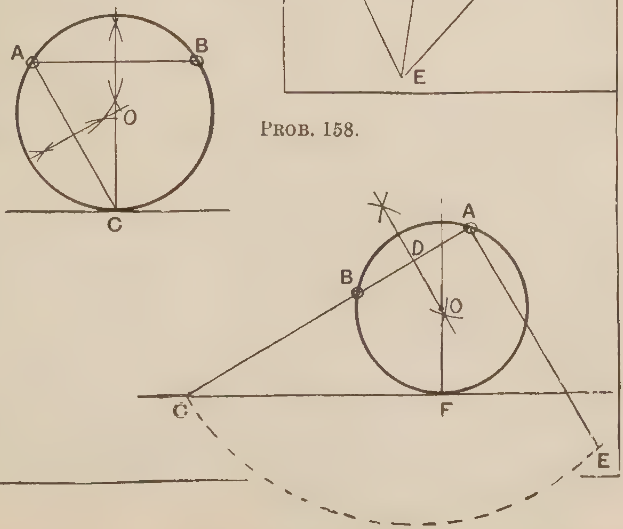


PROB. 158.



PROBLEM 159.—From a given point without a circle to draw a straight line cutting the circumference in two points, so that the segment intercepted between the two points shall equal a given distance.

Let A be the given point and B the given distance.

From a point C in the circumference set off C D equal to B, and draw the chord C D. Find the centre O of the given circle, and from it draw O E perpendicular to C D. With centre O and radius O E describe a circle. From A draw A G touching the described circle. Then F G will equal the given distance B, because all chords of a circle equidistant from the centre are equal. (EUC. III. 14.)

PROBLEM 160.—Describe a circle of 1·25″ diameter touching each pair of adjacent lines *oa*, *ob*, *oc*, *od*, produced if necessary. Describe two circles touching the three circles.

Describe a circle touching each pair of lines, having a radius of $\frac{5}{8}''$ by Problem 123. The only two circles that can touch the three are the circles touching them externally and internally.

From *o* draw a line passing through the centre of one of the circles. With centre *o* and radii *o e*, *o f* describe the circles.

Notes—1. If the angles between each pair of adjacent lines are unequal, then find the centre for the touching circles by bisecting the lines joining the centres of the circles. The point where the two bisecting lines meet will be the centre.

2. In working problems of a similar character to Problems 146–160, the following facts should be constantly borne in mind:

a. 'If two circles touch one another either internally or externally, the straight line, or the straight line produced, which joins their centres passes through the point of contact.' (EUC. III. 11 and 12.)

b. 'Equal straight lines in a circle are equally distant from the centre; and those which are equally distant from the centre are equal.' (EUC. III. 14.)

c. 'If a straight line touch a circle, the straight line drawn from the centre to the point of contact is perpendicular to the line touching the circle.' Also the converse of this. 'If a straight line touch a circle, and from the point of contact a straight line be drawn at right angles to the touching line, the centre of the circle shall be in that line.' (EUC. III. 18 and 19.)

3. There is a large variety of problems of this kind, and in cases of difficulty a sketch of the required figure should be made, assuming it to be completed. Then by working backwards step by step, endeavour to find out what principles must be used to secure this result.

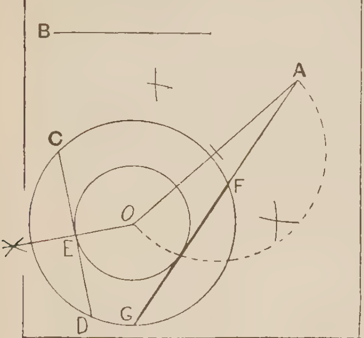
PROBLEM 161.—The given figure is made up of circular arcs, all of $\frac{3}{4}''$ radius. Draw it full size. (*Sc.* 1889.)

First draw the equilateral triangle A B C of $1\frac{3}{8}''$ sides. With centres A, B, C and radius $\frac{3}{4}''$ describe arcs intersecting at D, E, F, giving three of the centres. The points *a*, *b*, *c* are obtained in each case by describing two circles tangential to those already obtained. Join the points D, E, F, and produce the lines each way. Set off on each line $\frac{3}{4}''$ as shown, and from the centres thus obtained describe arcs completing the figure.

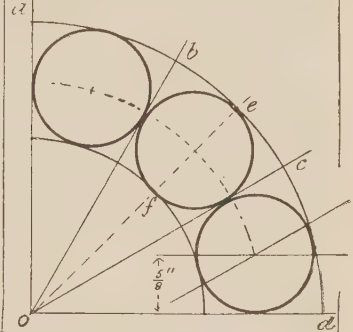
PROBLEM 162.—Draw the given geometrical pattern.

Describe a circle of $3\frac{1}{2}''$ diameter. From the extremity of the diameter set off $\frac{1}{8}''$ and describe the inner circle. Divide the circumference into six equal parts, and draw the diameters. From *a* set off $1\frac{3}{4}''$ and describe one of the smaller circles. Set off the centres for the other two inner circles and describe them. From the same centres describe the remaining circles. Omit the parts where the lines are not continuous.

PROB. 159.



PROB. 160.



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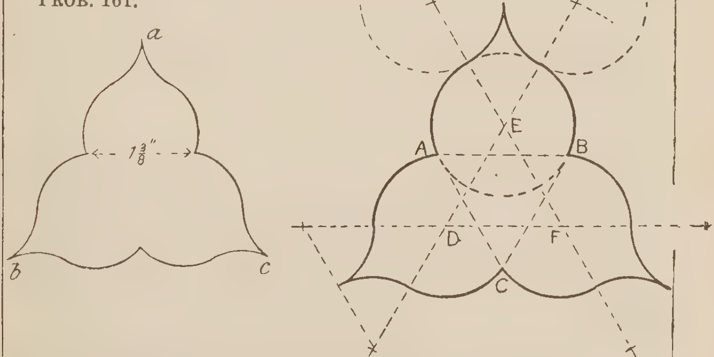

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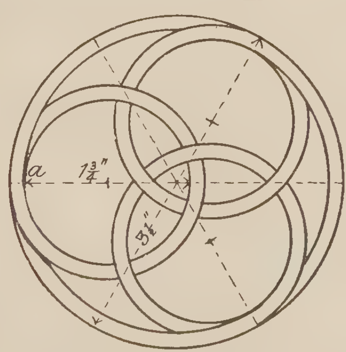

PROBLEM 163.—Draw the geometrical pattern shown, adhering strictly to the figured dimensions, and showing the construction lines.

It will be seen that the figure is formed by six equal circles, each touching two other circles. But when six equal circles are inscribed in a circle, a seventh equal circle may always be inscribed touching the six circles internally. The problem then resolves itself into describing a circle of $\frac{3}{4}''$ radius, placing six equal circles about it (*Prob.* 144), and omitting those portions of the circles which are not needed. Describe a circle of $\frac{3}{4}''$ radius, divide its circumference into 12 equal parts. Find the centres for the circles, and complete as shown.

Note.—Problems 161–163 indicate a few of the numerous ways in which the preceding problems on circles may be applied.

PROBLEM 164.—To inscribe a semicircle in an isosceles triangle.

Bisect the angle A C B by the line C D. Bisect the angle C D B by the line D E.

Draw E F parallel to A B, and on it describe a semicircle.

PROBLEM 165.—To inscribe three equal semicircles in an equilateral triangle having their diameters adjacent, and each touching one side of the triangle.

Bisect each angle of the triangle by the lines A D, B E, and C F.

Bisect the angle C F B by the line F G. From G draw G J, G H parallel to A B, B C. Join J and H. On the lines H G, G J, J H describe semicircles forming a *trefoil* of semicircular arcs.

PROBLEM 166.—To inscribe four equal semicircles in a square, having their diameters adjacent, and each touching one side of the square.

Draw the diagonals and diameters of the given square. Bisect one of the angles at A, and obtain the inner square. Describe a semicircle on each side of this square, forming a *quatrefoil* of semicircular arcs.

PROBLEM 167.—To inscribe within any regular polygon as many semicircles as the figure has sides, each touching one side and having their diameters adjacent.

Let A B C D E be the given polygon, in this case a pentagon. Divide the polygon into equal isosceles triangles. Inscribe a semicircle in each as before, forming a *cinquefoil* of semicircular arcs.

Notes.—1. In Problems 165–173 notice that the foiled figures are made by *semicircular* arcs. In Problems 174–177 they are formed by *tangential* arcs. The problems relating to the inscription and circumscription of circles and foiled figures are exceedingly useful in geometrical design, as they furnish the leading lines for window tracery and ornamental forms of various kinds.

2. Problems 165–168 depend upon the same principles of construction as those used in Problem 164.

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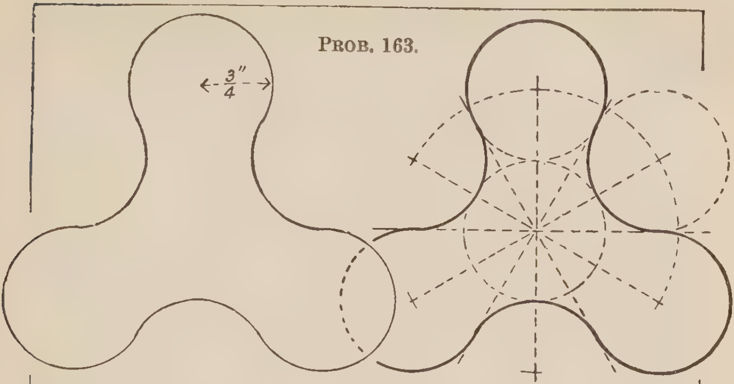

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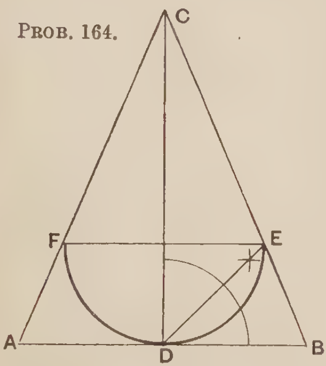

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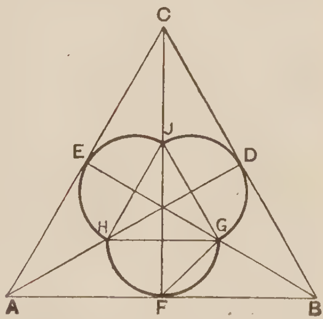

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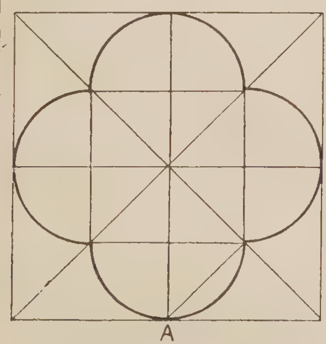

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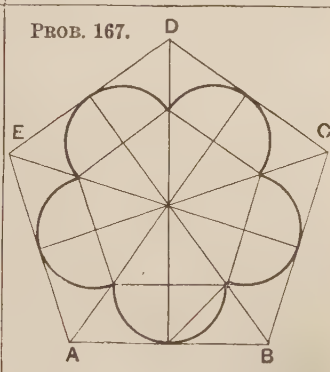

PROBLEM 168.—To inscribe any number of equal semicircles in a circle, having their diameters adjacent, and each touching the circumference.

Divide the circumference of the given circle into twice as many parts as there are semicircles required. (In this case four.) Draw the diameters. Draw a tangent at A and bisect the angle. Set off the distance O B on the alternate diameters, and draw the square. On each side of the square describe a semicircle.

PROBLEM 169.—To inscribe a semicircle in a square.

Draw the diagonals of the square, and on one of them A B describe a semicircle. From O the centre of the square draw O C at right angles to the side of the square. Draw C F, and from D where the line C F cuts the side of the square draw D E parallel to C O. Through E draw a line parallel to A B, and on it describe the required semicircle.

PROBLEM 170.—To inscribe four equal semicircles in a square, having their diameters adjacent, and each touching two sides of the square.

Draw the diagonals and diameters of the given square, and inscribe a semicircle in one of the four squares thus obtained by the preceding problem. Complete the inner square, and describe a semicircle on the other three sides.

PROBLEM 171.—To inscribe a semicircle in a trapezion, or kite.

Draw the diagonals of the given figure. On A B, the shorter diagonal, describe a semicircle. From O draw O C perpendicular to the side of the trapezion. Draw C F, and from D draw D E parallel to C O. Through E draw a line parallel to A B, and upon it describe the required semicircle.

PROBLEM 172.—To inscribe three equal semicircles in an equilateral triangle, having their diameters adjacent, and each touching two sides of the triangle.

Bisect each angle of the triangle by lines dividing the triangle into three equal trapezions. Inscribe a semicircle in the trapezion A F B G by the preceding problem. Complete the inner equilateral triangle, and upon the other two sides describe semicircles.

PROBLEM 173.—To inscribe in any regular polygon a number of equal semicircles, having their diameters adjacent, and each touching two sides of the polygon.

Let the given polygon be a regular hexagon. Draw the diameters and diagonals of the figure, cutting it into six equal trapezions, and inscribe a semicircle in each, as shown.

Notes.—1. Problems 170–173 depend upon the same principles of construction as those used in Problem 169.

2. Notice that in all cases the line joining the points C and F is drawn from C to the angle of the quadrilateral which is opposite to the semicircle described upon A B.

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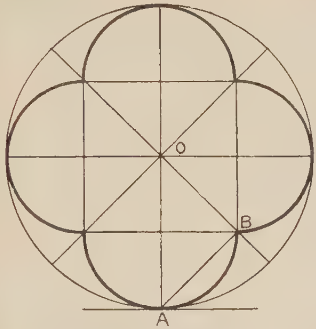

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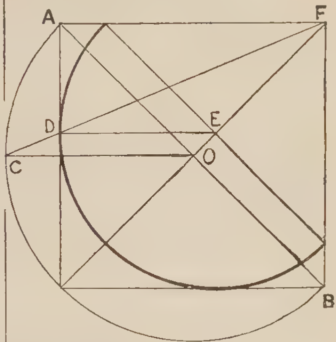

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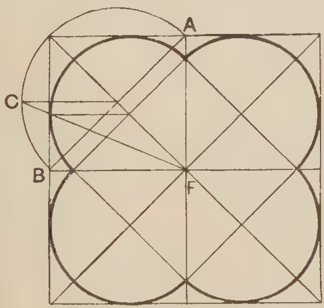

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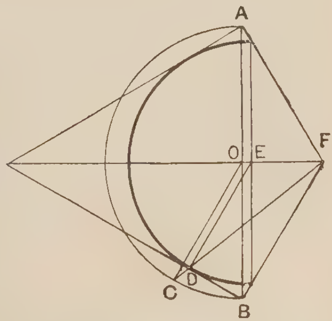

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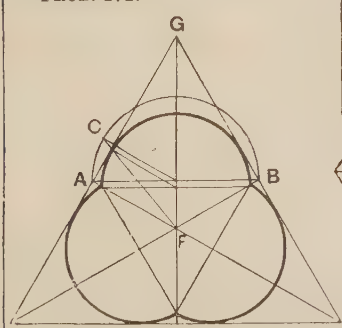

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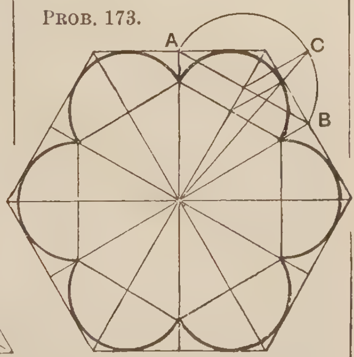

PROBLEM 174.—To describe a trefoil of tangential arcs, the radius of the arc being given.

Construct an equilateral triangle having each of its sides double the given radius. From each angle with the given radius describe the arcs.

PROBLEM 175.—To describe a quatrefoil of tangential arcs, the radius being given.

Construct a square having each of its sides double the given radius and describe the arcs as shown.

Note.—The same principle may be employed in the construction of all foiled figures formed by tangential arcs.

PROBLEM 176.—About any regular polygon, to construct a foiled figure of tangential arcs.

Let the given polygon be a regular hexagon. Bisect one side to obtain the radius, and proceed as in the preceding problems.

Note.—These problems are identical with the problems connected with the describing of a number of equal circles each touching two others. For example, in Problem 140, if the centres of the circles be joined, and the portions of the circles enclosed by the square thus formed be removed, we have the figure of Problem 175 inscribed in an octagon.

PROBLEM 177.—To draw the given geometrical pattern to the figured dimensions.

The inner arcs are identical with *Prob.* 176. Construct a hexagon having each side double the given radius ($\frac{5}{8}''$), and from each angle describe the tangential arcs. From the same centres with a radius of $\frac{3}{4}''$ describe the outer arcs.

CHAPTER XII

THE INSCRIPTION AND CIRCUMSCRIPTION OF RECTILINEAL FIGURES

PROBLEM 178.—To inscribe or describe an equilateral triangle in or about a circle.

Draw a diameter A B, and find its centre. Set off A C, A D equal to the radius. Draw B C, B D, C D.

For the triangle about the circle find the points of the inscribed triangle, and through each draw a parallel to the opposite side.

PROBLEM 179.—To inscribe or describe a square in or about a circle.

Draw two diameters at right angles to each other, and join the ends. For the square about the circle, with centres A, B, C, D, and radius A O, describe arcs intersecting without the circle and join the points; or through the points A, B, C, D draw parallels to the diagonals.

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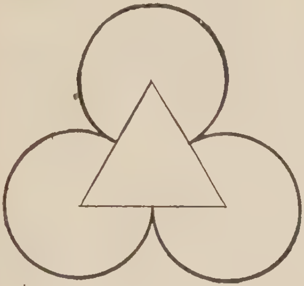

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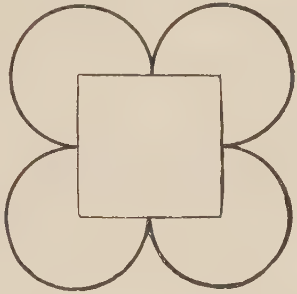

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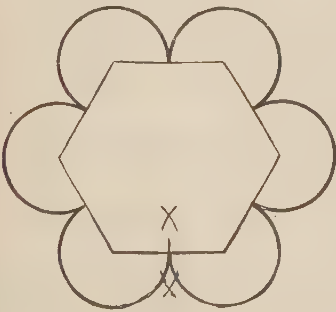

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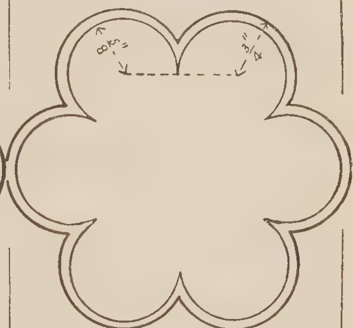

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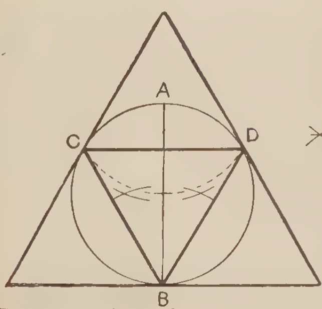

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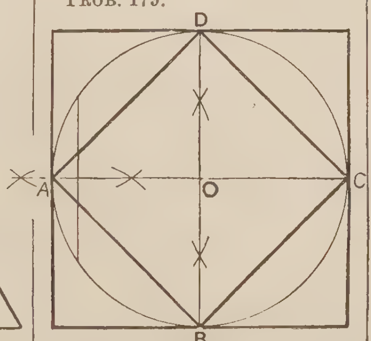


PROBLEM 180.—To inscribe an equilateral triangle in a square.

Draw the diagonal A B. On A B construct an equilateral triangle. From C draw C E, C D, parallel to the sides of the triangle. Join D and E.

PROBLEM 181.—To inscribe an equilateral triangle in a pentagon.

Find B, the middle point of one side, and join it with A, the opposite angle. On each side of A B construct an angle of 30° (*Prob.* 33), and join C D.

Notes.—1. This method may be applied to the square by making an angle of 30° on each side of the diagonal.

2. The triangle may also be obtained, as in Problem 180, by describing an equilateral triangle on E F, and drawing parallels from A.

PROBLEM 182.—To describe an equilateral triangle about a square.

On one side A B of the given square describe an equilateral triangle A B E. Produce E A, E B to meet the side C D produced.

PROBLEM 183.—About a given square to describe a triangle similar to a given triangle.

Let A B C D be the given square and E F G the given triangle. This problem depends upon exactly the same principle as the preceding. On A B construct a triangle similar to the triangle E F G. Produce the sides to meet C D produced.

PROBLEM 184.—About a given triangle to describe another triangle similar to a given triangle.

On A B construct a triangle similar to the triangle D E F.

Through C draw a line parallel to A B, and produce G A, G B to meet it.

PROBLEM 185.—Within a given triangle to inscribe another triangle similar to a given triangle.

On A C, a side of the given triangle, construct a triangle similar to the triangle D E F. Draw G B. From H draw H J and H K parallel to G C and G A. Join J and K. Then H J K is the required triangle.

PROBLEM 186.—Within a given circle to inscribe a triangle similar to another triangle.

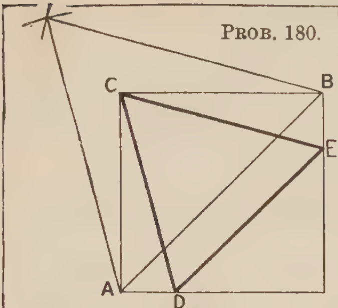
At any point D in the circumference of the given circle draw a tangent. Make the angle E D F equal to the angle A B C, and the angle G D H equal to the angle B C A. Join F H. Then D F H is the required triangle, the angle at F being equal to the angle at C, and the angle at H to the angle at B. (Euc. iv. 2.)

PROBLEM 187.—Within a given circle to inscribe a quadrilateral figure similar to a given one.

Note.—A quadrilateral figure can only be inscribed in a circle when the sum of the opposite angles equals two right angles.

Let the given quadrilateral have angles of 100°, 70°, 80°, and 110°, as shown.

Through any point E draw a tangent F G. Make the angle H E G equal to the angle A D C, G E J equal to C A B, and F E K equal to C A D. Draw J H, H K.



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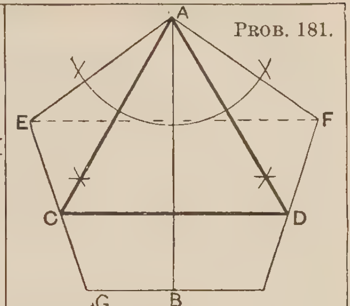

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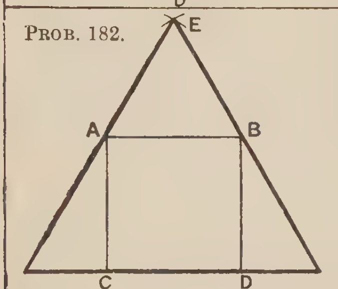

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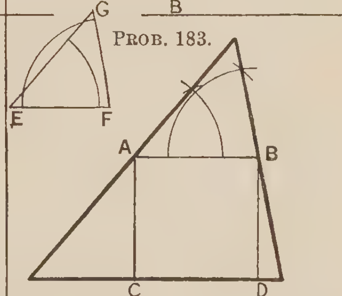

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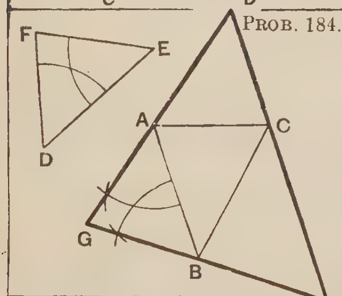

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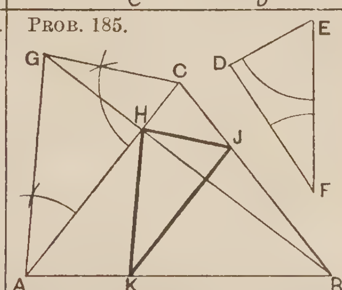

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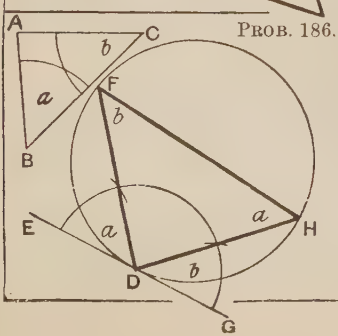

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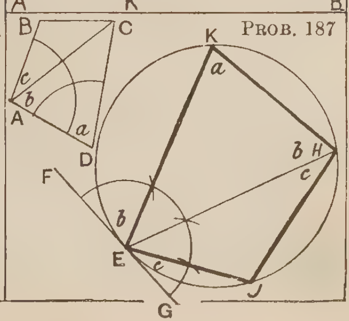


PROB. 187

PROBLEM 188.—About a given circle to describe a triangle similar to a given triangle.

Produce the base A B of the given triangle. Find the centre O of the given circle, draw any radius O G, and produce it. Construct the angle F O G equal to the exterior angle C B D, and the angle H O G equal to the exterior angle C A E. Produce O H, O F, and draw tangents as shown. (EUC. IV. 3.)

Note.—Angle F O G + angle F J G = angle C B D + angle C B A = two right angles, because the angles at F and G are right angles.

PROBLEM 189.—In a given square to inscribe an isosceles triangle, the base being given.

Draw the diagonals A C and B D of the given square, and on A C set off A F equal to the given base E. Draw F G parallel to A B and G H parallel to A F. Draw D H and D G.

Note.—G H = A F. (EUC. I. 34.)

PROBLEM 190.—To describe a square about an isosceles triangle,

Bisect the base B C of the given triangle by the line A D. On B C describe a semicircle. Then A D will be a diagonal of the required square. Draw D C and D B of indefinite length, and from A draw parallels to meet them.

PROBLEM 191.—In a given hexagon to inscribe an isosceles triangle, the base being given.

Draw the diagonal A B of the given hexagon. Draw C D, one of the diameters, at right angles to A B. Set off D F equal to E. Draw F G parallel to B D, and G H parallel to C D. Draw A G and A H.

PROBLEM 192.—Within a given circle to inscribe an isosceles triangle, the base being given.

Draw two diameters of the given circle, A B and C D, at right angles to each other. Make O F equal to half the given base E. Draw F G parallel to A B, and G H parallel to C D. Draw A G and A H.

Note.—A similar method may be used for inscribing an isosceles triangle in a square, rhombus, or polygon.

PROBLEM 193.—To inscribe a square in a triangle.

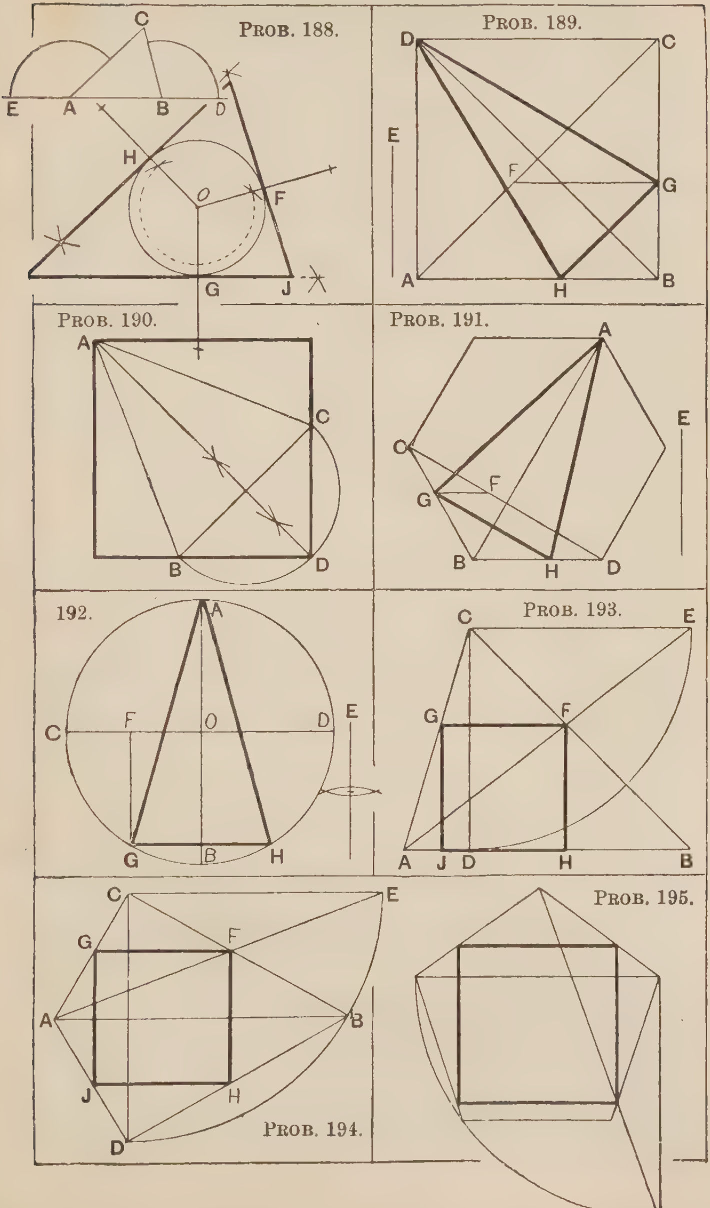
Draw C D perpendicular to A B. From C draw C E parallel to A B and equal to C D. Draw A E. From F draw F G parallel to A B, and F H parallel to C D. From G draw G J parallel to C D. Then G F H J is the required square.

PROBLEM 194.—To inscribe a square in a trapezion.

Draw the diagonals A B, C D. Draw C E parallel to A B, and equal to C D. Draw A E. From F draw F G parallel to A B, and F H parallel to C D. From G draw G J parallel to C D, and join J and H.

PROBLEM 195.—To inscribe a square in a pentagon.

If two sides of the pentagon be produced until they meet, a trapezion will be formed. The same construction as in *Prob.* 194 will then apply.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C F O D E
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A J D H B
C E
G F
A B
J H
PROB. 194.
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PROB. 195.

PROBLEM 196.—To inscribe a square in a sector.

Join B and C. Draw C D perpendicular to C B and equal to it.

Draw A D, and from E draw E G parallel to B C, and E F parallel to C D.

Draw G H and F H parallel to E F and E G.

PROBLEM 197.—To inscribe a square in a segment.

Bisect the chord A B of the segment. Draw B D equal to A B and perpendicular to it. Draw D C. From E draw E F parallel to B D and E G parallel to A B. Draw G H parallel to E F.

PROBLEM 198.—Within a given square to inscribe another square, one angle to touch a side at a given point.

Let A be the position of one angle. Draw the diagonals of the given square.

With centre O and radius O A describe a circle. Join the points A, B, C, and D.

Note.—If the length of the diagonal be given, proceed in a similar manner, taking half the given diagonal as radius.

PROBLEM 199.—To inscribe a square in a rhombus.

Draw the diagonals, and bisect the angles thus formed.

Join A, B, C, and D.

PROBLEM 200.—To inscribe a square in a hexagon.

Draw the diagonal A B, and bisect it at right angles by the diameter C D.

Complete as in the preceding problem.

Note.—To inscribe a square in an octagon, join the alternate corners.

PROBLEM 201.—To inscribe a rhombus in a parallelogram, having one of its angles at a given point.

Let A be the given point. Draw the diagonals of the parallelogram.

From A draw A B passing through the centre O of the parallelogram.

Bisect A B by a line C D at right angles to it. Draw A C, C B, B D, and D A.

PROBLEM 202.—To inscribe a regular hexagon in an equilateral triangle.

Bisect each of the angles of the given triangle by the lines A E, B F, and C D. With centre O and radius O A describe a circle. Draw D E, E F, and F D.

PROBLEM 203.—Within a given triangle to inscribe a rectangle, the length of one side being given.

Let A B C be the given triangle and D the given side of the rectangle.

Set off A E equal to D. Draw E F parallel to A C.

From F draw F G parallel to A B. Draw F H and G J perpendicular to A B.

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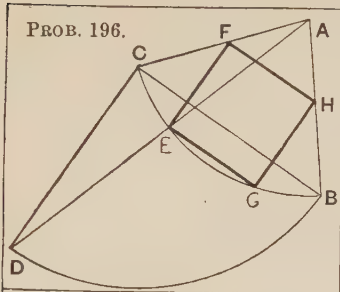

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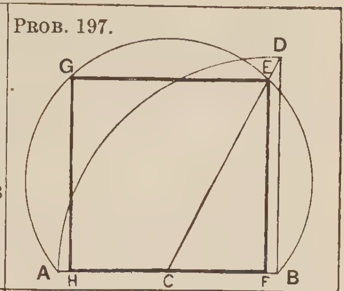

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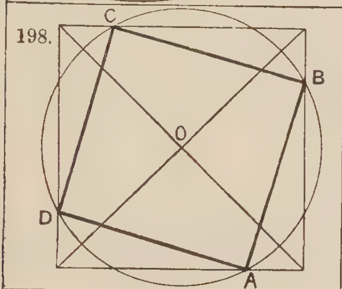

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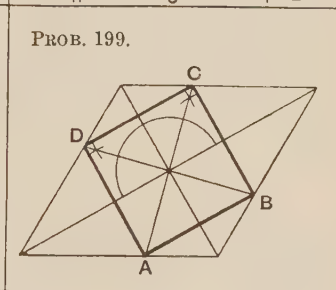

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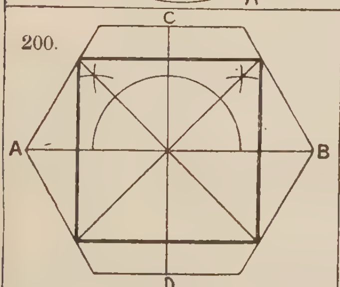

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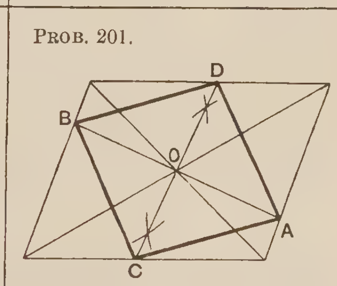

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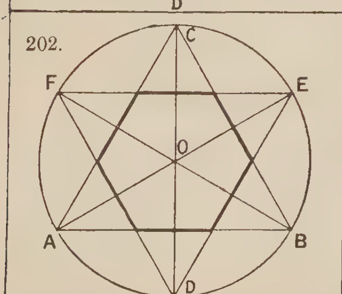
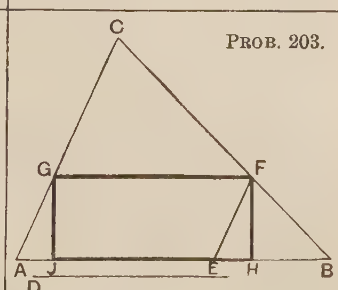


PROB. 203.



PROBLEM 204.—Within any given quadrilateral to inscribe a parallelogram, having given the position of one angle.

Let E be the position of one angle. Draw the diagonals A C, B D. Draw E F parallel to A C, E H and F G parallel to B D. Join G and H. E F G H will be the required parallelogram.

PROBLEM 205.—Within any given quadrilateral to inscribe a parallelogram, having given the length of one side.

Let E be the length of one side. Draw the diagonals A C, B D. On one of them set off A F equal to E.

From F draw F H parallel to A B. Draw H G parallel to A C, H K and G J parallel to B D. Join J K.

Notes.—1. G H=A F. (Euc. I. 34.)

2. The same construction will apply for inscribing a rectangle in a square, rhombus, or trapezoid.

PROBLEM 206.—Within a given triangle or any regular polygon to inscribe another similar figure, having its sides parallel to and equidistant from those of the given figure, the length of one side being given.

Let A B C be the given triangle, and D the length of one of the sides of the required triangle. Bisect the angles and obtain the centre E. Set off A F equal to D. Draw F G parallel to A E, G H parallel to A B, H J parallel to A C, and G J parallel to B C.

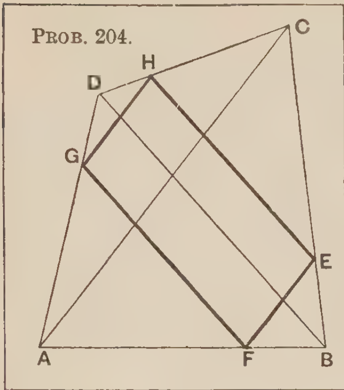
Note.—The construction for the inscription of a hexagon within a hexagon is also shown, A being the length of one side of the inscribed figure.

PROBLEM 207.—About a given triangle or any regular polygon to describe another similar figure, having its sides parallel to and equidistant from those of the given figure, the length of one side being given.

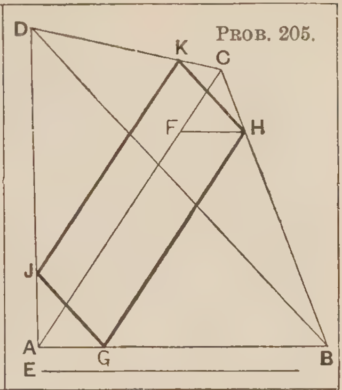
Let A B C be the given triangle, and D the length of one of the sides of the required triangle.

Find the centre as before, and produce the lines bisecting the angles. Produce A B, and from A set off A F equal to D. Draw F G parallel to A E, G H parallel to A B, H J parallel to A C, and G J parallel to B C.

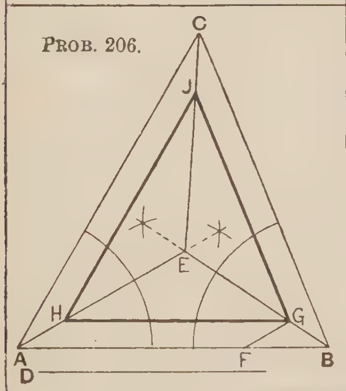
Note.—The construction for the description of a square about a square is also shown.



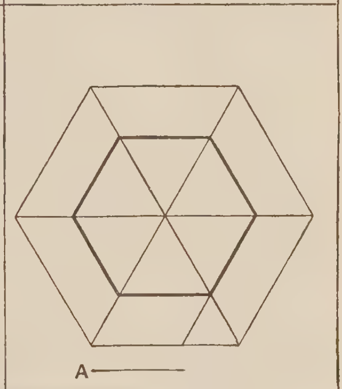
Prob. 204.

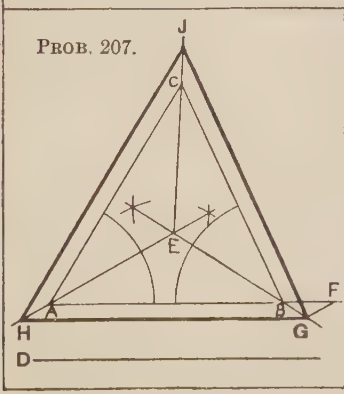


Prob. 205.

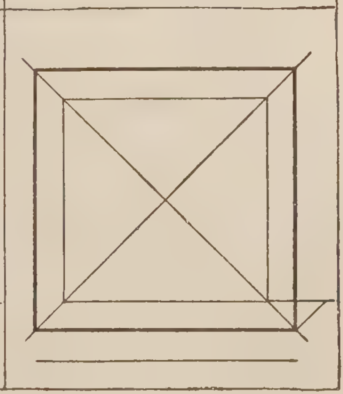


Prob. 206.





Prob. 207.



EXERCISES

CHAPTER XI

1. To describe a circle touching two lines and passing through a point between them. (*Art.*) (*Prob.* 125.)

2. Describe a circle of 3″ diameter, and within it inscribe six equal semicircles. (*Art.*)

3. Describe two circles touching each other and having radii of 1″ and $\frac{3}{4}$″ respectively. Draw a third circle having a radius of $\frac{1}{2}$″ to touch the other two circles. (*Art.*) (*Prob.* 152.)

4. Within a given equilateral triangle of 3″ sides inscribe three equal semicircles, having their diameters adjacent and each touching one side of the triangle. (*Art.*)

5. Within a given circle of $1\frac{1}{2}$″ radius inscribe four equal circles. (*Art.*)

6. Within a square of $2\frac{7}{8}$″ sides inscribe four equal semicircles each touching two sides of the square. (*Art.*)

7. Within a square of $2\frac{1}{2}$″ sides inscribe four equal circles, each touching two sides and two circles. (*Art.*)

8. Describe a circle of $\frac{3}{4}$″ radius which shall touch a given circle and a given straight line. (*Art.*) (*Prob.* 148.)

9. Construct a quatrefoil of tangential arcs of $\frac{1}{2}$″ radius. (*Art.*)

10. Describe a circle to enclose two other given circles. (*Art.*)

11. Describe a triangle with sides of 3″, 2″, and $2\frac{1}{2}$″ respectively, and within it inscribe three circles each touching one side and two circles. (*Prob.* 132.)

12. Within a given isosceles triangle inscribe two equal circles touching each other and two sides of the triangle. (*Art.*)

13. Two parallel lines **A B**, **C D** are 2″ apart and $1\frac{1}{2}$″ long. Describe a circle to pass through points **A** and **B** and touch **C D**. (*Art.*)

14. Within a given pentagon inscribe five tangential arcs each touching two sides of the pentagon. (*Art.*) (*Note, Prob.* 139.)

15. About a square of 1″ sides describe four equal circles each touching a side of the square and two circles. (*Art.*) (*Prob.* 145.)

16. Describe a circle $\frac{3}{4}$″ radius to pass through a point **A** and touch a given straight line. (*Art.*)

17. Construct a rhombus of $2\frac{3}{4}$″ sides and within it inscribe four equal circles each touching one side and two circles.

18. Describe a triangle about a circle of 1″ diameter, having angles of 30° and 105°.

19. About a circle of $\frac{1}{2}$″ radius place five equal circles each one touching two others and the given circle.

20. Draw two right lines meeting at an angle of 38°. Describe a circle of $\frac{3}{4}$″ radius to touch these lines. (*Sc.*, 1877.)

21. Draw three equal circles of ·75″ radius, each touching the other two. (*Sc.*, 1870.)

22. Two circles of 1″ and ·5″ radius respectively have their centres 2·5″ apart. Draw a circle of 1·5″ radius to touch both, but to contain the smaller one (*Sc.*, 1871.) (*Prob.* 153.)

23. Draw three circles, each touching the other two, their radii being ·5″, ·75″, and 1″, respectively. (*Sc.*, 1876.) (*Prob.* 154.)

24. Construct an equilateral triangle of $2\frac{1}{4}$″ sides. On each side as diameter describe a circle. Circumscribe the three circles by a circle. (*Sc.*, 1877.)

25. Describe a circle of $2\frac{1}{2}$″ radius touching two given circles of 1″ and $\frac{3}{4}$″ radius, and having their centres $2\frac{1}{4}$″ apart. (*Sc.*, 1877.)

26. Inscribe a circle in a rhombus of 2″ side and $2\frac{1}{4}$″ diagonal. (*Sc.*, 1878.)

27. Draw three circles of 1″, 1·25″, and 1·5″ diameter, each touching the other two externally. Draw a circle which shall be touched internally by the largest and smallest of these three circles. (*Sc.*, 1881.) (*Prob.* 152.)

28. Draw two circles touching the same straight line at points 2·5″ apart and touching one another, the radius of the smaller circles to be 1″. (*Sc.*, 1882.) (*Prob.* 150.)

29. Construct a square of $4\frac{1}{4}$″ side, and place in it four equal circles, each touching one side and two diagonals. (*Sc.*, 1883.)

30. Draw two lines cutting each other at 57°, and describe four circles of $2\frac{1}{8}$″ diameter, each touching both lines. (*Sc.*, 1883.)

31. Describe a circle passing through two given points a and b and touching a given line $c\,d$. The line joining $a\,b$ is not parallel to $c\,d$. (*Sc.*, 1884.) (*Prob.* 158.)

32. Describe a circle passing through a point p and touching a line $a\ b$ in a given point c. (*Sc.*, 1886.) (*Prob.* 121.)

CHAPTER XII

1. Draw a triangle and within it inscribe a square.

2. Within a square of 2″ sides inscribe the largest possible isosceles triangle having its base $\frac{1}{2}$″ long. (*Art.*) (*Prob.* 189.)

3. Draw a trapezion with sides of 1″ and 2″ respectively, and within it inscribe a square. (*Art.*)

4. Within a square of 2″ sides inscribe the largest possible equilateral triangle. (*Art.*)

5. About a regular pentagon of 1″ sides describe a similar figure, having its sides parallel and equidistant to those of the given figure, and $1\frac{1}{2}$″ in length. (*Art.*)

6. Within an equilateral triangle of 3″ sides inscribe a similar figure, base $1\frac{1}{4}$″. (*Art.*)

7. About a circle of $1\frac{1}{2}$″ diameter describe an equilateral triangle. (*Art.*)

8. Within a circle of $1\frac{1}{4}$″ radius inscribe a triangle having angles of 30° and 60°. (*Art.*)

9. Construct a parallelogram, sides $2\frac{1}{2}$″ and $1\frac{3}{4}$″, included angle 50°, and within it inscribe a rhombus having one angle touching one side of the parallelogram at a point $\frac{1}{2}$″ from one of the corners. (*Art.*)

10. About an isosceles triangle describe a square. (*Prob.* 190.)

11. Within a given circle of 3″ diameter inscribe an isosceles triangle having its equal sides $2\frac{1}{2}$″ long. (*Art.*)

12. Construct an isosceles triangle, the two equal sides to touch the circumference of a given circle at two given points **A** and **B**; the angle made by the radii from **A** and **B** to be 100°.

13. Draw a triangle two of whose angles are 50° and 65°, and the radius of the inscribed circle 1″. (*Sc.*, 1870.)

14. Within a square of 3″ sides inscribe an octagon, so that the alternate sides of the octagon shall coincide with the sides of the square. (*Sc.*, 1871.)

15. Construct a quadrilateral base 3″, base angles 90° and 75°, sides 2″ and 2″. Within it inscribe a parallelogram having a side of 2″.

CHAPTER XIII

AREAS

Before beginning the problems on areas, the following principles should be thoroughly understood; and it will be of considerable service to the student to go through Euclid's demonstrations of these principles :—

1. The **area** of a plane figure is the amount of **surface** enclosed by its boundary, or perimeter. It depends upon both the shape and the perimeter of the figure.

2. **Parallelograms** upon the **same base**, and between the **same** parallels, are equal. (EUC. I. 35.)
A B C D = A B D E. (Fig. 1.) F G H J = F G K L. (Fig. 2.)

3. **Parallelograms** upon **equal bases**, and between the same parallels, are equal. (EUC. I. 36.)

M N O P = Q R S T, because each of them is equal to M N S T. (Fig. 3.)

4. **Triangles** upon the **same base**, and between the same parallels, are equal. (EUC. I. 37.)

A B C = A B D. (Fig. 4.)

5. **Triangles** upon **equal bases**, and between the same parallels, are equal. (EUC. I. 38.)

A B C = D E F. (Fig. 5.)

6. If a parallelogram and a triangle be upon the same base and between the same parallels, the parallelogram shall be double of the triangle. (EUC. I. 41.)
A B C D = twice A B C. (Fig. 6.) E F G H = twice E F J. (Fig. 7.)

7. The square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides. (EUC. I. 47.)
The sq. C B D E = the sq. A B F G + the sq. A H J C. (Fig. 8.)

Note.—The same principle applies to other figures constructed upon the sides of a right-angled triangle as long as they are similar. (EUC. VI. 31.)

8. The area of a triangle is equal to the area of a rectangle upon the same base, but having half the altitude.
Triangle A B C = rectangle A B E F. A E = $\frac{1}{2}$ C D. (Fig. 9.)

9. **Parallelograms** and **triangles** upon the **same base** have their areas in the same ratio as their altitudes.

A B E F = twice A B C D, because the altitude B E = twice the altitude B G. (Fig. 10.)

A B D = three times A B C, because the altitude D E = three times the altitude C E. (Fig. 11.)

10. **Parallelograms** and **triangles** of the **same altitude** are to one another as their bases. (EUC. VI. 1.)

E B F G = $\frac{2}{3}$ A B C D, because E B = $\frac{2}{3}$ A B. (Fig. 12.)

D B E = $\frac{1}{4}$ A B C, because D B = $\frac{1}{4}$ A B. (Fig. 13.)

11. The **areas** of **similar** figures are proportional to the squares on their homologous, or corresponding, sides. (EUC. VI. 19, 20.)

A B C D E : F G H J K as $AB^2 : FG^2$. (Fig. 14.)

12. The **areas** of **circles** are proportional to the squares on their diameters.

FIG. 1.

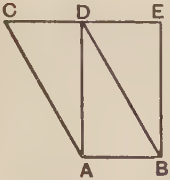


FIG. 2.

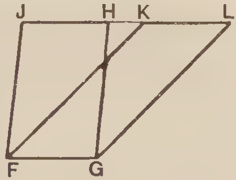


FIG. 3.

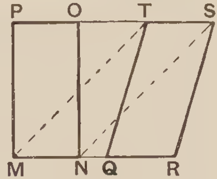


FIG. 4.

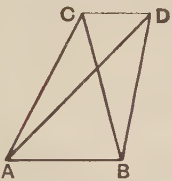


FIG. 5.

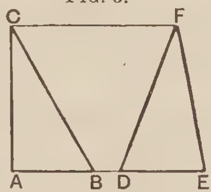


FIG. 6.

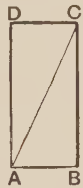


FIG. 7.

FIG. 8.

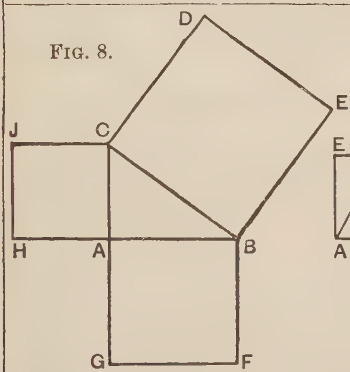


FIG. 9.

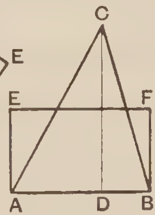


FIG. 10.

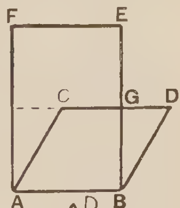


FIG. 11.

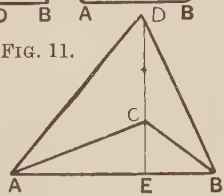


FIG. 12.

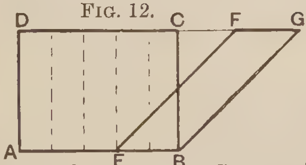
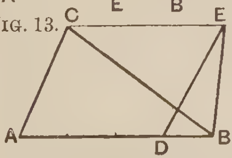


FIG. 13.



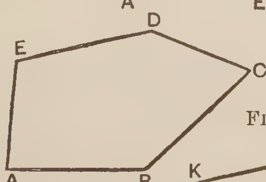
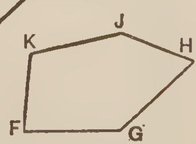


FIG. 14.



PROBLEM 208.—To construct a triangle equal in area to any parallelogram.

Set up twice the altitude of the given figure and join with the extremities of the base as shown in the first two figures, or double the base and keep the same altitude as in the third figure. The triangle A B C in each case is equal to the given parallelogram. The second figure shows the construction if an isosceles triangle be required.

PROBLEM 209.—To construct a parallelogram equal in area to a triangle.

Draw the altitude of the given triangle, and bisect it by the line D E parallel to A B. At A and B draw perpendiculars. Then A B D E is the rectangle equal to the given triangle. A B F G is a rhombus equal to A B C.

PROBLEM 210.—To construct a triangle equal in area to a given trapezium.

Let A B C D be the given trapezium. Draw the diagonal D B, and from C draw C E parallel to D B to meet A B produced in E. Draw D E. Then A D E is the required triangle.

Note.—D B E = D B C (Euc. I. 37). Add A B D to each. Then A D E = A B C D.

PROBLEM 211.—To construct a triangle equal in area to an irregular pentagon.

Draw D A, D B. From E draw E F parallel to D A and meeting A B produced in F. Draw D F. From C draw C G parallel to D B and meeting A B produced in G. Draw D G. Then F D G is the required triangle.

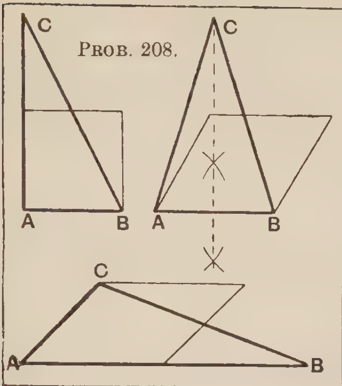
PROBLEM 212.—To construct a triangle equal to any irregular polygon.

Let A B C D E F be the given polygon. Draw E A. From F draw F G parallel to A E. Draw E G. (The figure G B C D E = A B C D E F.) Draw D B, and from C draw C H parallel to D B. Join D and H. (G H D E = G B C D E.) Draw E H, and from D draw D J parallel to E H. Join E and J. Then G E J is the required triangle.

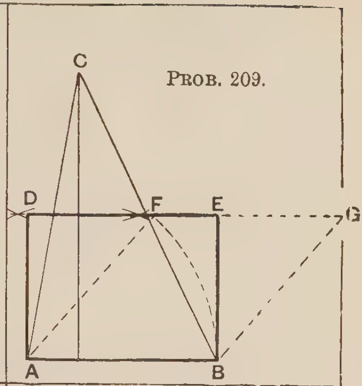
PROBLEM 213.—To construct an isosceles triangle equal to a trapezium, one side to be common to both figures.

Let A B C D be the given trapezium and A B the side common to both figures. Draw A D, and from C draw C E parallel to A D and meeting B D produced. Join A and E. Then the triangle A B E is equal to the trapezium. To get an isosceles triangle equal to it, bisect the base by the perpendicular F G, and through E draw E G parallel to A B. Draw G A and G B. Then A B G is the required triangle.

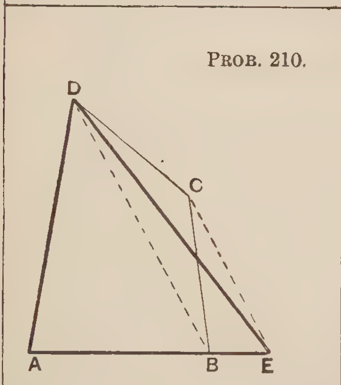
Note.—In the case of a pentagon proceed in a similar manner, first converting the pentagon into a trapezium on A B.



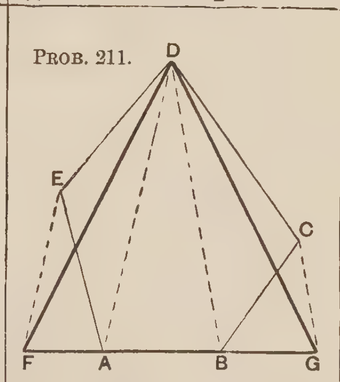
Prob. 208.



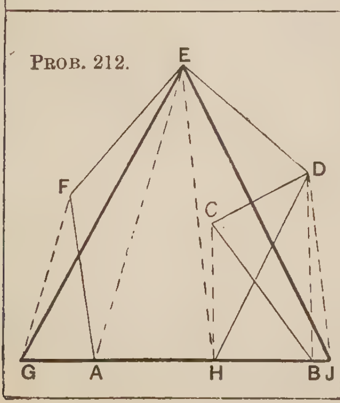
Prob. 209.



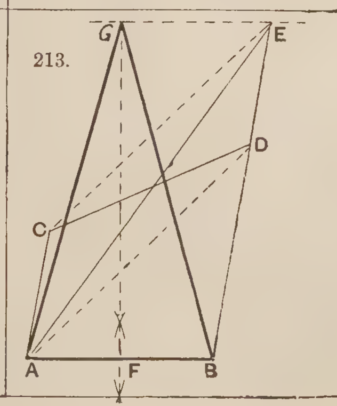
Prob. 210.



Prob. 211.



Prob. 212.



213.

PROBLEM 214.—To construct a triangle equal in area to the sum of two given triangles.

Let A B C and D E F be the given triangles. Make the triangle C E F equal to the given triangle D E F, forming an irregular pentagon, A B E F C. Draw A F, and from C draw C H parallel to A F, and meeting A B produced. Join F and H. Draw F B, and from E draw E G parallel to F B. Join F and G. Then F G H is the required triangle.

PROBLEM 215.—To construct a triangle equal to any regular polygon.

Let A B C D E be a regular pentagon. Find the centre, and divide the polygon into five equal triangles. Make the base G H equal to five times the base A B. Draw F G, F H. Then F G H is the required triangle.

Notes.—1. Another method is to proceed as in Problem 211.

2. Where the polygon has a larger number of sides, the altitude of the triangle may be doubled, and the base made equal to half the perimeter of the given polygon.

PROBLEM 216.—To construct a triangle equal in area to a circle. (Approximate.)

Draw A B, the diameter of the circle. Divide the radius A C into 7 equal parts. Draw A D perpendicular to A B, and make it $3\frac{1}{7}$ times A C. Draw B D. A B D is the required triangle.

PROBLEM 217.—On a given base to draw a triangle equal in area to another given triangle.

Let A B C be the given triangle, and D the given base. On A B or A B produced set off A E equal to D. Draw C E, and from B draw B F parallel to C E. Join F and E. Then A F E is the required triangle. (B F C = F B E.)

PROBLEM 218.—To construct a triangle equal in area to a given triangle, having its vertex in a given point, and its base in the same straight line as that of the given triangle.

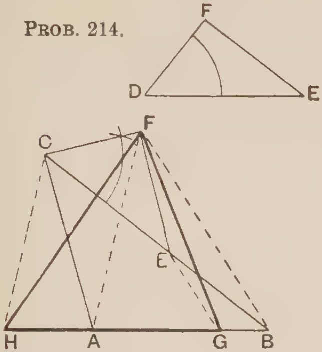
Let A B C be the given triangle, and D the given point. Draw A D. From C draw C E parallel to A B, and join B and E. Then the triangle A B E = the triangle A B C. The Problem now resolves itself into the same as the previous Problem—that is, to construct a triangle on A D equal to the triangle A B E. Draw D B, and from E draw E F parallel to D B. Join D and F. Then A D F is the required triangle.

PROBLEM 219.—To construct a triangle of a given altitude equal in area to another given triangle.

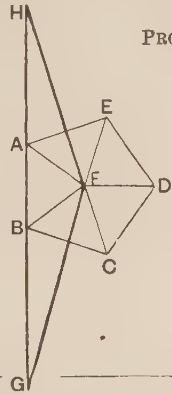
Let A B C be the given triangle, and D the given altitude. Draw C E, the altitude of the given triangle, and on it mark off E F equal to D. Draw F A, and from C draw C G parallel to F A. Join F and G. Draw F B, and from C draw C H parallel to F B. Join F and H. Then G F H is the required triangle.

Note.—A F G = A F C, and B F H = B F C.

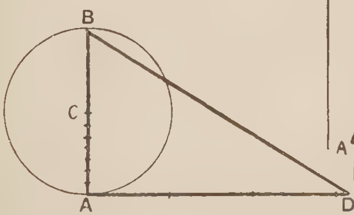
PROB. 214.



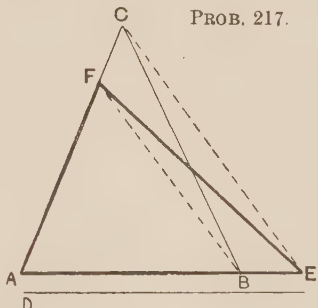
PROB. 215.



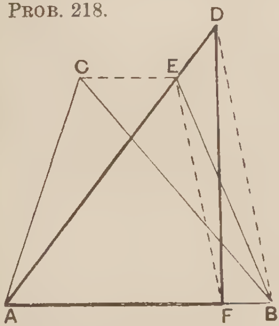
PROB. 216.



PROB. 217.



PROB. 218.



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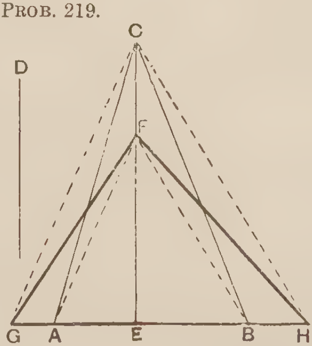


PROBLEM 220.—To construct a square equal in area to the sum of three squares.

Let A, B, and C be the lengths of the sides of the given squares.

Draw E F equal to A, and E D at right angles to E F, and equal to B. Join F and D.

Then the square on D F is equal to the sum of the squares on D E and E F. (EUC. I. 47.) Draw D G at right angles to D F, and equal to C. Join F and G. On G F describe a square.

PROBLEM 221.—To construct a square equal in area to the difference between two given squares.

Let A and B be the sides of the given squares. Draw two lines at right angles to each other. Make C D equal to B. With C as centre and A as radius mark off E. On D E construct the square.

Notes.—1. If the square on C E = the sum of the squares on C D and D E, then the square on D E must be equal to the difference of the squares on C E and C D.

2. The same principles may be applied to the circle, or any rectilineal figure. An equilateral triangle, polygon, &c., may be constructed equal in area to the sum or the difference of two similar figures.

PROBLEM 222.—To describe a circle equal in area to the sum of two given circles.

Draw C D and D E perpendicular to each other, and equal to A and B, the diameters of the given circles. Draw C E, the diameter of the required circle.

Note.—To describe a circle equal in area to the difference between two circles, proceed as in Problem 221. C D will be the smaller diameter, C E the larger, and D E the diameter of the required circle.

PROBLEM 223.—To construct a triangle similar to a given triangle, but having twice its area.

Let A B C be the given triangle. Draw A D perpendicular to A B, and equal to it. Make B E equal to B D. From E draw E F parallel to A C, and meeting B C produced. Then E B F is the required triangle.

Note.—The same principle may be applied to any other rectilineal figure. A trapezium is shown similar to, and double the area of, a given trapezium.

PROBLEM 224.—To construct a trapezium similar to a given trapezium, having half its area.

Let A B C D be the given trapezium. Bisect A B, and describe a semicircle. Draw B H, and make B E equal to it. Draw B D, and from E draw E F parallel to A D, and from F draw F G parallel to D C. Then B E F G is the required figure.

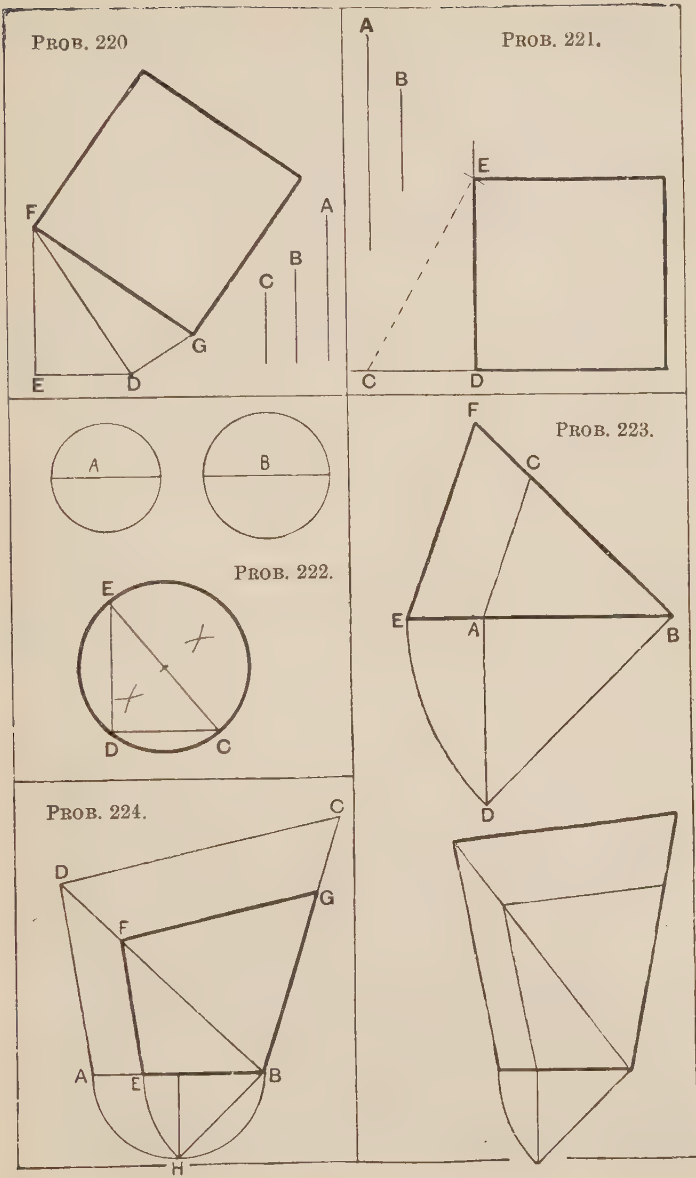
Note.—The same principle applies to other rectilineal figures. The triangle B E F is half the triangle B A D.

PROBLEM 225.—To construct a square having twice the area of a given square.

Draw the diagonal, and describe a square on it.

PROBLEM 226.—To construct a square having half the area of a given square.

On half the diagonal describe a square.


PROB. 220
F
G
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PROB. 221.
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PROB. 222.
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PROB. 223.
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PROB. 224.
C
D
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H

PROBLEM 227.—To construct a square equal in area to any given parallelogram.

Let A B C D be the given parallelogram. Produce A B, and make B F equal to the altitude of the parallelogram. Find B G, a mean proportional between A B and B F. (*Prob.* 29.) On B G describe the square.

PROBLEM 228.—To construct a square equal in area to a triangle.

Make a rectangle equal to the triangle. (*Prob.* 209.) Find a mean proportional between the base and the height of the rectangle, and on it describe the square.

PROBLEM 229.—To construct a square equal in area to a trapezium.

Draw the diagonal. On one side of it make a rectangle equal to the triangle A D C, and on the other side a rectangle equal to the triangle A B C. Proceed as in the preceding problem.

PROBLEM 230.—To construct a square equal in area to any polygon.

First obtain a triangle equal in area to the polygon, as shown in Problems 211 and 212; then construct a rectangle equal in area to the triangle, and proceed as above.

PROBLEM 231.—To construct a rectangle of a given perimeter, and equal in area to a given square.

Let the perimeter equal $4\frac{1}{2}$ inches, and the side of the given square 1 inch. Divide half the perimeter into 2 parts, whose mean proportional shall equal 1 in., as follows:—Draw A B, $2\frac{1}{4}$ inches long, and at B draw B C perpendicular to A B, and 1 inch long. Describe a semicircle on A B. Through C draw C D parallel to A B, and from D draw D E parallel to C B. Then A E and E B will be two of the sides of the rectangle. Complete the figure as shown.

PROBLEM 232.—To construct a rectangle equal in area to a square, and having its sides in a given ratio.

Let the ratio of the sides be as 2 : 3. Produce the base of the given square. From A set off to any convenient unit A B and A C, equal to 2 and 3 units respectively. Find A D, the side of a square equal to the rectangle contained by A B and A C. From E draw E F and E G parallel to D C and D B. A F and A G will be the sides of the rectangle. Complete the figure as shown.

PROBLEM 233.—To construct a rectangle equal in area to a square, having the difference between two adjacent sides given.

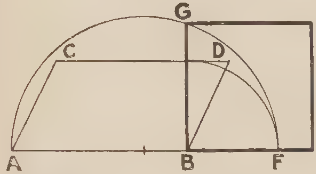
Let A B be the difference between the two adjacent sides. Bisect A B in C. With centre C and radius C D describe a semicircle. Then A F and A E will be the sides of the rectangle.

PROBLEM 234.—On a given base to construct a rectangle equal in area to a given rectangle.

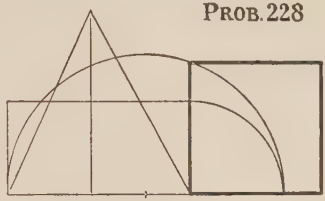
Let A B C D be the given rectangle, and E the given base. Produce A B, and make A F equal to E. Join C and F, and from B draw B G parallel to C F. A G is the other side of the rectangle.

Note.—A C : A G as A F : A B. (Euc. VI. 2.) But when quantities are in proportion the product of the *extremes* equals the product of the *means* (p. 18): therefore A C × A B = A G × A F.

PROB. 227.



PROB. 228



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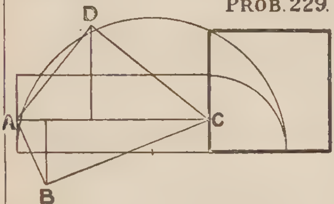

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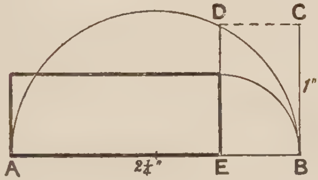

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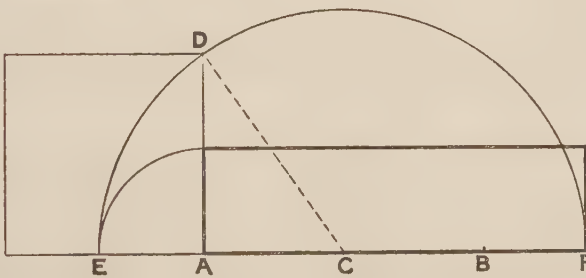

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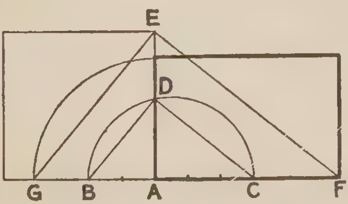

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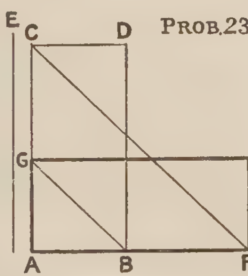


PROBLEM 235.—To divide a triangle into any number of equal parts by lines drawn from one of its angles.

Divide A B into the required number of equal parts (say 3). Draw C 1 and C 2. The three triangles thus formed are equal to each other. (EUC. I. 38.)

PROBLEM 236.—To divide a parallelogram into any number of equal parts by lines drawn from one of its angles.

Divide two adjacent sides into the same number of equal parts as the figure has to be divided into (say 3.) Join D 1, D 1.

PROBLEM 237.—To bisect a triangle by a line drawn from a point in one of the sides.

Let **D** be the given point. Bisect **A B** in **E**, and draw **C E**. Join **D** and **E**, and from **C** draw **C F** parallel to **D E**. Join **D** and **F**. Then **D F** bisects the triangle.

PROBLEM 238.—To bisect a parallelogram by a line drawn from a point in one of the sides.

Let **E** be the given point. Find the centre, and draw **E F** through it. Then **E F** bisects the parallelogram.

Note.—This construction will apply for any position of the point.

PROBLEM 239.—To bisect a trapezium by a line drawn from one of its angles.

Let A B C D be the given trapezium. Draw the diagonals. Bisect A C in E. Draw D E, E B, dividing the trapezium into two equal areas. Through E draw F G parallel to B D. Join D and F. Then D F bisects the trapezium.

Note.—The triangle D B F = the triangle D B E.

PROBLEM 240.—To divide a triangle into any number of equal parts by lines drawn from a point in one of the sides.

Let D be the given point. Divide the side C B, in which the given point is situated, into as many equal parts as the triangle has to be divided into (say 3). Draw A D. From 1 and 2 draw 1 E, 2 F, parallel to A D. Draw D E, D F, dividing the triangle as required.

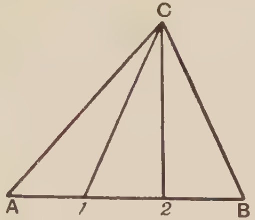
Note.—The triangle D E 1 = the triangle A E 1.

PROBLEM 241.—To divide a parallelogram into any number of equal parts by a line drawn from a point in one of the sides.

Let E be the given point. Divide A B into the required number of equal parts (say 4) in 1, 2, 3. Draw 1 1, 2 2 parallel to A D and bisect them in F and G. Through F and G draw E H and E J. Bisect the trapezium E J C B by the line E K. (*Prob.* 239.) Then the lines E H, E J, E K divide the parallelogram into 4 equal parts.

Note.—If the point E were placed so that E K would fall upon the side D C, then a line 3 3 might have been bisected, and E K drawn through the points of bisection, as in 1 1 and 2 2.

PROB. 235.



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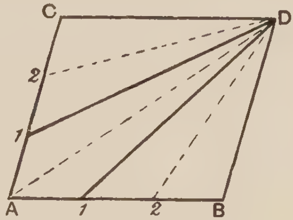

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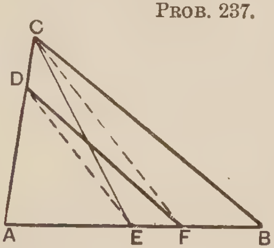

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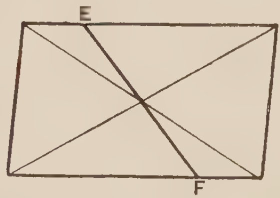

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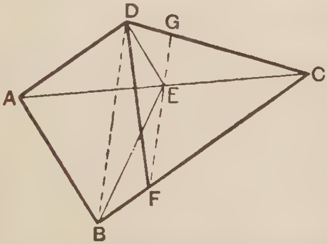

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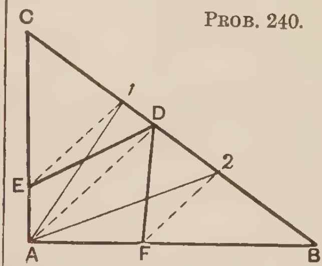

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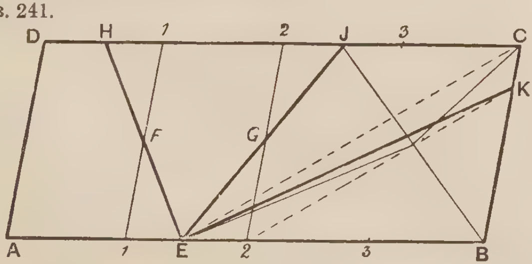

PROBLEM 242.—To divide an irregular polygon into any number of equal parts by lines drawn from one of the angles.

Let A B C D E be an irregular polygon; it is required to divide it into 3 equal parts by lines drawn from the angle at D.

Construct the triangle D F G equal to the polygon. (*Prob.* 211.)

Divide the base, F G, into 3 equal parts. Draw D 1. It is evident that the triangle D 1 F is ⅓rd of the triangle D F G, and consequently ⅓rd of the given polygon. As the point 2 does not fall upon the base A B of the polygon, draw 2 *d* parallel to the diagonal D B. Draw D *d*. The lines D 1 and D *d* divide the polygon as required.

Note.—The triangle D B 2 = the triangle D B *d*. (Euc. I 37.)

PROBLEM 243.—To divide a triangle into any number of equal parts by lines drawn from a point within the triangle.

Let D be the given point. Divide the base A B into as many equal parts as required (say 3). Draw D 1, D 2, and D C. From C draw C E parallel to D 1, and C F parallel to D 2. Draw D E and D F. The lines D C, D E, and D F divide the triangle into 3 equal parts.

Notes.—1. The triangle E 1 C = the triangle E D C. If A E C be added to each, then A 1 C = A E D C. But A 1 C is ⅓rd of the triangle A B C. Therefore A E D C will equal ⅓rd of the triangle A B C.

2. If the line D F does not fall upon A B, then proceed as shown in Problem 243 a. Obtain D E as in the preceding problem. Draw 2 F parallel to B C. Join D and B, and from F draw F G parallel to D B. Draw D G. Then D C, D E, and D G divide the triangle into 3 equal parts.

Proof.—The triangle B F C = the triangle B 2 C. (Euc. I. 37.) Triangle F G D = triangle F G B. Add to each the triangle C F G. Then C F G + F G D = C F G + F G B—that is, C D G = C F B. But C F B = C 2 B. Therefore, C D G = C 2 B.

PROBLEM 244.—To bisect a triangle by a line drawn parallel to one side.

Bisect C B by the perpendicular E F, and describe a semicircle. With centre C and radius C F describe the arc F G. From G draw the line G H parallel to A B. This line bisects the triangle.

Note.—C F, which equals C G, is a mean proportional between the side C B and its half, C E. (Euc. VI. 8, Cor.) The triangle C G H : triangle C B A as $CG^2 : CB^2$—that is, as C E : C B, or as 1 : 2.

PROBLEM 245.—To bisect a triangle by a line perpendicular to the base.

Draw A D perpendicular to B C, and bisect B C in E. Find a mean proportional, C F, between the *larger* segment of the base C D, and the half C E. Make C G equal to C F, and draw G H perpendicular to the base. Then G H bisects the triangle.

PROBLEM 246.—To divide a triangle into any number of equal parts by lines drawn parallel to one of the sides.

Let the triangle A B C be divided into 3 equal parts. Divide B C or B A into 3 equal parts. Describe a semicircle on B C, and erect perpendiculars at 1 and 2. Make B *d* equal to B D, and B *e* equal to B E. Parallels drawn from *d* and *e* will divide the triangle as required.

Note.—B D is a mean proportional between B C and its third, B 1, and B E is a mean proportional between B C and two-thirds, B 2. B *d* F : B C A as $Bd^2 : BC^2$—that is, as B 1 : B C, or as 1 : 3.

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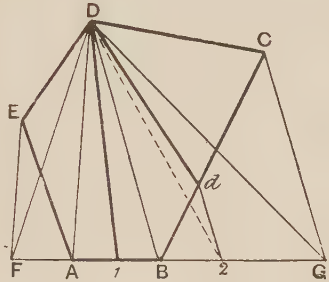

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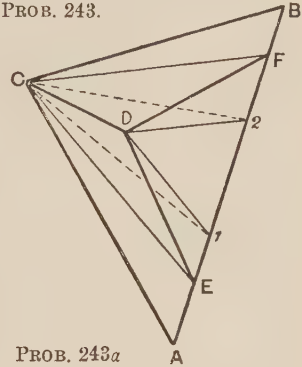

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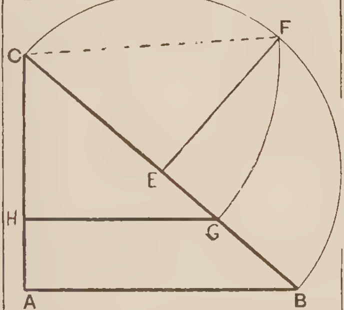

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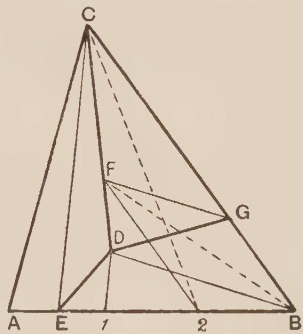

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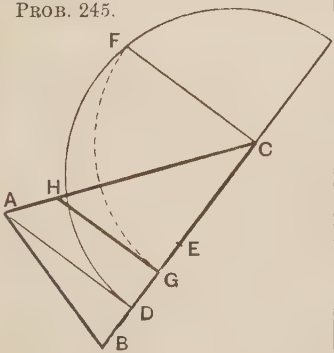

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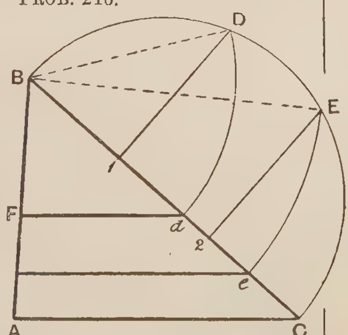


PROBLEM 247.—To divide a parallelogram into any number of equal parts by lines parallel to the diagonal.

Let it be required to divide the parallelogram A B C D into 5 equal parts by lines drawn parallel to the diagonal D B. Divide D C into 5 equal parts. Describe a semicircle on D C, erect perpendiculars at the alternate parts, 2 and 4, and proceed as in Problem 246.

Note.—Each of the triangles A B D and D B C is divided into 5 equal parts. By drawing the alternate lines only, the whole parallelogram is divided into 5 equal parts.

PROBLEM 248.—To divide a circle into any number of equal parts (say 3) by concentric circles.

Divide the radius A O into 3 equal parts. On it describe a semicircle, and erect perpendiculars at 1 and 2. O B and O C will be the radii of the required circles.

Note.—O B is the mean proportional between the radius O A and its third O 1.

PROBLEM 249.—To divide a circle into any number of parts (say 3) equal in area and perimeter.

Divide the diameter of the circle into twice as many parts as there are equal areas required. With centres 1, 2, 4, and 5 describe semicircles.

PROBLEM 250.—To divide a triangle into 2 parts, having a given ratio to each other, by a straight line drawn through a given point in one of its sides.

Let D be the given point, and the given ratio as 3 : 2.

Divide C B into 3 + 2 equal parts. Join A 2, which divides the triangle into 2 parts in the ratio of 3 to 2. Draw D A, and from 2 draw 2 E parallel to D A. Join D E. Then D E divides the triangle as required.

Note.—The triangle E 2 A = the triangle E 2 D. Therefore, E C D = A C 2.

PROBLEM 251.—To divide a parallelogram into 2 parts, having a given ratio to each other, by a straight line drawn from a given point in one of the sides.

Let E be the given point, and the given ratio as 3 : 1.

Divide A B into 4 equal parts. From 1 draw a line, 1 F, parallel to A D, cutting the parallelogram into 2 parts having the required ratio. Bisect 1 F, and from E draw a line through the point of bisection. This line divides the triangle as required.

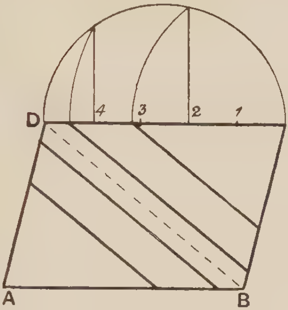
PROBLEM 252.—To construct a square, the area being given.

Let the required area be 2½ square inches. Construct a rectangle, sides 1 inch and 2½ inches. This rectangle will contain 2½ square inches. Find a mean proportional, B E, to the two sides. On B E construct the square.

Notes.—1. It is not necessary to construct the rectangle, but only to find the mean proportional between the dimensions of the two sides which would contain a rectangle of the given area.

2. The figure is drawn to a smaller scale.

PROB. 247.



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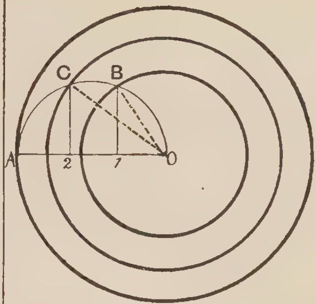

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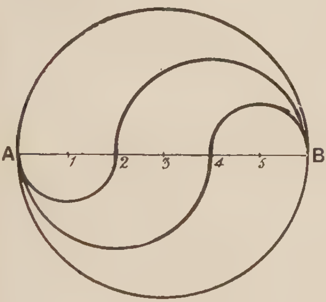

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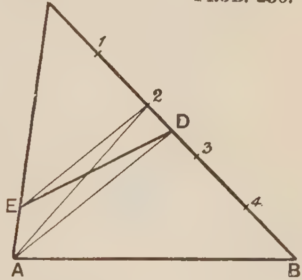

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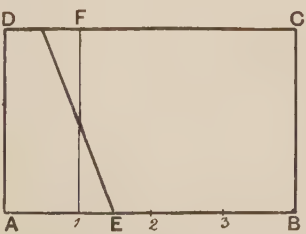

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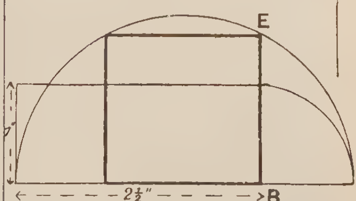
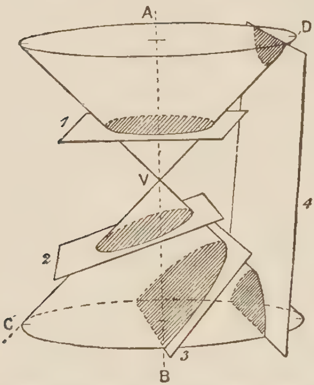


CHAPTER XIV

PLANE CURVES

Some of the curves used in geometrical and mechanical drawings, such as the **ellipse, parabola,** and **hyperbola**; **cycloids,** the **involute,** various **spirals,** &c., cannot be described by the ordinary compasses. These curves are obtained by finding a number of points in the required line, and then tracing the curve through these points by hand, or with the help of French curves.

The **ellipse, parabola,** and **hyperbola** are known as **conic sections,** because they are formed when a right circular cone is intersected by a plane in various positions. In dealing with conic sections it is preferable to consider the cone to be generated in the following manner: Let **A B** and **C D** be two straight lines intersecting at **V.** If **A B** remain fixed and **C D** revolve around it, always making a constant angle with **A B,** then the surface of a right circular cone will be generated.



A B is the **axis, C D** the **generator,** and **V** the **vertex** of the cone. It will be seen that the cone thus formed consists of two symmetrical portions, one on each side of the vertex.

The various positions of the cutting plane to form the conic sections are shown on the figure.

1. Here the plane is perpendicular to the axis, and the curve formed is the **circle.**

2. The plane is inclined, but still passes through opposite sides of the cone, and the curve formed is the **ellipse.**

3. The plane is further inclined until it makes an angle with the axis, **A B, equal** to that made by the generator. The curve formed is the **parabola.**

4. The plane is still further inclined so that the angle made by it with the axis is **less** than the angle made by the generator. The curve formed is the **hyperbola.** It will be noticed that the plane now cuts both portions of the cone, and the hyperbola consequently has two branches.

Note.—As will be seen from the above, the circle and ellipse are closed curves, while the parabola and hyperbola are open and unlimited.

In the construction of these curves it is better to regard them as being traced by a point moving on a plane surface according to some fixed law. In the circle, for instance, the moving point always keeps the same distance from a fixed point, the centre of the circle.

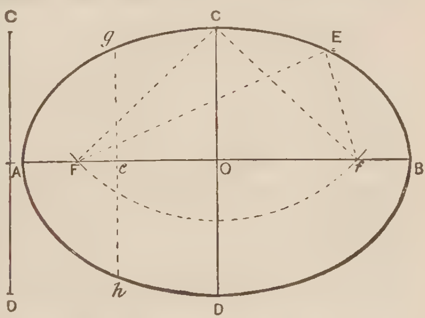
THE ELLIPSE

This curve is traced by a point moving so that the distance from a fixed point or **focus** is **less** than its distance from a fixed line called the **directrix**.

An ellipse has two **foci** (*Prob.* 253, **F** and *f*) and two directrices. The line passing through the two foci is called the **transverse** or **major axis** (**A B**, *Prob.* 253). The line at right angles to the major axis, and bisecting it, is called the **conjugate** or **minor axis** (**C D**, *Prob.* 253). Any line passing through the centre and terminated by the curve is a **diameter**. Any straight line perpendicular to the major axis, as *e g*, is an **ordinate** ; *h g* is called a **double ordinate**. If any point in the curve be joined to the foci by two lines, these two lines are, together, equal to the major axis ; for example **F C** + C *f* and **F E** + **E** *f* are each of them equal to **A B**. A **tangent** is a line touching the curve in one point. A **normal** is a perpendicular to a tangent at the point of contact.

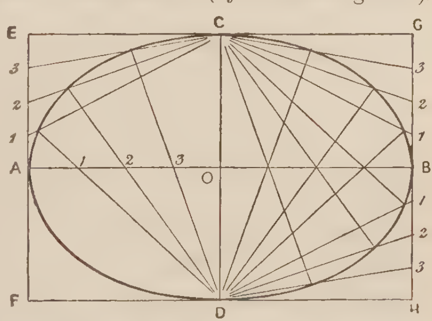
PROBLEM 253.—To describe an ellipse, the major and minor axes being given. First Method (by means of a piece of thread).

Let **A B** and **C D** be the axes. Bisect **A B** and **C D**. Make **O C**, **O D**, each equal to half **C D**. With radius **A O** and centre **C** describe an arc cutting **A B** in **F** and *f*; these points are the **foci**. Take three pins, and stick them in firmly at the points **C**, **F**, and *f*. Tie a piece of thread round these pins, as shown by the lines **C F**, **F** *f*, *f* **C**. Remove the pin at **C**, and replace it with a pencil. Move the point of the pencil round, keeping the thread tightly stretched. The curve described by the pencil-point will be an ellipse.

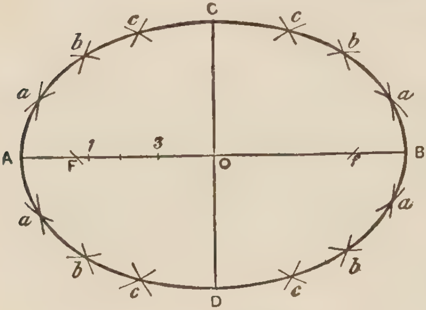


PROBLEM 254.—The same. Second Method (by intersecting lines).

Place the axes at right angles to each other, as before, and through the extremities draw parallels forming a rectangle. Divide **A E** into any number of equal parts (say 4). Set off these parts on **A F**, **B G**, and **B H**. Join each of these points with **C** and **D**. Divide **O A** and **O B** into the same number (4) of equal parts. Draw lines from **D** through 1, 2, and 3, on each side to meet **C** 1, **C** 2, **C** 3. In the same manner draw lines from **C**, through the same points, to meet **D** 1, **D** 2, **D** 3. Through the points thus obtained draw the curve.



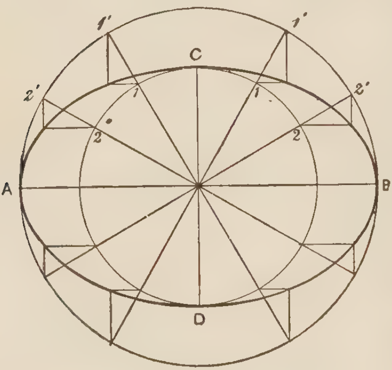
PROBLEM 255.—The same. Third Method (by intersecting arcs).



Place the axes as before. With radius **A O** and centre **C** obtain the foci. In **F O** take any number of points, as 1, 2, 3. With radius **B** 1 and centres **F** and *f* describe four arcs at *a*. With radius **A** 1 and the same centres intersect these arcs. With radius **B** 2 and centres **F** and *f* describe four arcs at *b*, and with radius **A** 2 intersect these arcs. With radius **B** 3 and centres **F** and *f* describe arcs at *c*, and intersect them with radius **A** 3. Through the points thus obtained draw the ellipse.

Note.—Describe all the arcs from the foci, **F** and *f*.

PROBLEM 256.—The same. Fourth Method.

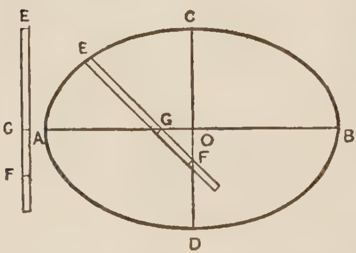


Place the axes as before, and describe a circle on each. Divide one of the quadrants into any number of parts, and obtain corresponding points on the others by producing the radii. From the points 1 and 2 on the smaller circle draw parallels to **A B**, and from the points 1′ and 2′ on the larger circle draw parallels to **C D** to meet the lines drawn parallel to **A B**. Through the intersections of these parallels draw the curve.

Note.—All the problems on curves should be drawn to a much larger scale than shown.

PROBLEM 257.—The same. Fifth Method (by means of a straight-edge or a paper trammel).

This is an exceedingly useful method for practical purposes. Set up the axes **A B** and **C D** as before. Take a piece of paper (or a long, flat ruler, if the figure be large), and make **E F** equal to **A O**, and **E G** equal to **C O**. Place it so that **G** may be on the major and **F** on the minor axis. Then **E** will be a point on the curve. By shifting the paper, and always keeping **G** on the major and **F** on the minor axis, a number of points in the curve may be obtained. Draw the curve through the points.

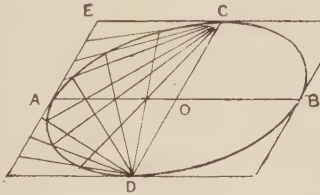


Note.—If a point in the curve, as **E**, and the major axis, were given, the ellipse could be described as follows:—Draw a line, **C D**, at right angles to **A B**. With radius **A O** and centre **E** cut the line **C, D** in **F**. Then the distance **E G** will equal half the minor axis. This being determined, complete the ellipse by any of the methods given.

PROBLEM 258.—To describe an ellipse passing through any three points not in the same straight line.

Let A, B, C be the given points. Join A and B, and bisect in O.

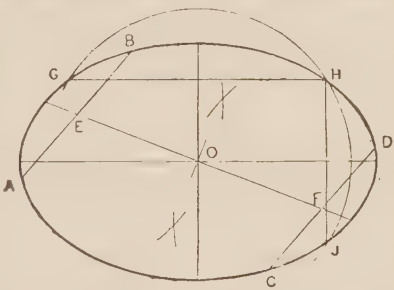
Draw C O, and produce, making O D equal to O C. Through A and B draw parallels to C D, and through C and D draw parallels to A B, forming a parallelogram. Divide A O and A E into the same number of equal parts, and proceed as in Problem 254.



Note.—This problem offers solutions to the following problems:—To describe an ellipse about a triangle (the points A, B, and C, if joined, form a triangle); to inscribe an ellipse in a rhombus or rhomboid.

PROBLEM 259.—To find the centre, axes, and foci of a given ellipse.

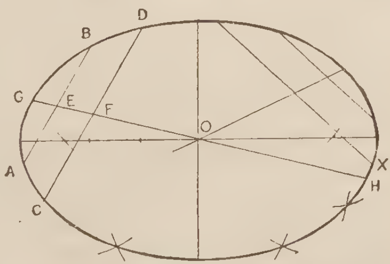
Draw any two parallel chords, A B and C D. Bisect them both in E and F. The line drawn through E and F will be a diameter. Find the centre, O. With centre O describe an arc cutting the ellipse in G, H, and J. Draw G H and H J. Parallels to G H and H J, through O, will give the axes.



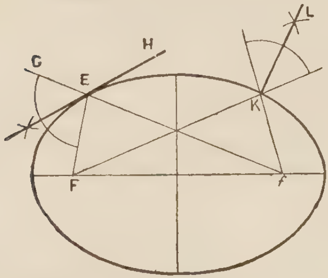
PROBLEM 260.—A portion of the curve of an ellipse being given, to complete it.

Let C D X be the given portion.

Draw two chords, A B and C D, and bisect them in E and F. Through E and F draw a line. If this line be terminated by the curve of the ellipse at each end, obtain the axes as in Problem 259, find the foci, and complete the ellipse by obtaining points in the curve as already shown. If the line be not terminated, then draw two other parallel chords, and bisect them as in the first pair. The line drawn through the points of bisection will intersect the line drawn through E and F in O, the centre of the ellipse. Make O H equal to O G, and obtain the axes and points in the curve as before.



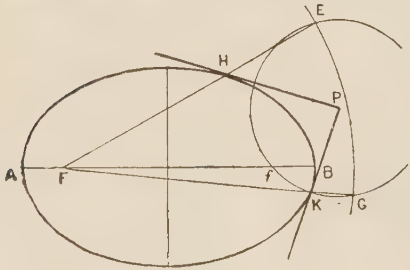
PROBLEM 261.—To draw a tangent and a normal to an ellipse from given points in the curve.



1. The **tangent**. Find the axes and foci (*Prob.* 259). Join the foci with **E**, the given point, and produce *f* **E**. Bisect the angle **F E G**. The line **E H** is the required tangent.

2. For the **normal** at **E** draw a perpendicular to the tangent. If another point, **K**, be given, then join the foci with **K** and produce both lines. Bisect the angle thus formed. **K L** is the required normal or perpendicular.

PROBLEM 262.—To draw a tangent to an ellipse from a point without the curve.



Let **P** be the given point. Find the axes and foci of the ellipse (*Prob.* 259). With centre **P** and radius **P** *f* describe a circle. With centre **F** and radius equal to the major axis **A B** intersect the first circle in **E** and **G**. Draw **F E** and **F G** cutting the ellipse in **H** and **K**. Lines drawn from **P** through **H** and **K** will be tangents to the ellipse.

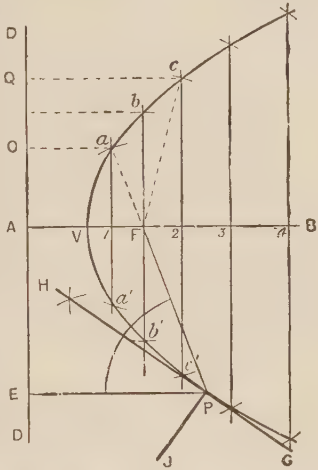
THE PARABOLA

This curve is traced when a point moves so that its distance from the **focus** (**F**, *Prob.* 263) always **equals** its distance from the fixed line called the **directrix** (**D D**, *Prob.* 263). Thus **F** a = a **O**, **F** c = c **Q**, &c.

The path of a projectile forms a parabolic curve. The curve is also largely used in graphic methods for determining the stress upon beams, girders, &c.

The **axis**, **A B**, is a line drawn through the focus, perpendicular to the directrix, and consequently divides the curve into two symmetrical parts. The point where the curve meets the axis is called the **vertex** (**V**, *Prob.* 263). A perpendicular from any point on the curve to the axis is an **ordinate**, as a 1, c 2, &c. $a\,a'$, $b\,b'$, &c., are **double ordinates**. The double ordinate $b\,b'$, through the focus, is called the **latus rectum**. The part of the axis between the ordinate of a point and the vertex of the curve is called the **abscissa**; thus 2 **V** is the abscissa of the point c.

PROBLEM 263.—To draw a parabola, the focus and directrix being given.



Let **F** be the focus, and **D D** the directrix. Through **F** draw the axis **A B** perpendicular to **D D.** Bisect **F A** in **V.** Then **V** is a point on the curve, because it is equally distant from the focus and directrix. Take any points, 1, **F**, 2, 3, 4, &c., and through them draw perpendiculars to the axis (ordinates). From centre **F** with radius **A** 1 cut the ordinate through 1 in a, a'. With radius **A F** and centre **F** cut the ordinate in b, b'. With radius **A** 2 and centre **F** obtain the points c, c'. Any number of points may be obtained similarly. Through the points draw the curve.

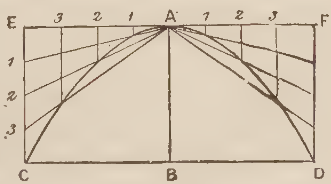
PROBLEM 264.—To draw a tangent and a normal from a point in the curve.

In Problem 263 let **P** be the point. Join **P** with the focus **F.** Draw **P E** parallel to **A B.** Bisect the angle **E P F.** Then **G H** is the required **tangent.**

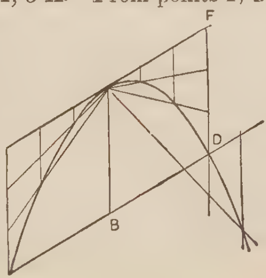
For the **normal** draw **P J** perpendicular to the tangent.

Note.—In a parabolic arch, the joints of the stones are obtained by drawing normals to the curve.

PROBLEM 265.—To draw a parabola, when the axis and an ordinate are given.



Let **A B** be the axis and **B C** the ordinate. Make **B D** = **B C.** The problem is now to draw a parabola through the points **C, A,** and **D.** Construct the rectangle **C E F D.** Divide **A E** and **E C** each into the same number of equal parts (say 4). Set off these parts on **A F** and **F D.** Draw 1 **A**, 2 **A**, 3 **A.** From points 1, 2, 3 in line **E F** draw parallels to **A B**, meeting the other lines as shown. Through the points of intersection draw the curve.

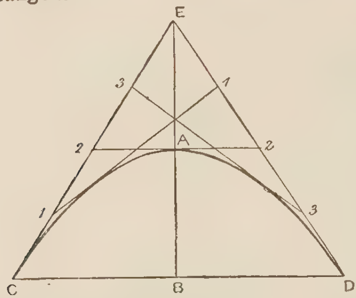


PROBLEM 266.—To inscribe a parabola in any parallelogram.

Proceed as in the previous problem. The figure shows a parabola inscribed in a rhomboid.

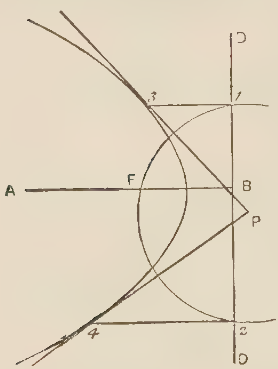
Note.—The curve may be extended by continuing the divisions on lines **B D** and **F D.**

PROBLEM 267.—To draw a parabola by means of intersecting tangents.



Given the height, **A B**, and a double ordinate, **C D**. Produce **B A** and make **A E** = **A B**. Join **E** with **C** and **D**. Divide **E C** and **E D** into any number of equal parts (say 4 or 6). Draw 1 1, 2 2, 3 3. These lines are tangents to the parabola, which touches them midway between the points of intersection.

PROBLEM 268.—To draw a pair of tangents to a parabola from a point outside the curve, the focus and directrix being given.



Let **P** be the given point. Draw the axis **A B**. With centre **P** and radius **P F** describe a circle cutting the directrix **D D** in 1 and 2. From these points draw 1 3 and 2 4 parallel to **A B**. From **P** through 3 and 4 draw the tangents.

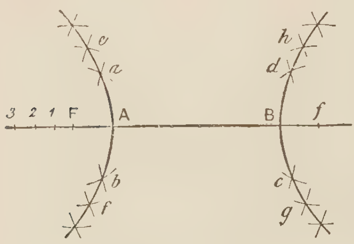
THE HYPERBOLA

This curve is traced when a point moves so that its distance from the **focus** is always **greater**, in a constant ratio, than its distance from the **directrix.**

The variation in pressure and volume of steam, when it expands in the cylinder of an engine, and the relative changes in pressure and volume of a gas when the temperature remains constant, may be graphically represented by this curve.

PROBLEM 269.—To describe the hyperbola, the major axis and the foci being given.

The hyberbola, like the ellipse, has two foci. The major axis is the distance between the two branches of the curve.

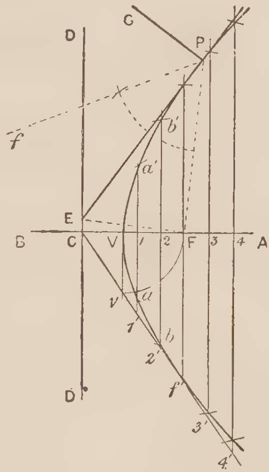


Let **A B** be the major axis, and **F**, *f* the foci. In **A B** produced and beyond **F** take any number of points, 1, 2, 3, &c. From the foci **F**, *f*, with radius **A** 1, describe four arcs at *a*, *b*, *c*, and *d*; and from the same centres, with radius **B** 1, intersect these arcs. With radius **A** 2 and the same centres describe arcs at *e*, *f*, *g*, and *h*, and intersect them with radius **B** 2. Proceed similarly with radii **A** 3 and **B** 3.

Note.—Compare this construction with that of the ellipse in Prob. 255.

PROBLEM 270.—To describe the curve of a hyperbola, the focus, directrix, and vertex being given. Also at a point in the curve to draw a tangent and a normal.

Let **F** be the focus, **D D** the directrix, and **V** the vertex. Join **F V**, and produce both ways. Draw **V** v perpendicular to **A B** and equal to **V F**. Draw **C** v and produce. Take any points, 1, 2, **F**, 3, 4, &c., on **A B**, and through them draw perpendiculars on each side, meeting **C** v in 1′, 2′, f', 3′, 4′, &c. From centre **F** with radius 1 1′ cut the double ordinate through 1 in a, a'. With radius 2 2′ and centre **F** obtain points b, b'. Proceed in a similar manner for the other points. Through the points thus obtained draw the curve.

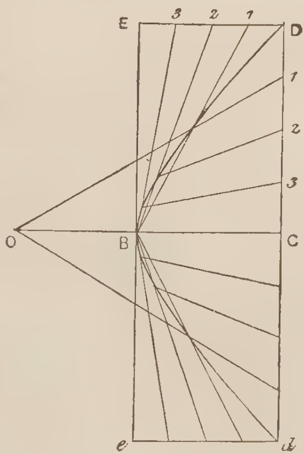


For the **tangent**, join **F** with the given point **P**. Draw **F E** perpendicular to **P F**, meeting the directrix in **E**. Then **E P** will be the required tangent.

For the **normal**, draw **P G** perpendicular to the tangent.

For the other branch of the curve make the angle **E P** f = **E P F**. To obtain the other focus draw **P** f until it meets **A B** produced. Set off distances equal to **F V** and **F C**, and proceed as for the first branch.

PROBLEM 271.—To draw the curve of the hyperbola when half the major axis, the abscissa, and an ordinate are given.



Let **O B** be half the major axis, **B C** the abscissa, and **C D** the ordinate. Make **C** d = **C D**, and complete the parallelogram **D E** e d. Divide **D E** and **C D** into the same number of equal parts (say 4). Transfer these parts to **C** d and d e. Draw **B** 3, **B** 2, **B** 1 on each side of **B C**. Draw **O** 1, **O** 2, &c., to meet these lines. Through the points of intersection draw the curve. A similar construction to the left of **O** will give the other branch.

CYCLOIDAL CURVES

When a circle rolls along a straight line, and always remains in the same plane, a point on the circumference describes the curve known as the **cycloid.**

If the circle rolls along the outside of another circle, both circles keeping in the same plane, the curve traced by a point is the **epicycloid.** If the circle rolls along the inside of a circle, both circles keeping in the same plane, the curve traced by a point is the **hypocycloid.** The moving circle is the **generating circle,** the line upon which it rolls is the **director or base,** and the point tracing the curve is the **generator.**

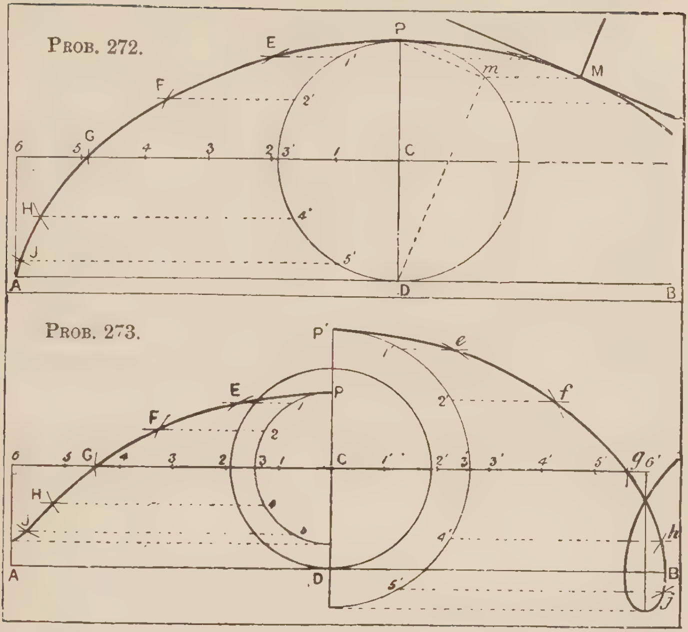
PROBLEM 272.—To draw a cycloid, the generating circle and the director being given. Also to draw a tangent and a normal from a point in the curve.

From **D**, the point where the generating circle touches the director **A B**, draw the diameter **D P**. Through **C**, the centre of the circle, draw a line, **C** 6, parallel to **A B**; this line will be the path of the centre of the circle while rolling. Make **D A** = half the circumference of the circle by setting off $3\frac{1}{7}$ times the radius **C D**. While the circle rolls from **D** to **A**, point **P** will trace half the cycloid. From **A** draw a perpendicular **A** 6. Divide both the semi-circumference and **C** 6 into the same number of equal parts (say 6). Through 1′, 2′, 4′, 5′ draw parallels to **A B**. From centre 1, with radius **C P**, cut the parallel from 1′ in **E**, and from centres 2, 3, 4, &c., with the same radius, cut the parallels from 2′, 3′, &c., in points **F**, **G**, **H**, and **J**. Through these points draw the semicycloid. It is evident that, as the centre of the circle has moved from **C** to 1, 2, &c., the point **P** will have fallen to the level of the parallels through 1′, 2′, &c. To obtain the other half, produce the parallels 1′ **E**, &c., and set off equal distances to the right of **D P**.

To draw a **tangent** at **M**. Draw **M** *m* parallel to **A B**. Join *m* with **P** and **D**. The tangent will be parallel to **P** *m*.

The **normal** will be parallel to **D** *m*.

If the generator be not in the circumference of the circle, the curve is called a **trochoid**, and is obtained in a similar manner.



PROBLEM 273.—To draw a trochoid, the director, generating circle, and generator being given.

1. When the generator **P** is within the given circle. Draw the diameter of the generating circle, and set off the semi-circumference as in the previous problem. Describe a circle passing through **P**. Divide this circle and **C** 6 into the same number of equal parts. Describe arcs with radius **C P** from points 1, 2, 3, &c., in the line **C** 6 to meet parallels from 1, 2, 3, &c., in the circle. Through the points thus obtained draw the curve. This is called the **inferior trochoid.**

2. When the generator **P**′ is without the circle. Proceed as before. Divide the circle described through **P**′ and **C** 6 into the same number of equal parts. Describe arcs from 1′, 2′, 3′, &c., to meet parallels from 1′, 2′, 3′, &c., in the circle. Through *e*, *f*, *g*, &c., draw half the curve. This is called the **superior trochoid**, and forms a loop as shown, the points of which may be obtained by continuing the centres along **C** 6′.

PROBLEM 274.—To draw the epicyloid, the director and the generating circle being given.

Let the arc **A B** be the director, and **P** 3′ **D** the generating circle. Join **O**, the centre of the director, with **D**, the point where the generating circle touches the director, and produce to **P**. Then **D P** will be the diameter of the generating circle. Cut off the arc **D A** = the semi-circumference of the generating cirle **P** 3′ **D**. This is best done as follows :—The angle **D O A** bears the same ratio to 180° that the radius of the generating circle does to the radius of the director. Hence the following proportion for all cases: As **O D** : **D C** :: 180° : **D O A**. Thus if **O D** = 3 and **D C** = 1, the angle **D O A** = $\frac{180°}{3}$ = 60°. Draw **O A**, making 60° with **D O**, and produce. From centre **O** describe an arc through **C**. This will evidently be the path of the centre of the circle when rolling, and 6 will be the position of the centre when **P**, the generating point, has reached **A**. Divide **C** 6 into any number of equal parts (say 6), and divide the semicircle **P D** into the same number of equal parts, 1′, 2′, 3′, &c. From centre **O** describe arcs from 1′, 2′, 3′, &c. From centre 1, with radius **C D**, cut the arc from 1′ in **E**. From centre 2, with the same radius, cut the arc from 2′ in **F**. In the same manner cut the arcs from 3′, 4′, &c. Through the points thus obtained draw the curve from **P** to **A**. The other half can be obtained similarly.

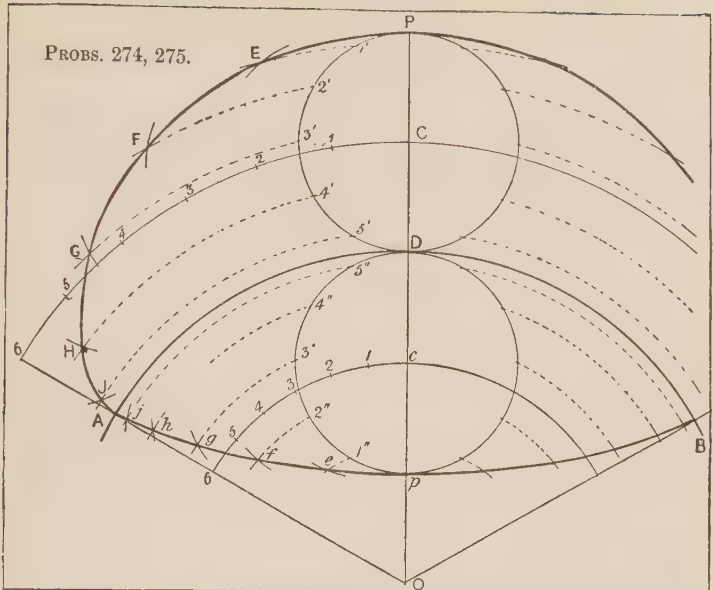
PROBLEM 275.—To draw the hypocycloid, the director and the generating circle being given.

Let **A B** be the director, as in the last figure, and **D** *p* the generating circle. Proceed exactly as in the previous problem. Number the points 1″, 2″, &c., from *p*. From 1, 2, 3, &c., in *c* 6 cut the arcs from 1″, 2″, &c. Draw the half-curve **A** *p* through the points *e*, *f*, *g*, *h*, *j*. The other side can be similarly obtained.

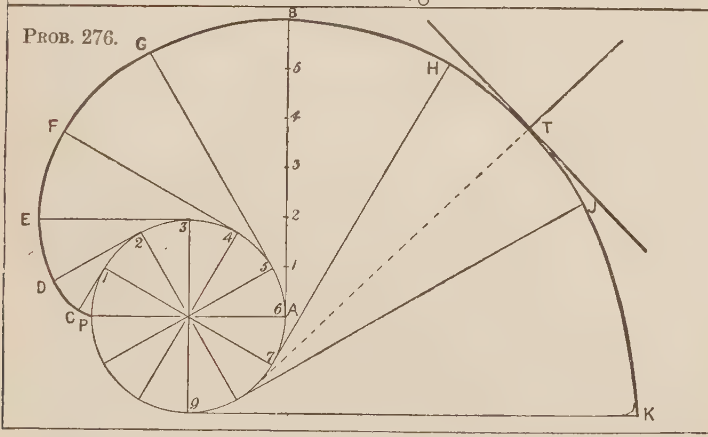
PROBLEM 276.—To draw the involute of a given circle. Also to draw a tangent to the curve at any given point.

If a perfectly flexible thread be unwound from a circle and kept constantly stretched, the extremity of the thread describes a curve known as the **involute** of the circle.

Let **A P** be the circle, and **P** the generating point. Draw the diameter **A P**. At **A** draw the tangent **A B**. Make **A B** = the semi-circumference of the circle. (Radius × 3·1416, taken from a scale.) Divide **A B** and the semi-circumference into the same number of equal parts (say 6). Draw tangents to the circle at points 1, 2, 3, 4, 5, 6, &c. Make 1 **C** = **A** 1, 2 **D** = **A** 2, 3 **E** = 3 parts, &c. To obtain points beyond **B**, proceed in the same manner; 7 **H** = 7 divisions, 9 **K** = 9 divisions. Through the points **C**, **D**, **E**, **F**, &c., draw the curve.



Probs. 274, 275.



Prob. 276.

To draw a **tangent** at **T**. Draw a line from **T** tangential to the circle. This line is the **normal**; the tangent is perpendicular to it.

Note.—The involute of the circle, and the cycloid, epicycloid, and hypocycloid, are employed in shaping the teeth of wheels.

SPIRALS

The **spiral** is a curve which gradually recedes from, or approaches to, a fixed point or centre called its **pole**, by some definite law. Thus in the **Archimedean spiral** the radii increase in succession from the pole by equal distances ; in the **Logarithmic spiral** the lengths of the radii are in geometrical progression.

A line drawn from the pole to any point of the curve is a **radius**. The curve may consist of any number of **turns** or **convolutions**.

PROBLEM 277.—To construct an Archimedean spiral, the longest radius and the number of convolutions being given.

Let **O A** be the longest radius, the number of convolutions being two. Describe a circle with radius **O A**. Divide **O A** into as many equal parts as convolutions required—in this case two. Divide the circle into any number of equal parts (say 8), and draw the radii **O B**, **O C**, &c. Divide **A** a into the same number of equal parts (8). Make **O** b = **O** 7, **O** c = **O** 6, &c., each radius of the spiral diminishing by one part. Draw the first convolution through the points b, c, d, &c. The second turn, being parallel to the first, is most easily obtained by setting off the distance **A** a from each of the points b, c, d, &c.

Note.—The spiral is of great service in designing cams—contrivances much used in machinery involving complicated and irregular movements.

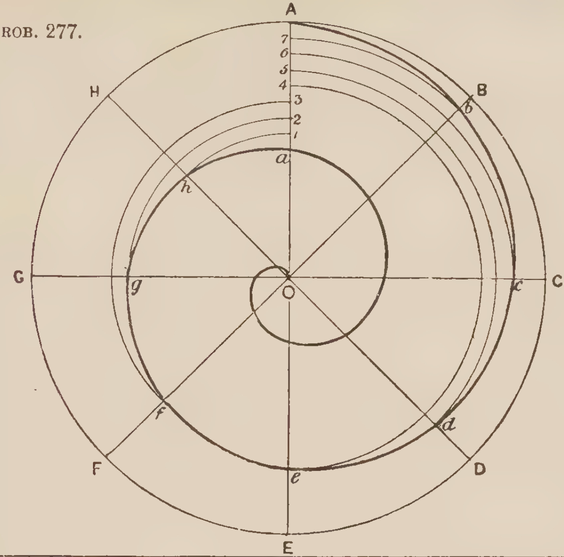
PROBLEM 278.—To construct a Logarithmic spiral, the greatest radius, the angle between consecutive radii, and the ratio of succeeding radii, being given.

The greatest radius is **O A**, the angle between the consecutive radii is 30°, and the ratio of one radius to that which follows it is $\frac{7}{8}$. Describe a circle with radius **O A**, and set off the radii at angles of 30°. For the lengths of successive radii construct a supplementary figure. Draw **O** a and **O** b at any angle. Make **O** a = **O A**. Set off **O** b equal to $\frac{7}{8}$ of **O** a, and join $a\,b$. Make **O** 1 = **O** b, and from 1 draw 1 2′ parallel to $a\,b$, then **O** 2 = $\frac{7}{8}$ of **O** 1. In the same manner **O** 3 = $\frac{7}{8}$ of **O** 2, &c. Set off successively **O** 1′, **O** 2′, &c., on the radii = **O** 1, **O** 2, &c. Through these points draw the curve.

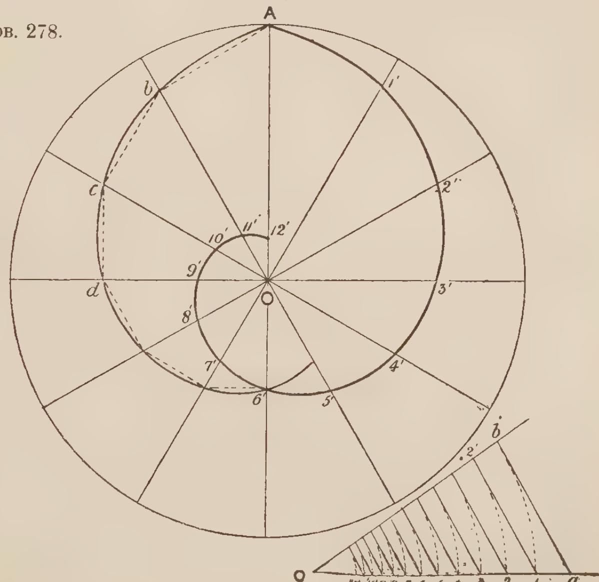
This spiral cuts all the radii at the same angle ; hence it is also known as the **Equiangular spiral**. The angle **A** 1′ **O** = the angle 1′ 2′ **O**. This gives a ready way of constructing the spiral as shown on the left of the figure. Let the angle at which the curve meets the radii be 90°. From **A** draw **A** b making 90° with radius **O** b. From b draw $b\,c$ making 90° with **O** c, &c. Through the points b, c, d, &c., draw the curve.

Note.—This spiral may also be used to determine graphically the powers and roots of numbers.

PROB. 277.



PROB. 278.



PROBLEM 279.—To draw the Ionic volute, the cathetus, or greatest radius, being given.

This forms part of the capital of the Ionic column. There are various methods of obtaining the curve, of which that known as *Goldmann's* method is perhaps the most satisfactory. The greatest radius, **A B**, is called the **cathetus.** The circle **D C** is the **eye** of the volute. The portion between the inner and outer curves, **B E**, is the fillet. The proportions of these parts are as follows: **A B** = 9, **C D** = 2, **B E** = 1.

For Examination purposes only draw the outer curve, unless the inner curve also is asked for. Always draw the problem to a large scale.

Divide the cathetus **A B** into *nine* equal parts. With centre **A** and radius equal to one of the parts describe the eye **C D**. Bisect **A C** and **A D** in 1 and 4, and obtain the square 1 2 3 4. (A larger drawing is given above.) Draw **A** 2 and **A** 3, and trisect them. (Use dividers.) From the points of trisection, 6, 7, 10, 11, draw lines obtaining the smaller squares, 5 6 7 8 and 9 10 11 12. With centre 1 and radius 1 **B** describe the first quadrant, to meet 12 produced in **F**. With centre 2 and radius 2 **F** describe the second quadrant. Proceed similarly for the other quadrants, taking the centres successively as numbered. The last arc should finish at **C**, and be described from centre 12.

To obtain the inner curve, draw $c\,b$ = **C B**, and perpendicular b 1 = **A** 1. Join 1 and c. Make $b\,e$ = **B E**, and draw perpendicular $e\,f$. On each side of **A** set off $e\,f$, and obtain the dotted square. Trisect as before for the smaller squares. The corners of the dotted squares will be the centre for the inner curve.

Notes.—1. If the height of the volute be given, divide it into 8 equal parts, and on the fourth part from the bottom describe the eye.

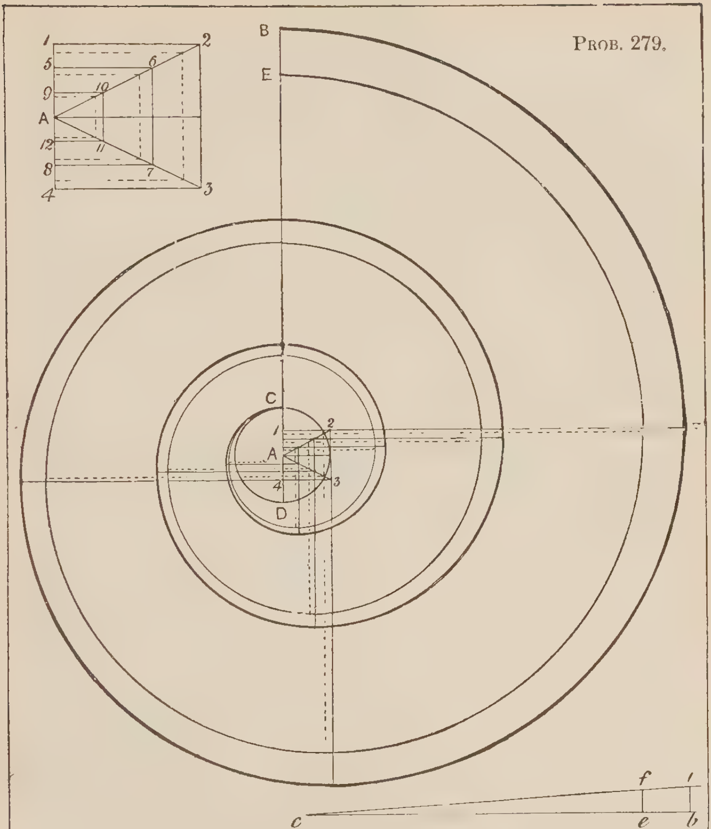
2. The greatest possible care must be taken in the construction of the squares for the centres, as the slightest error will prevent the volute from ending properly.

PROBLEM 280.—To draw a spiral scroll by means of semicircular arcs.

Let the spiral be described upon **A B**, and consist of three convolutions. Divide **A B** into six equal parts. Bisect 3 4 in **O**. With centre **O** and radius **O** 3 describe a semicircle. With centre 3 and radius 3 4 describe another semicircle. With alternate centres, **O** and 3, describe the remaining semicircles

PROBLEM 281.—To describe a continuous curve of tangential arcs passing through a number of given points.

Let **A**, **B**, **C**, **D** be the given points. Join the points, and bisect the lines by perpendiculars at **E**, **F**, and **G**. Take any convenient point, 1, in **E** 1, and describe an arc on **C D**. From 1, through **C**, draw 1 2, meeting the perpendicular from **F**. With centre 2 and radius 2 **C** describe the arc upon **C B**. Draw 2 **B** to meet the perpendicular from **G**. With centre 3 describe the remaining arc.



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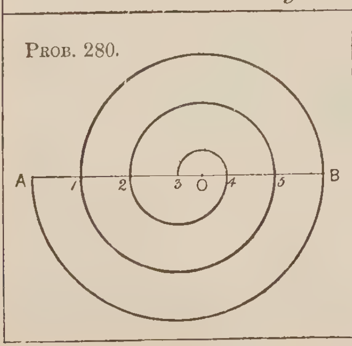

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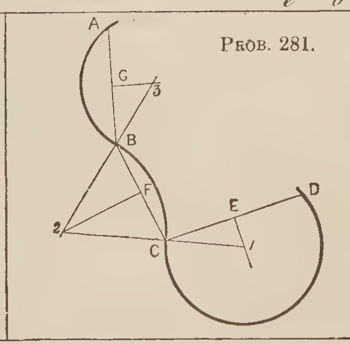


PROB. 281.

EXERCISES

CHAPTER XIII

1. Construct a square equal in area to a rectangle 2" by $1\frac{1}{2}$".

2. Draw a semicircle 3" in diameter, and divide it into 3 equal parts by means of concentric semicircles.

3. Construct a square equal in area to the sum of three squares of 1", 1·5", and 2·25" respectively. (*Sc.*, 1877.)

4. Construct a square of $2\frac{1}{2}$" sides, and through one corner draw a line cutting off $\frac{1}{3}$ of its area. (*Sc.*, 1886.) (*Prob.* 236.)

5. Draw a triangle, sides $1\frac{1}{2}$", 2", and $1\frac{1}{2}$" respectively. On a base of $1\frac{3}{4}$" construct an isosceles triangle of equal area. (*Sc.*, 1889.) (*Prob.* 217.)

6. Draw any irregular hexagon having one re-entering angle, and construct a similar figure whose sides are to those of the given figure as 7 : 3. (*Sc.*, 1889.)

Note.—Divide one side into 3 equal parts, and on 7 of these parts construct the similar figure.

7. In a square of 3" sides inscribe another square having $\frac{3}{4}$ the area of the given square. The corners of the required square must lie in the sides of the given one. (*Sc.*, 1870.)

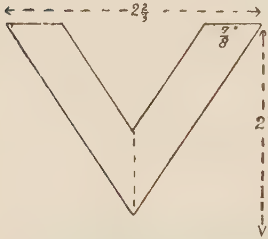
Note.—A mean proportional between the side of the given square and $\frac{3}{4}$ of the side will be the side of the required square. From this side obtain the diagonal, and from the centre of the given square, with half the diagonal as radius, describe a circle. The points where the circle cuts the sides will be the corners of the required square.

8. Construct a parallelogram having sides 4" and 1·2", and the included angle 50°. Determine a rhombus of equal area, and having the same included angle.

Note.—The side of the rhombus will equal the mean proportional between the two sides of the parallelogram.

9. Reduce the given figure (Fig. 1) to a square of equal area. (*Sc.*, 1887.)

Fig 1.



Note.—Divide the figure into 2 equal parts. Obtain a rectangle equal to 1 part, double it, and get a square equal to the rectangle.

10. Divide an equilateral triangle of $2\frac{1}{2}$" sides into 4 equal parts by perpendiculars to one side.

Note.—Bisect the triangle, and apply Problem 244 to each half.

11. Draw an irregular pentagon, and bisect it by a line drawn from one angle.

Note.—First convert the pentagon into a triangle.

12. The side of a rhombus is 3" long, and one angle is 75°. Construct the figure, and divide it into 3 equal parts by lines drawn from one angle. (*Prob.* 236.)

13. Construct an equilateral triangle of $1\frac{1}{2}$" sides, and a rectangle of equal height and area.

14. Construct a square equal in area to an equilateral triangle of 1" sides.

15. Divide a triangle whose sides are $2\frac{1}{2}$", 3", and 4" respectively, into 3 equal parts, by a line drawn from the middle of the longest side.

16. Construct a triangle, base 3", and base angles 45° and 75°. On the same base construct an isosceles triangle equal to it in area.

17. Draw a rectangle equal in area to a square of 1·75" side, making the shorter side 1·25" long. (*Sc.*, 1882.)

18. Having given a circle of 1" radius, draw another $\frac{1}{3}$ of its area. (*Sc.*, 1876.)

19. On a line, **A B**, 2" long, as base, construct a triangle, **A C B**, whose altitude is $2\frac{1}{2}$" and angle **A B C** 105°. On **A B** as base describe a second triangle, **A D B**, equal to the triangle **A C B**, and having the side **B D** parallel to **A C**.

20. Draw a quadrilateral figure **A B C D**, with the following dimensions:—**A B** = 2", **B C** = 1·5", **A D** = 1·5". The diagonal **B D** = 2", the diagonal **A C** = 2·5". Find the length of the side of a square equal in area to the quadrilateral.

CHAPTER XIV

1. Construct a semi-ellipse, major axis 3″, and half the minor axis 1″. (*Art.*)

2. Find the centre, axes, and foci of a given ellipse. (*Art.*) (*Draw the ellipse by means of thread and pins.*)

3. Describe an ellipse by means of intersecting arcs. Axes $3\frac{1}{2}''$ and 2″. (*Art.*)

4. Construct a rhombus, side $2\frac{1}{2}''$, diagonal $4\frac{1}{4}''$. In this rhombus inscribe an ellipse. (*Sc.*, 1889.) (*Prob.* 258.)

5. Describe an ellipse, the longer diameter of which is half as long again as the shorter, and at any point on the curve draw a normal to it.

6. Draw a spiral curve composed of five semicircles, whose diameters are, successively, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, and 3 inches.

7. Draw an undulating, continuous curve of six arcs of circles, each containing 90°, and described with a radius of 1″.

8. The major axis of an ellipse is 3″ long, and the foci 1″ from the centre; describe the curve, and show how to draw a tangent to it.

9. Construct a triangle, sides $3\frac{1}{2}''$, 2″, and $2\frac{3}{4}''$, and about it describe an ellipse. (*Prob.* 258.)

10. Construct an ellipse having its conjugate diameters each $3\frac{1}{2}''$ and intersecting at 70°.

11. Draw a half-ellipse with axes 4″ and 2·5″. Draw a sufficient number of normals to the curve, and produce them externally. Through points on these, 0·5″, from the curve, draw a second curve parallel to the first.

12. Draw a parabolic arch, height $2\frac{1}{2}''$, width 5″.

13. A point moves on a plane surface so that every point of its path is equidistant from a fixed line and a fixed point. Draw the curve.

14. The major axis of a hyperbola is 2″, and its focus is $\frac{1}{2}''$ from one extremity of the major axis. Draw the curve, and show how to obtain a tangent from a point in the curve.

15. The focus, directrix, and vertex of a hyperbola are given. Draw the curve.

16. The radius of a circle is 1″. Draw the cycloidal curve traced by a point on the circumference during one revolution.

17. A circle of 1″ radius rolls round a circle of 3″ radius. Draw the epicycloid traced by a point on the generating circle.

18. Draw the involute of a circle of 1·75″ diameter. The curve to be shown from its starting-point on the circumference of the circle till it cuts the produced diameter at that point. (*Sc.*, 1878, *Ad.*)

19. The line *a b* (Fig. 2) represents a piece of thread unwound from the given circle. Draw the curve traced by the extremity *a* when the thread is wound back on to the circle. (*Prob.* 276.)

20. Construct an Archimedean spiral of three convolutions, longest radius 3″.

21. Draw the spiral which would cut all its radii at an angle of 90°.

22. Draw the outer curve of Goldmann's volute, the cathetus being 4″.

23. The focus of a parabola is $\frac{3}{4}''$ from the directrix. Describe the curve making the axis $2\frac{1}{2}''$.

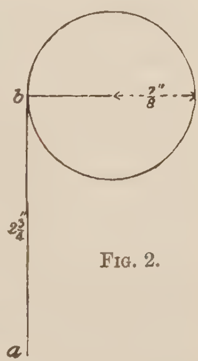


FIG. 2.

24. Draw a line **A B**, 1″ in length. At **A** draw a line **A D**, $\frac{3}{4}''$ long, making an angle of 120°, and at **B** draw a line **B C**, $1\frac{1}{4}''$ long, making an angle of 100°. Show how to describe a continuous curve of tangential arcs passing through the points **D A B C**.

CHAPTER XV

SOLID GEOMETRY

PLANS, ELEVATIONS, AND SECTIONS OF SOLIDS IN SIMPLE POSITIONS

THE preceding chapters have dealt with *Plane Geometry*—that is, with the representation of figures having *length* and *breadth* only. When we require to represent objects having *length*, *breadth*, and *thickness*, two drawings are necessary: one to show the length and breadth, and the other the height, or thickness. This representation of length, breadth, and thickness is termed **Solid Geometry**.

To illustrate this, fold a piece of paper at right angles, place it on a board or table adjoining a wall, so that one-half of the paper rests on the table, and the other half against the wall. Take a simple object, such as a box, place it on the paper, and trace its form on the horizontal fold (Fig. 1, *a b c d*); this drawing will show its length and breadth, and will be the **plan** of the box. Next, without moving the box, trace its form on the vertical fold (Fig. 1, A′ B′ *a′ b′*); this will show its thickness, or height, and will be the **elevation** of the box.

The surface, or plane, upon which the object stands is called the **horizontal** or **ground plane**, and the drawing, or **plan** on it *shows the space covered by the object on the ground, and its position with regard to the upright plane.* This upright surface is called the **vertical plane**, and the drawing traced on it is termed the **elevation**. The line represented by the crease in the paper where the two planes intersect, or cut each other, is called the **intersecting** or **ground line**.

If the paper be now opened out, the two drawings will be seen upon one surface (Fig. 2), with the line X Y showing the intersection of the two planes. The second drawing (A′ B′ *a′ b′*) is termed a **projection**, because each point of the object is projected, or thrown upon the part of the vertical plane exactly opposite to it; thus, *a′* is the projection of *a*, and the line joining *a* and *a′* is called a **projector**.

When the projection is effected by *parallel lines at right angles* to the vertical and horizontal planes, as in Fig. 2, it is called **orthographic projection**, and every part of the object is represented its correct size *by scale*, no matter how distant. In **perspective projection**, the rays of light converge towards one point, and the back lines of the object are represented slightly shorter than the corresponding front ones, as, being farther off, they appear less.

All perpendiculars and parallels must be carefully drawn, and each point should be lettered correctly as it is obtained. The system of lettering is as follows:—The same point, whatever its position, is denoted by the same letter; thus, if A represents an actual point, then *a* shows its plan, and *a′* its elevation. The student, in case of difficulty, should place the object in the required position, remembering, first, that by looking *vertically downwards* the *plan* will be seen; second, that the *elevation* is seen by looking *horizontally forwards*; and third, that *every point in the plan is exactly under the corresponding point in the elevation.*

Fig. 1.

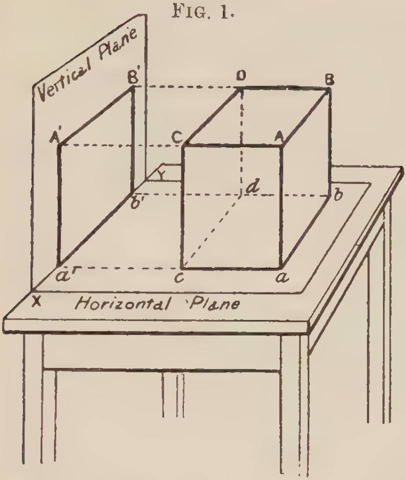
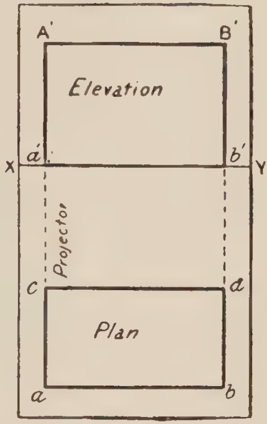


Fig. 2.



The following are the simplest solid forms :—

A **cube** is a solid figure contained by six equal squares.

A **prism** has its two ends formed by equal, similar, and parallel plane figures, and each of its sides parallelograms.

A **pyramid** has a plane figure for its base, and each of its sides are triangles meeting at a point above the base, called the **vertex**, or **apex**.

Notes.—1. Prisms and pyramids are named from the shapes of their ends and bases ; thus, a square prism has square ends, a hexagonal pyramid has a hexagon for its base ; &c.

2. The line passing through the middle of the solid is termed the **axis**. When the axis is perpendicular to the base, or ends of the solid, it is termed a **right** prism, or pyramid. When the axis is not perpendicular, the prism, or pyramid, is termed **oblique**.

A **sphere** is generated by the revolution of a semicircle about its diameter; every part of its surface is equally distant from the centre.

A **cone** is generated by the revolution of a right-angled triangle about its perpendicular.

A **cylinder** is generated by the revolution of a rectangle about one of its sides.

A **tetrahedron** is a solid contained by four equal equilateral triangles.

An **octahedron** is contained by eight equal equilateral triangles.

A **dodecahedron** is contained by twelve equal and regular pentagons.

An **icosahedron** is contained by twenty equal equilateral triangles.

If the upper part of a pyramid or cone be cut away, the portion left is called the **frustum**, and is said to be **truncated**.

Note.—Many teachers prefer to begin the study of solid geometry with the cube. Problem 291.

POINTS, LINES, AND PLANE FIGURES

The diagrams in this chapter are drawn to a small scale, and, where dimensions are not given, should be copied about *three times the size.* The *projectors* should be drawn with *very fine lines*, and the *edges* which are *not visible* should always be *dotted.*

PROBLEM 282.—To represent the plan and elevation of a point in the following positions:—1. Touching both planes. 2. One inch in front of the vertical plane, and two inches above the horizontal plane.

1. As the given point touches both planes, its position must be on the intersecting line, **X Y**.

2. Rule a projector of indefinite length. Set off 1″ below **X Y**, and 2″ above it. Then a and a' will be the plan and elevation of the point.

PROBLEM 283.—To represent the plan and elevation of a line in the following positions:—1. Standing vertically, 1″ in front of the vertical plane. 2. Parallel to the horizontal plane, 2″ above it, and at right angles to the vertical plane. 3. Parallel to both planes, and 2″ from them. 4. Lying on the ground, at an angle of 30° to the vertical plane. 5. Parallel to the vertical plane, 1″ in front of it, and making 45° with the horizontal plane.

1. Draw a line standing vertically on **X Y**; this will be the elevation. Its plan will be the point a projected 1″ below **X Y**.

2. Draw the plan $a\,b$ at right angles to **X Y**. The elevation will be the point a' projected 2″ above **X Y**.

3. Draw parallel lines 2″ above and 2″ below **X Y**; both the plan and the elevation in this position will show the full length of the line.

4. In this case the plan will show the full length of the line. Draw $a\,b$ at 30° with **X Y**. As the line is on the ground, its elevation must lie in **X Y**. Project from a and b, giving the elevation $a'\,b'$.

5. Here the elevation must be obtained first, as that will be the true length of the line. Draw $a'\,b'$ at 45°, and project the plan as shown.

Note.—The student should take a pencil, and hold in the positions indicated, if he has any difficulty in realising them.

PROBLEM 284.—The line represented in slightly more difficult cases:—

1. $a'\,b'$ is the elevation of a line inclined at 50° to the vertical plane. Determine its plan.

Project from a' and b'. At a make an angle of 50°, and draw a line to meet the projector from b'. Then $a\,b$ is the plan.

2. $a\,b$ is the plan of a line $2\frac{1}{2}$″ long; to obtain its elevation.

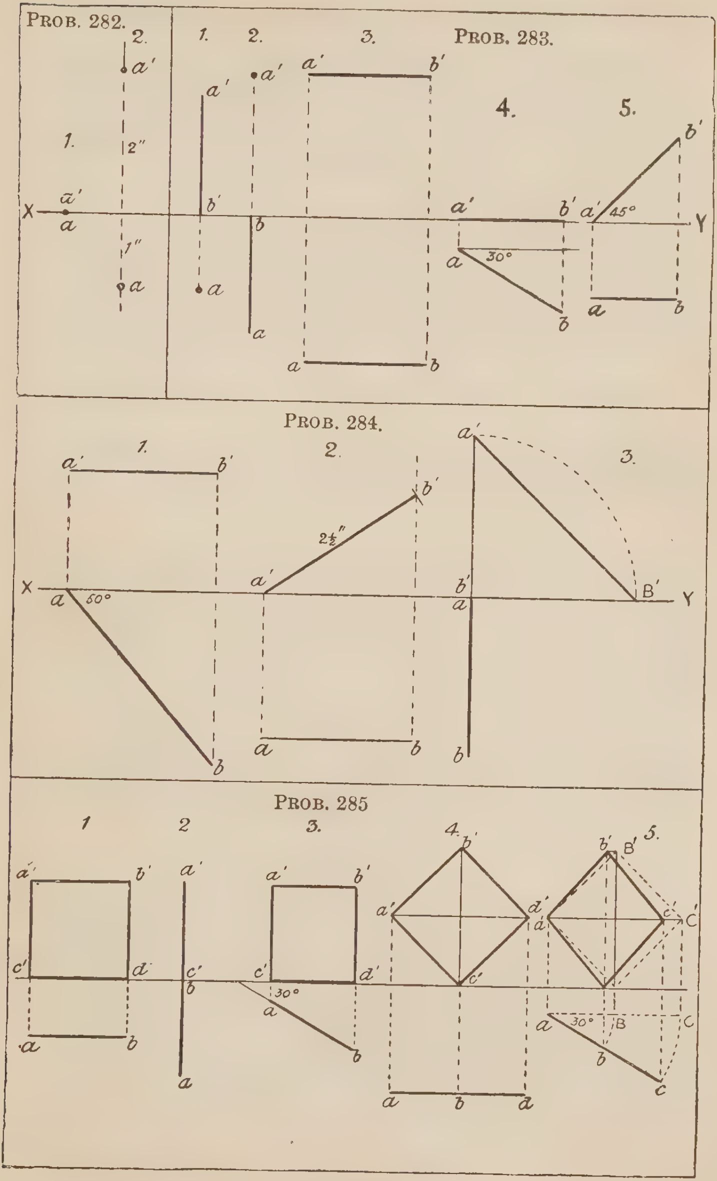
Project from a and b. With a' as centre and $2\frac{1}{2}$″ as radius, cut the projector from b in b'. Then $a'\,b'$ is the elevation.

3. $a'\,b'$ is the elevation of a line touching both planes, and making equal angles with them. Draw its plan, and find its true length.

As it makes equal angles with both planes, it will be at an angle of 45° with both, and the plan will equal the elevation. To obtain the true length, the object must be turned at right angles, when a' B′ will represent the actual line.

PROBLEM 285.—To represent a square of 2″ sides standing vertically, with its side on the horizontal plane:—1. Parallel to the vertical plane. 2. At right angles to both planes. 3. At an angle of 30° to the vertical plane. 4. Standing with its diagonal vertical, and parallel to the vertical plane. 5. With one diagonal vertical, and the other at 30° to the vertical plane.

In 1, 2, and 3, the figures will explain themselves. In 4, construct the elevation first, and then deduce the plan. In 5, draw the elevation and plan, as if parallel to the vertical plane, as shown in dotted lines; then rotate the plan into position $a\,b\,c$, at 30° to the vertical plan, and project a fresh elevation. Obtain the heights from the first elevation.


PROB. 282.
2"
1"
PROB. 283.
30°
45°
PROB. 284.
50°
2½"
PROB. 285
30°
30°

PROBLEM 286.—To draw the plan and elevation of a square of 2″ sides :—

1. Lying horizontally, with one edge parallel to the vertical plane. 2. Lying horizontally, with one edge making 30° with the vertical plane. 3 With its plane inclined at 45° to the ground, and one edge at right angles to the vertical plane. 4. With one diagonal at right angles to the vertical plane, and the other inclined at 45° to the horizontal plane.

In 1 and 2, first draw the plan in the required position, and project the elevations as shown. In 3, draw the elevation *a′ b′* at 45° with X Y, and project for the plan. Make *c a* equal to *a′ b′*.

In 4, first draw the plan *a* B C D and the elevation *a′* B′ D′ of the square when lying horizontally. Next, turn the elevation at an angle of 45° with the horizontal plane, as *a′ b′ d′*. Project from *a′ b′ d′* to meet parallels from C, D, and B, giving the plan *a b c d*.

PROBLEM 287. —To draw the plan and elevation of a square of 2½″ sides :—

1. When one of its diagonals is parallel to both planes, and the other makes 40° with the horizontal plane. 2. With two of its sides parallel to both planes, and its surface making 45° with the horizontal plane.

1. First obtain the plan *a* B C D and the elevation *a′* B′ D′ when placed horizontally, and from this obtain the elevation *a′ b′ d′*, and the plan *a b c d* when one diagonal is at right angles to the vertical plane, and the other is inclined to the horizontal plane at 40°. The position required is at right angles to this. Place the plan *a b c d* so that the diagonal *c b* is parallel to X Y. Project from this to meet parallels from *b′* and *d′*, giving the elevation *a′ b′ c′ d′*.

2. As in the last figure, draw the plan and elevation when the square is horizontal, and obtain from it the elevation and plan when inclined at 45°. Turn the plan at right angles, so that *a c* and *b d* may be parallel to both planes. Project the elevation, obtaining its height as in the previous figure.

EXERCISES

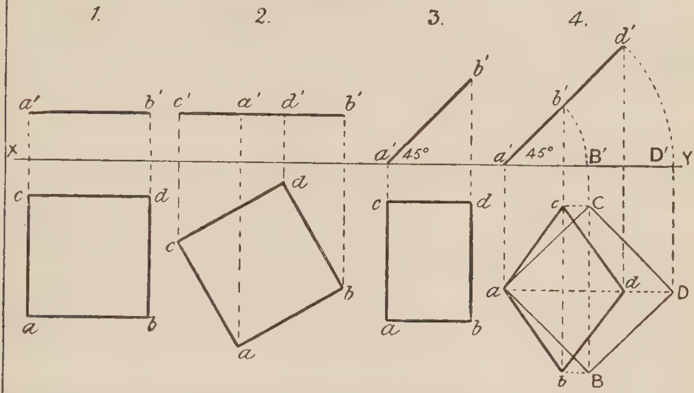
1. Draw the plan and elevation of a line 3″ long inclined at equal angles to both planes.

2. Represent the plan and elevation of an equilateral triangle of 3″ sides standing vertically on one side, with its plane at an angle of 45° to the vertical plane.

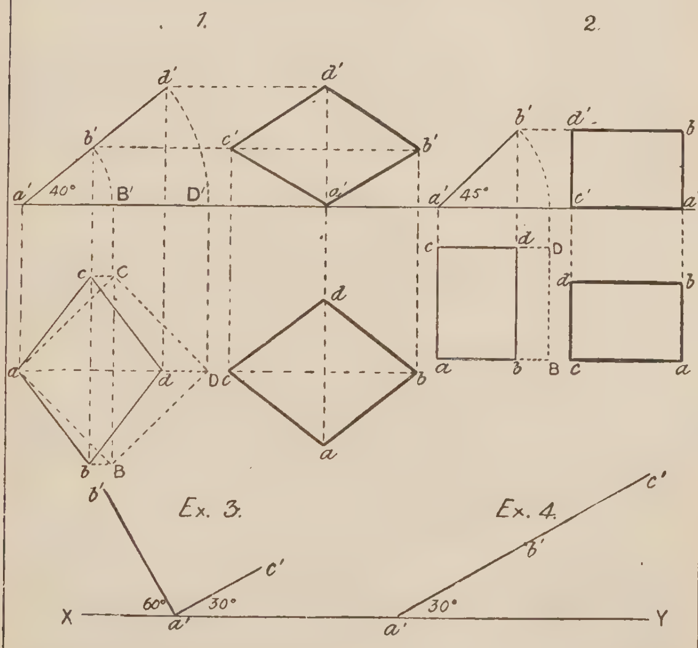
3. *a′ b′* and *a′ c′* (*Ex.* 3) are the elevations of two squares of 3″ and 2″ sides respectively, having their centres in a line parallel to the V. P. The nearest edge of the larger square touches the vertical plane. Draw their plans.

4. *a′ b′ c′* is the elevation of a square with one diagonal parallel to the vertical plane and inclined at 30° to the H. P. Draw the plan. The diagonal *a′ c′* is 3″ long.

PROB. 286.



PROB. 287.



PROBLEM 288.—To draw the plan and elevation of a hexagon of 2″ sides in the following positions:—

1. Lying on the horizontal plane, with one side at an angle of 20° with the vertical plane. 2. With a diagonal parallel to the vertical plane, and inclined at 45° to the horizontal plane. 3. Standing vertically on one edge, with its plane inclined at 50° to the vertical plane.

1. Draw a line making 20° with X Y, and on it construct a hexagon of 2″ sides. This will be the plan. Project each angle to X Y for the elevation.

2. Draw a hexagon of 2″ sides, *a* B C D E F, with one diagonal parallel to X Y. This will be the plan when lying horizontally. Project to X Y, and turn this elevation at an angle of 45°, giving the required elevation, *a′ b′ c′ d′*. From each point in this elevation draw a projector to meet the parallel from the corresponding angle of the first plan. Then *a b c d e f* will be the required plan.

3. Draw elevation *a′* B′ C′ D′ E′ F′ and plan *a* D of the hexagon when its plane is parallel to the vertical plane. Turn the plan at an angle of 50°, and from this project the required elevation, *a′ b′ c′ d′ e′ f′*.

PROBLEM 289.—To draw the plan and elevation of a hexagon of 2″ sides, having two of its edges parallel to both planes, its nearest edge 1″ from each plane, and its surface inclined forwards, at 60° to the horizontal plane.

Obtain the elevation and plan of a hexagon 1″ from each plane when two sides are at right angles to the vertical plane, as in Fig. 2, *Prob.* 288. Now turn the plan at an angle of 90°, and place it so that *a b* is 1″ from X Y; and as the plane of the hexagon slopes forward, *d e* will be farthest from X Y. From this plan project to meet parallels from the first elevation, giving *a′ b′ c′ d′ e′ f′*, the required elevation.

Note.—The plans and elevations of other plane surfaces may be solved in a similar manner, and should be worked as exercises by the student.

PROBLEM 290.—To draw the plan and elevation of a square of 2¼″ sides, one edge being in the horizontal plane, making 45° with the vertical plane, and the surface of the square being inclined at 40° to the horizontal plane.

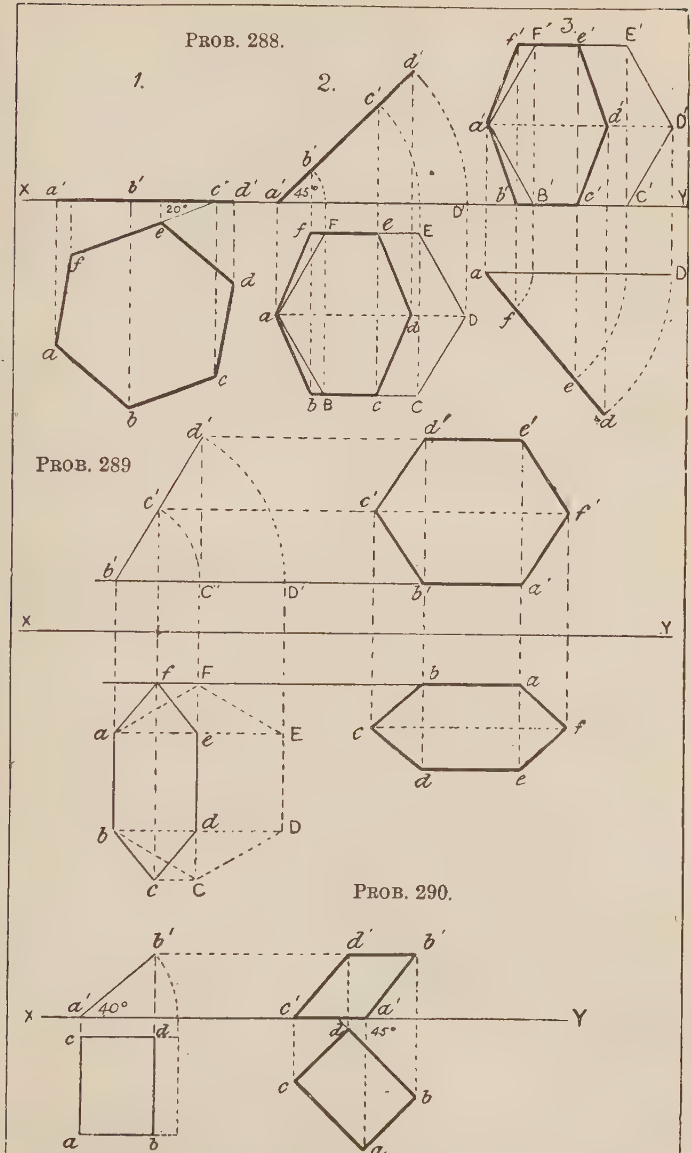
First draw the elevation, *a′ b′*, and plan, *a b c d*, when one edge is at right angles to the vertical plane and the surface is inclined at 40° to the horizontal plane.

Now place the plan, *a b c d*, at the required angle of 45°, and project the elevation. As the points *a* and *c* are on the horizontal plane, project from them to the intersecting line, X Y. For the elevations of *b* and *d* project to meet the parallel from *b′*. Join the points thus obtained for the required elevation.

EXERCISES

1. Obtain the plan and elevation of an equilateral triangle of 3″ sides when in a similar position to the square in Problem 290.

2. A regular pentagon of 2″ sides stands with one side in the horizontal plane, and inclined to the vertical plane at 45°. Its surface is inclined to the horizontal plane at 30°. Draw its plan and elevation.


PROB. 288.
1.
2.
3.
20°
45°
X
Y
PROB. 289
X
Y
PROB. 290.
40°
45°
X
Y

THE CUBE

PROBLEM 291.—To draw the plan and elevation of a cube of 3″ sides:—

1. Standing on the horizontal plane, with one face parallel to the vertical plane. 2. With one face on the horizontal plane, and with a vertical face inclined at 30° to the vertical plane. 3. Standing on one edge with one face, making an angle of 30° with the horizontal plane, and its vertical faces parallel to the vertical plane.

1. The plan will be the square *a b c d*, having two of its sides parallel to X Y. The elevation will be a square of the same size.

2. Place the square *a b c d* at an angle of 30° to X Y for the plan. From each angle of the plan draw projectors. Make *d′ h′* equal to *a b*, and draw *d′ b′* parallel to X Y, completing the elevation.

3. As the vertical faces are parallel to the vertical plane, draw *e′ f′* at an angle of 30° with X Y, and on it construct a square. This will be the elevation. For the plan, draw projectors from each angle. Take a point, *d*, at any given distance in front of the vertical plane, and draw *d g* parallel to X Y. Make *a d* equal to *e′ f′*, and draw *a f* parallel to *d g*.

Note.—The line *e h* will be dotted, because, when the cube is viewed from above, it would not be seen. In Figure 2 the line *c′ g′* is dotted for a similar reason.

PROBLEM 292.—To draw the plan and elevation of a cube of 3″ sides when standing with its vertical faces at right angles to both of the planes, and with one face inclined to the horizontal plane at an angle of 30°.

Draw the elevation and plan when the vertical faces are parallel to the vertical plane, as in Problem 291, Fig. 3. The required elevation and plan will be then seen when the projections already obtained are viewed at right angles.

Turn the plan already found so that *g f* is parallel to X Y, and project to meet the parallels from *a′ b′ f′* for the elevation.

Note.—The student should carefully follow the lettering, and note that the same point keeps the same letter all through.

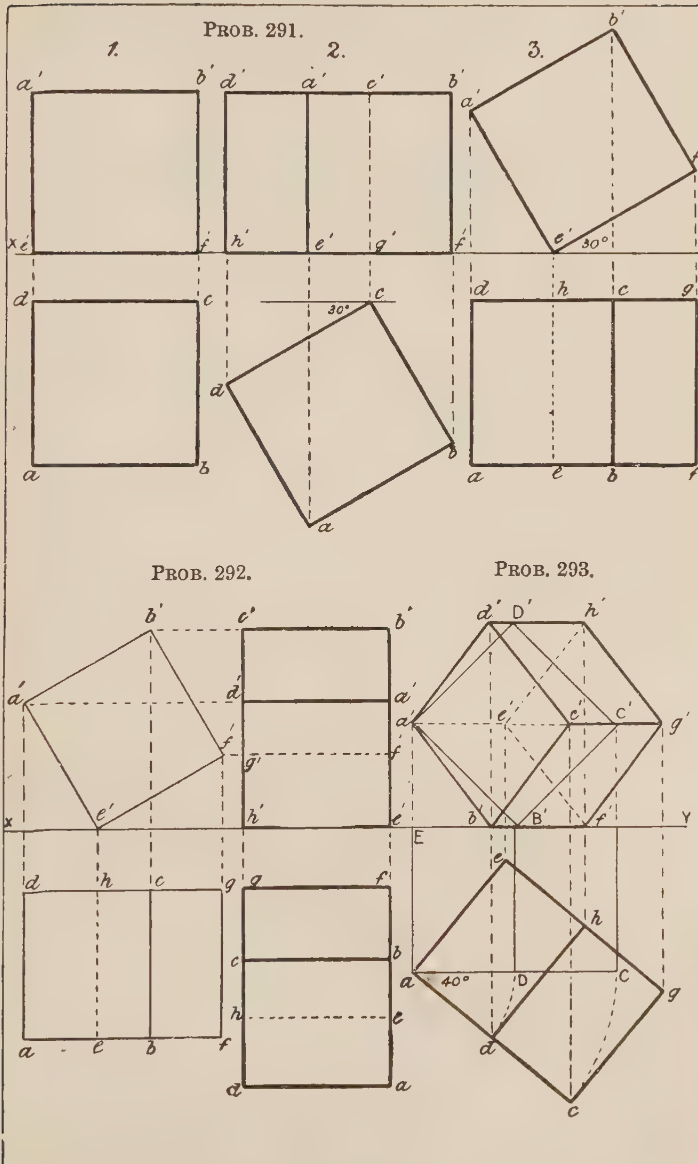
PROBLEM 293.—To draw the plan and elevation of a cube of 3″ sides standing on one edge, with its vertical faces inclined to the vertical plane at 40°, and its sloping faces inclined to the horizontal plane at equal angles.

Draw the elevation, *a′* B′ C′ D′, and the plan, E C, of the cube when its vertical faces are *parallel* to the vertical plane. Turn the plan E C so that *a c* is at an angle of 40° with X Y. For the elevation, first obtain the front face by drawing projectors from *d* and *c* to meet parallels from D′ and C′. Obtain the back square in a similar manner, and join the angles of the two squares.

Note.—The new plan may be placed at the side of the first one, if thought desirable, as in the previous problem.

ADDITIONAL PROJECTIONS

Very often the plan and elevation only of an object will not sufficiently represent it, and different views from other standpoints are necessary to explain its true form. These new views may be easily obtained by *changing*


PROB. 291.
1.
2.
3.
30°
30°
PROB. 292.
PROB. 293.
40°
X
Y

the position of the ground line and assuming fresh vertical or horizontal planes, instead of changing the position of the plan and elevation. The student should adopt this method of obtaining additional views at once. It may be of assistance at first in comprehending the new view, if the paper be turned so that the fresh ground line is horizontal. Probs. 294, 295, 300, 301, 303, 306, 318*a*, 359, 360, 361, 362, etc., furnish illustrations of this important principle.

PROBLEM 294.—To draw the plan and elevation of a cube of 3″ sides, standing on the horizontal plane, with one of its faces parallel to the vertical plane. Also draw a second elevation when one of its vertical faces makes an angle of 50° with the vertical plane.

Draw the plan and elevation as in Problem 291, Fig. 1. Now, instead of placing the plan at the given angle, turn the intersecting line, **X Y**, until it makes 50° with the vertical plane, and project at right angles from each point of the plan, making the height of the cube the same as in the original elevation.

PROBLEM 295.—To draw the plan and elevation of a cube of 3″ sides standing with its edge on the horizontal plane, its vertical faces inclined at 40° to the vertical plane, and one of its sloping faces making 30° with the horizontal plane; also another view of the object when its vertical faces are at right angles to the vertical plane.

Draw the plan and elevation when the vertical faces are parallel to the vertical plane. Now, instead of turning the plan into a position so that the vertical faces would be at the required angle, as in Problem 293, imagine a fresh vertical plane whose intersection, $X' Y'$, with the horizontal plane makes 40° with the vertical faces of the cube. If the paper be now turned so that $X' Y'$ be horizontal, the plan will be found inclined at 40° to the new intersecting line. From each point of the front face, $d\ a\ c\ b$, project at right angles to $X' Y'$, and make $a''\ b''\ c''\ d''$ the same heights above $X' Y'$ as $a'\ b'\ c'\ d'$ are above **X Y**. Proceed in a similar manner for the back face, and complete the elevation, dotting the lines which are not visible.

For the **second elevation**, draw a new intersecting line, $X^2\ Y^2$, at right angles to the vertical face, $d\ a\ c\ b$. Project at right angles for the elevation, and obtain the heights as before, b''' being the same distance above $X^2\ Y^2$ that b' is above **X Y**; &c.

Notes.—1. Compare the first projections of this figure with that obtained in Problem 293, and notice that one plan answers for both elevations, thus securing greater clearness and saving of time. For the second position, compare Problem 292.

2. This important principle may be applied to any solid.

PROBLEM 296.—To draw the projections of a cube when the plane of one of its faces is inclined to the horizontal plane, and at right angles to the vertical plane.

Note.—To understand this problem let the student place a book on the table, and on the cover place a small cube, or any similar object. The plan of the cube in this position will be a square; but if we open the cover of the book, still keeping the cube fixed on it, we shall notice that the shape of the plan changes, although the object still remains the same size.

Draw the plan, a **B C D**, of the cube when standing with its face in the horizontal plane, and project the points of this face to **X Y**. Draw $a'\ v'$ at the required inclination (say 45°) to the horizontal plane, and rotate $B'\ C'$ into it, giving $b'\ c'$. Draw perpendiculars to $a'\ v'$ at a', b', and c', and make them equal to **C D**. Then $e'\ c'$ will be the elevation of the cube. To obtain the plan, draw projectors from the top face of the cube to meet parallels from **B**, **C**, and **D**. Project the bottom face in a similar manner, and complete the plan, dotting the invisible edges.

Notes.—1. Any solid in a similar position may be projected in a similar manner by first drawing its true shape on the horizontal plane.

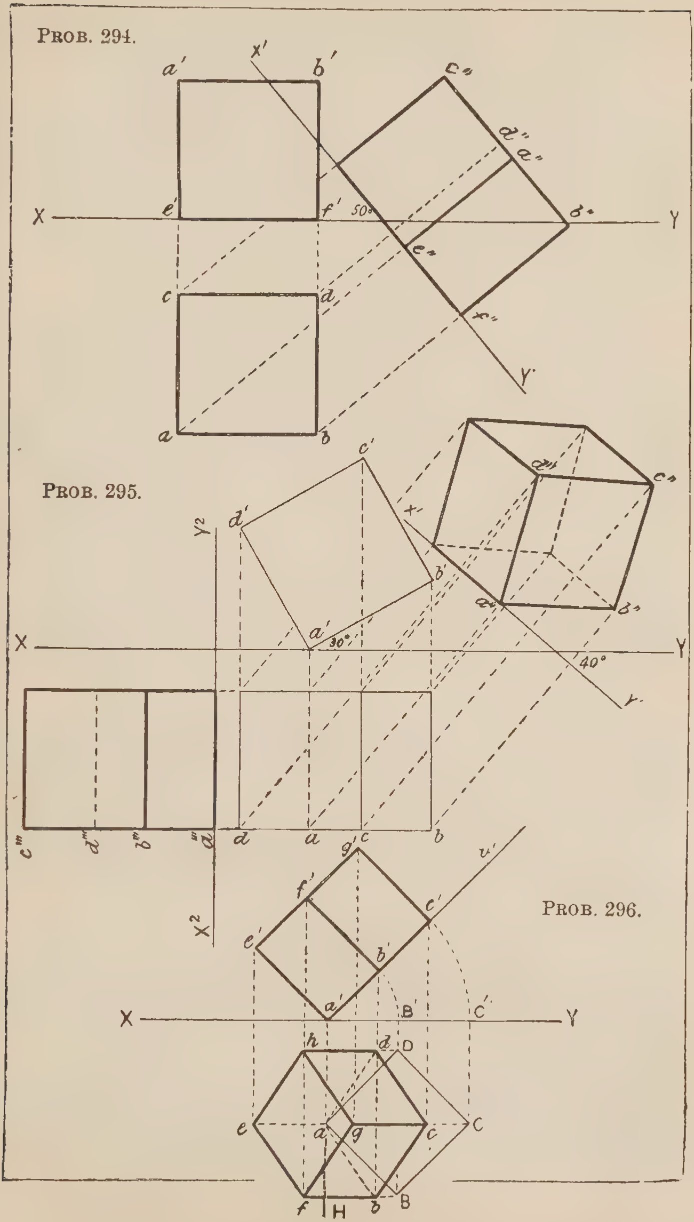
2. *All lines that are parallel to the vertical plane are seen their true length in the elevation, and all lines that are parallel to the horizontal plane are seen in their true length in the plan.*

3. The line $a'\ v'$, which represents where the inclined plane (upon which the cube is supposed to stand) cuts the vertical plane, is called the **vertical trace**; and a' **H**, which represents where this inclined plane would cut through the horizontal plane, is called the **horizontal trace.**

PROB. 294.

PROB. 295.

PROB. 296.



PROBLEM 297.—The elevation of a cube with its vertical faces at right angles to the vertical plane is given. Find the plan.

Let $a'g'$ be the elevation. Draw parallels to X Y from e', f', h'. Take any point H, and with $a'e'$ (the true length of the side of the cube) as radius, obtain points G and E. Draw E F parallel to H G, and join F G. Then E F G H will be the true shape of one of the faces of the cube. Project from c' and g', and obtain the width of the plan as shown.

Note.—The student will notice that when the cube is in this position the plan and elevation are of the same shape.

PROBLEM 298.—The plan of a cube with its vertical faces inclined to the vertical plane is given. Find the elevation.

Let $a g$ be the plan. We must first obtain the true shape of vertical face, as in Problem 297. Turn the plan so that its vertical faces would be parallel to the vertical plane as c D B A. Project from c, D, B, and A. Point d is on the horizontal plane, therefore mark D′ on X Y. With D′ as centre and $c g$ as radius, cut the projectors in c' and A′. Draw A′ B′ parallel to D′ c', and join B′ c', thus obtaining the elevation of the cube when its vertical faces are parallel to the vertical plane. Project from a, b, and d to meet parallels from A′ and B′. Join the points thus obtained, giving $a' b' c' d'$. Obtain the back face of the figure similarly, and complete the cube.

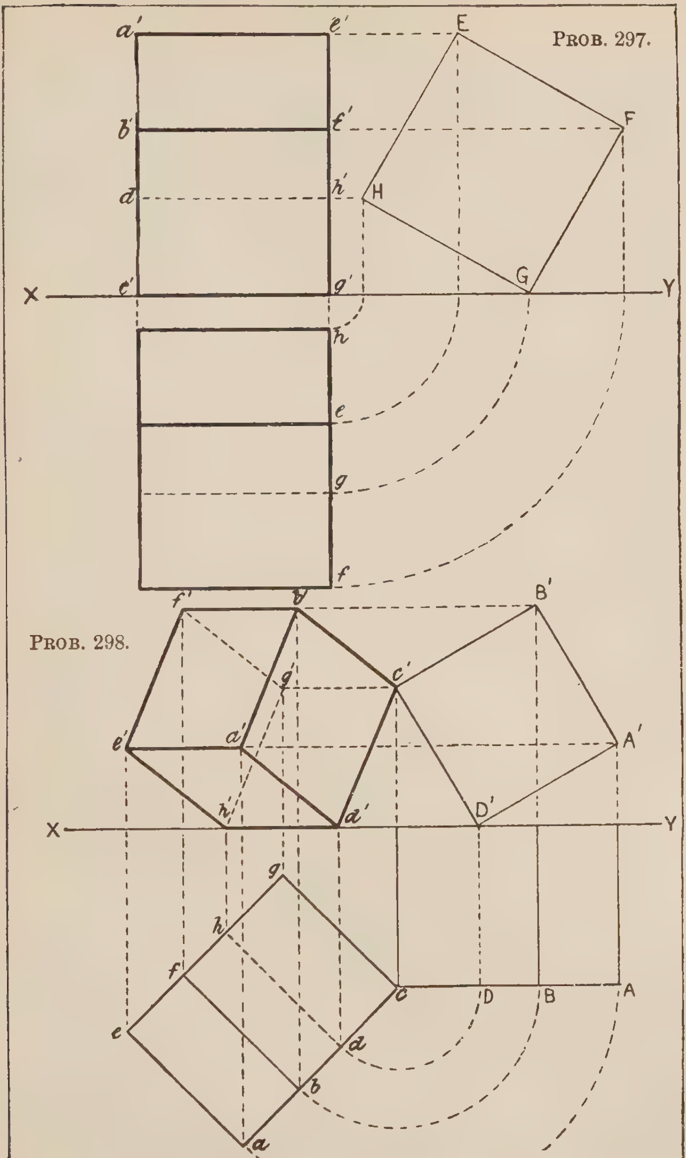
PROBLEM 299.—The elevation of a cube with its vertical faces inclined to the vertical plane at a given angle (say 45°) is given. To determine its plan.

Draw a line $a c$ (*Prob.* 298) at 45° to X Y, and project each point of the elevation to it, giving the points a, b, c, and d. Draw $a e$ at right angles to $a c$, to meet the projector from e', and complete the figure.

EXERCISES

1. Draw a square of 3″ sides standing on the horizontal plane with one side inclined at 60° to it. This square is the elevation of a cube. Obtain its plan and two fresh elevations, one on a plane inclined to the vertical plane at 30° to the right of the figure, and the other on a plane inclined to the vertical plane at 45° to the left.

2. Draw the plan and elevation of a square prism 4″ long and 2″ wide lying on one side. The length of the prism is inclined to the vertical plane at an angle of 30°. Also obtain a fresh elevation on a vertical plane inclined to one of the ends at 30°.


a'
e'
E
PROB. 297.
b'
f'
F
d'
h'
H
G
X
c'
g'
Y
h
e
g
f
f'
b'
B'
PROB. 298.
g'
c'
e'
a'
A'
h'
d'
D'
X
Y
g
h
f
c
D
B
A
e
d
b
a

PRISMS.

The **square prism**, or parallelopiped, is exactly similar to the cube in its projections, and presents no fresh points of difficulty.

PROBLEM 300.—To draw the plan and elevation of an equilateral triangular prism 3″ long, side of triangle 2″, in the following positions:—1. Standing on its end with one side inclined to the vertical plane at 20°. 2. Lying on its side with its ends parallel to the vertical plane. 3. Lying on its side with its ends at right angles to the vertical plane. 4. Standing on one edge with its axis inclined to the right at 30° with the vertical plane, and one of its rectangular faces inclined to the horizontal plane at 45°.

1. The plan will be an equilateral triangle with its side *a c* inclined at 20° to X Y. For the elevation, project as shown.

2. Draw the elevation, which will be the equilateral triangle, *a′ b′ c′*, and project the plan.

3. First draw the elevation A B C, when the end is parallel to the vertical plane. Draw a line at right angles to the vertical plane, and transfer the widths to this line, giving the points *a*, *b*, and *c*. From these points draw parallels to X Y, make them 3″ long and complete the plan. Project to meet the parallel from B for the elevation.

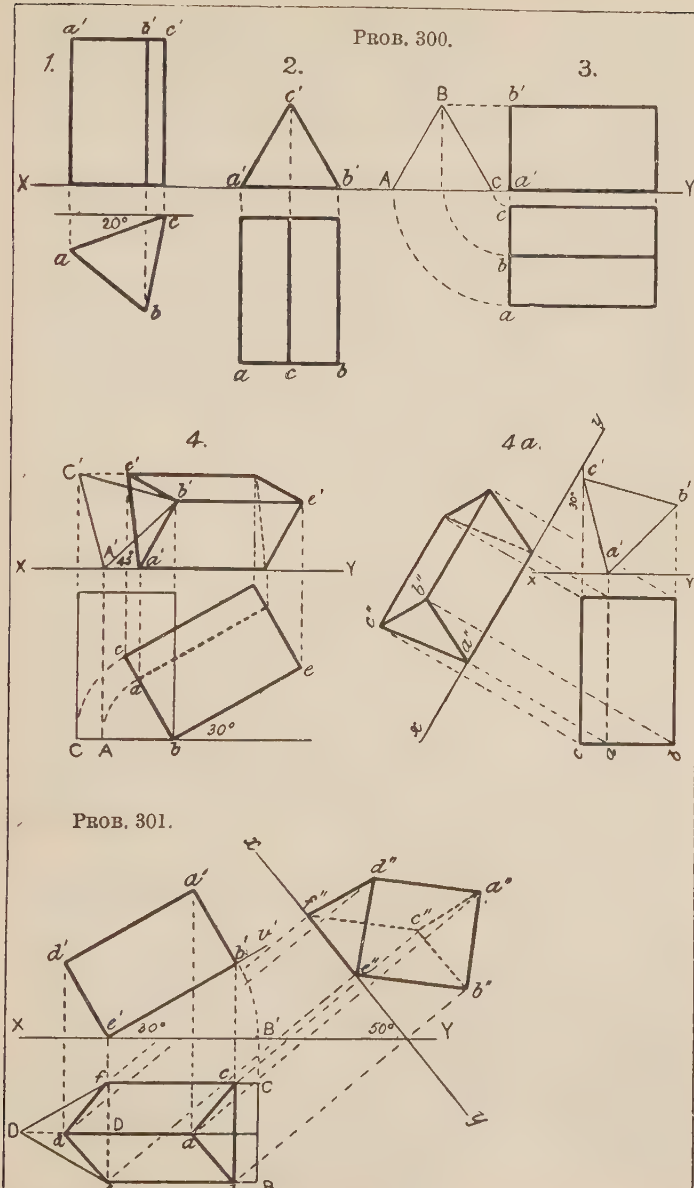
4. Draw the triangle A′ *b′* C′ with its side A′ *b′* inclined to X Y at 45°. This will be the elevation of the prism when its axis is at right angles to the vertical plane. From this obtain the plan. Turn this plan so that *b e* makes an angle of 30° to the right with the vertical plane. Draw projectors from each point of the ends to meet parallels from C′ and *b′*, and complete the elevation.

Note.—This may be solved by shifting the ground line instead of the plan, as in Problems 294 and 295. (*See* 4*a*.) Draw the elevation and plan when the axis is at right angles to V.P. Now as the prism is inclined to the right, draw the fresh ground line, *x y*, on the left side of the plan, and at an angle of 30° with its length. Project at right angles to *x y* and make *c″* and *b″* the same height above *x y* that *c′* and *b′* are above X Y.

PROBLEM 301.—To draw the plan and elevation of the same prism as above, when resting on one of the edges of its triangular ends, with one of its rectangular faces inclined at 30° to the horizontal plane, and its axis parallel to the vertical plane. Also a second elevation when the plan of its axis is inclined at 50° to the vertical plane.

Draw the plan of the prism *e* B C *f* when lying on the horizontal plane as in Problem 300, Fig. 3, and project to X Y. At *e′* draw the vertical trace *e′ v′* of the oblique plane on which the prism is supposed to stand. Make *e′ b′* equal to *e′* B′. At *e′* and *b′* draw perpendiculars, make them equal to D D, the height of the prism, and join *a′ d′* to complete the elevation. Project from each point of the elevation for the plan.

To obtain the **second elevation**, draw *x y* at an angle of 50° with X Y. Project from *d*, *e*, and *f* at right angles to *x y*. As *e* and *f* are on the horizontal plane, *e″* and *f″* will lie on *x y*. Make *d″* the same height above *x y* that *d′* is above X Y. Project from *a*, *b*, and *c*. Make *b″*, *c″*, and *a″* the same distance above *x y* that *b′* and *a′* are above X Y. Join the ends and complete the elevation.


PROB. 300.
1.
2.
3.
4.
4 a.
20°
30°
45°
PROB. 301.
30°
50°
X
Y

PROBLEM 302.—To draw the plan and elevation of any right prism. In this case a hexagonal prism 3″ long, sides of the hexagon 1½″, in the following positions:—1. Standing with one end in the horizontal plane and one of its sides inclined to the vertical plane at an angle of 50°. 2. Lying on one of its sides at right angles to the vertical plane. 3. Lying on one side with its axis parallel to the vertical plane.

1. Draw a line making 50° with X Y, and on it construct a regular hexagon of 1½″ sides. This will be the plan. The elevation may be found as shown.

2. On X Y construct a regular hexagon, this will be the elevation, and project the plan.

3. Proceed as in Problem 300, Fig. 3, by drawing the elevation A D when the end is parallel to the vertical plane. The plan will be the same as that in Fig. 2 turned at right angles, and the widths will be obtained in the same manner from the hexagon first constructed. Project from the plan to meet the parallels from C and D for the elevation.

Notes.—1. This method may be applied to all prisms. If a pentagonal prism be given, then first construct a regular pentagon, &c.

2. In the case of the hexagon it should be noted that A B is half F C, and A *f* is half A B. In this particular case the width of the prism could have been determined without the end elevation; for if the side be 1½″, then the full width *f c* would be 3″.

PROBLEM 303.—To draw the plan and elevation of any right prism (say octagonal), with its axis parallel to the horizontal plane, and inclined at 45° to the vertical plane.

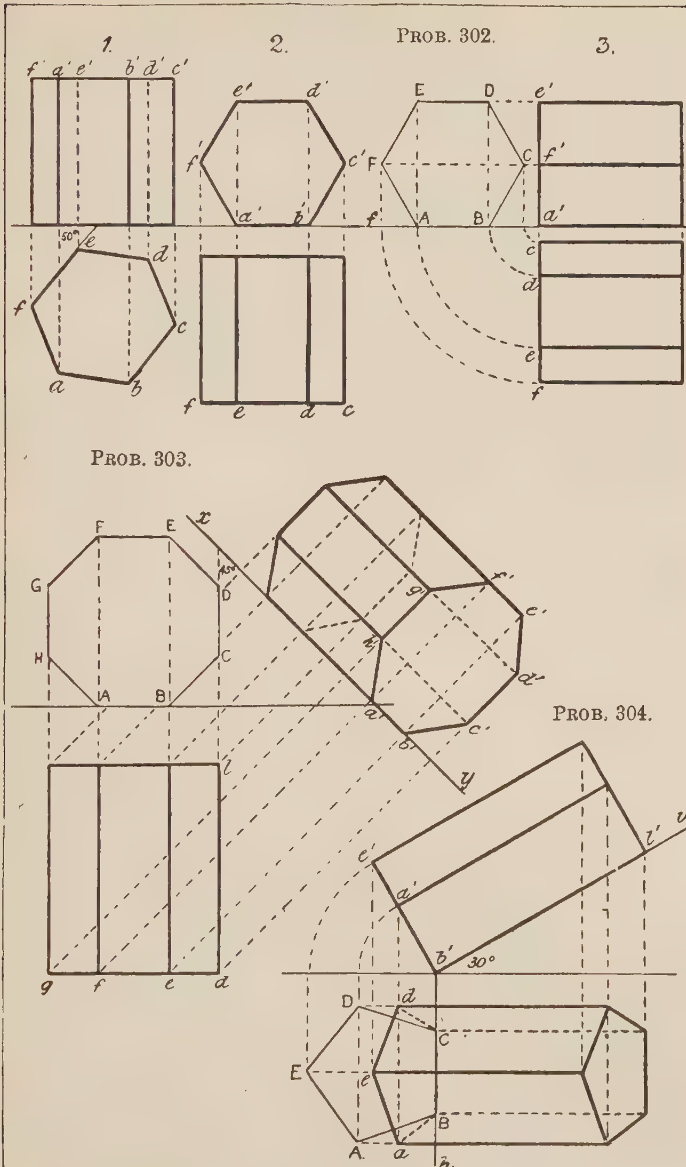
1st method.—Proceed as in Problems 293 or 300.

2nd method.—Let the sides of the octagon be 1″, and the length of the prism 3″.

Draw the end elevation A E. Project the plan *g l*. Draw *x y* at an angle of 45° to X Y. Imagine the paper turned so that *x y* is horizontal, then the plan *g l* will be seen to be at an angle of 45° to *x y*. Project at right angles to *x y* from *g*, *f*, *e*, and *d*. Make *c′*, *d′*, and *e′* the same height above *x y* that C, D, and E are above X Y, and from each point draw lines parallel to *x y*. Obtain the back end of the octagon by projecting to meet the parallels already drawn, and complete the figure.

PROBLEM 304.—To draw the projections of a prism (say pentagonal) when its axis is parallel to the vertical, but inclined to the horizontal plane.

Let the solid rest on one of the sides of the pentagon, and be inclined at 30° to the horizontal plane. Draw the traces *b′ h* and *b′ v′* of the plane on which the solid stands. On *b′ h* construct a regular pentagon A D, and from each angle draw lines parallel to X Y. Mark off *b′ l′* the required length of the prism, and erect perpendiculars at *b′* and *l′*. Project from E and D to X Y, and rotate these points to the perpendicular at *b′*. From *a′* and *e′* draw parallels to *b′ l′*, giving the elevation. For the plan, project from each point of the elevation to the parallels already drawn, as shown.


PROB. 302.
1.
2.
3.
PROB. 303.
PROB. 304.
30°

PYRAMIDS

PROBLEM 305.—To draw the plan and elevation of a square pyramid 3″ high in the following positions:—1. With its base in the horizontal plane and one of its edges making 60° with X Y. 2. With one of its triangular faces in the horizontal plane, and its base at right angles to the vertical plane.

1. Draw *d c* making 60° with X Y. On it construct a square, and draw the diagonals. This will be the plan. For the elevation project from each angle of the square to X Y, and from the centre *e* project the altitude, making *e′* three inches above X Y. Join *e′* with each of the points on X Y.

2. First obtain the plan and elevation, when the pyramid is standing on the horizontal plane with one edge parallel to X Y. Now turn the elevation so that one side rests on X Y. Then *a′ b′ e′* will be the required elevation. For the plan project from *a′*, *b′*, and *e′* to meet parallels from *c*, *b*, and E. (*See also Prob.* 362.)

PROBLEM 306.—To draw the plan and elevation of a pentagonal pyramid with its axis parallel to the vertical plane, and its base inclined to the horizontal plane at 45°. One of its edges to rest on the ground. Also an elevation of the pyramid when the plan of its axis is inclined to the vertical plane at 45°.

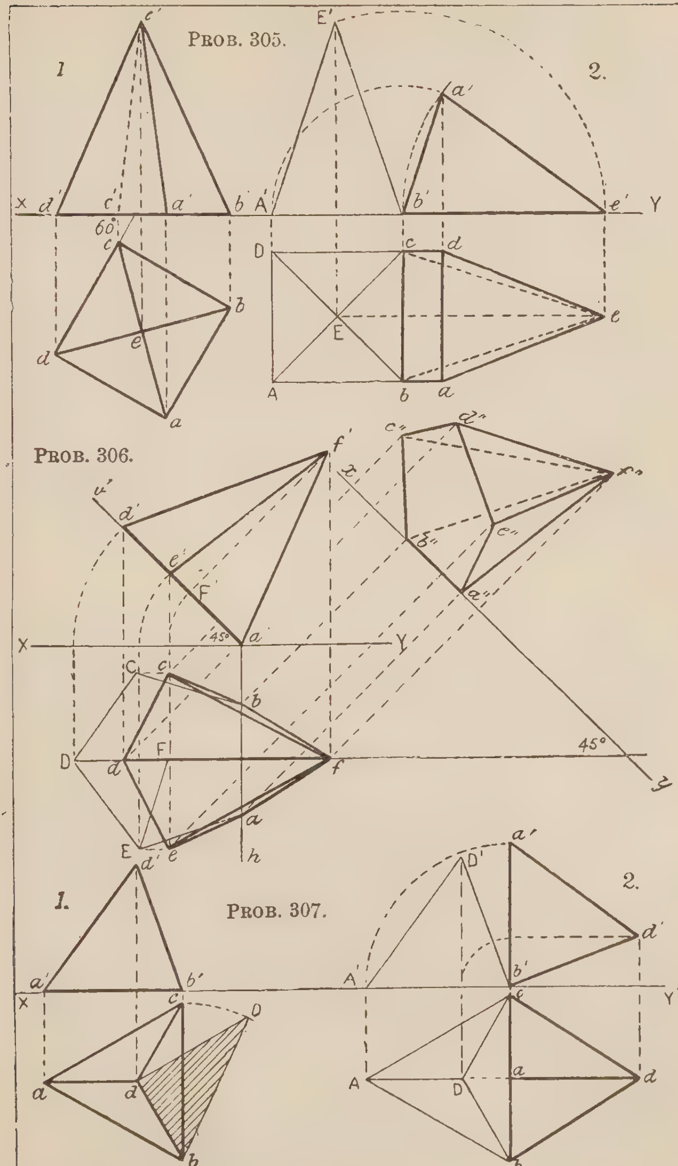
Draw the horizontal trace *a′ h* perpendicular to **X Y**, and the vertical trace *a′ v′* making 45° with it. Let the dimensions of the pyramid be as in the preceding problem. On *a′ h* construct the regular pentagon *a b* **C D E**, and find the centre **F**. Project from **D**, **C**, and **F**, to **X Y**, and turn these points into the vertical trace. At **F′** draw **F′** *f′* perpendicular to *a′ v′*, and join *f′* with *a′*, *e′*, and *d′*. This will be the elevation. For the plan, project from each point of the elevation to meet the parallels from **C**, **D**, **E**, and **F**. Join *f* with *a*, *b*, *c*, *d*, and *e*.

For the elevation, when the axis is inclined to the vertical plane, draw *x y* at 45° to the plan of the axis, and draw projectors from *a*, *b*, *c*, *d*, *e*, and *f*. Points *a* and *b* are on the ground, therefore their elevations *a″* and *b″* will be on *x y*. Make *e″* and *c″* the same distance above *x y* that *e′* is above **X Y**, and *d″* of corresponding height to *d′*. Join these points. Project *f″* as much above *x y* as *f′* is above **X Y**, and join with each point of the base.

PROBLEM 307.—To draw the plan and elevation of a tetrahedron of 2″ edge:—1. Standing with one face on the horizontal plane and one edge at right angles to the vertical plane. 2. Standing on one edge with its axis horizontal and parallel to the vertical plane.

1. Draw an equilateral triangle *a b c* with one side at right angles to X Y. Find the centre *d* and join with each angle of the triangle. This will be the plan. For the elevation project from each point of the plan as shown. To obtain the height, draw from *d* a line at right angles to *b d*. With centre *b* and radius *b c* cut the perpendicular in D. Then *d* D is the required height. *b d* D is the true size of a section from the axis of the figure to one angle.

2. Draw the plan A *b c* as before, and project to X Y. Turn the elevation so that the axis is horizontal, obtaining the altitude as before. For the plan project as shown.


PROB. 305.
1
2.
PROB. 306.
45°
45°
60°
PROB. 307.
1.
2.

PROBLEM 308.—To construct the plan and elevation of an octahedron:—1. When one of its axes is vertical. 2. When one of its triangular faces is in the horizontal plane.

Note.—The octahedron has eight faces, each of which is an equilateral triangle. It is formed of two square pyramids placed base to base, and all its edges are equal. It has three diagonals, all of equal length.

1. Draw $a\,d$ at any given angle to X Y. On it construct a square and draw the diagonals. This will be the plan. For the elevation draw a projector from e, and make the axis $e'\,e'$ equal to $a\,c$. Bisect $e'\,e'$ by $a'\,c'$ and draw projectors from each angle of the plan. Join the points a', b', c', and d', with the points $e'\,e'$.

2. As each face is an equilateral triangle, draw $a\,b\,c$; this will be the plan of the bottom face. Describe a circle about the triangle, and draw the hexagon $a\,e$. Draw $d\,e$, $e\,f$, $f\,d$.

Project the elevation of the face $a\,d\,b$. To obtain the height, with centre b' and radius equal to an edge of the solid, as $a\,b$, cut the perpendicular from d in d'. Join d' with a' and b'. From d' draw $d'\,e'$ parallel to X Y to meet the projector from e. Join e' and b'.

Note.—The student may easily make an octahedron to illustrate these positions. If $a\,b$ be the edge of the solid, develop eight equilateral triangles as shown, on cardboard or thick paper. Cut out the shape of the development, and by folding the edges of the triangles and gumming over with thin paper, the solid may be made.

THE CIRCLE

PROBLEM 309.—To draw the plan and elevation of a circle of 3″ diameter:—1. With its plane vertical and parallel to the vertical plane. 2. With its plane vertical and inclined to the vertical plane at 40°. 3. With its plane inclined to the horizontal plane at 45° and at right angles to the vertical plane.

1. The elevation will be the circle $a'\,b'$, and its plan the straight line $a\,b$.

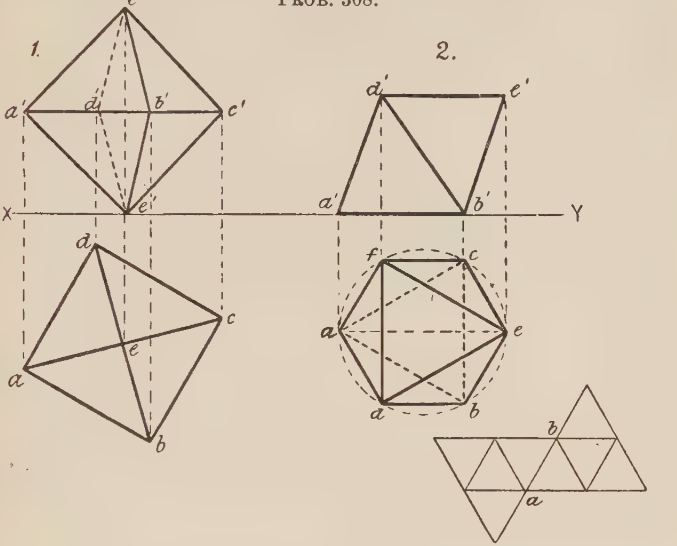
2. The plan will be the straight line $a\,b$ inclined at 40° to X Y. For the elevation, which will be an ellipse, describe a semicircle on $a\,b$. Draw the semi-diameter c C. Take any number of points in the curve as E and D equidistant from C, and draw perpendiculars E e, D d. From a, b, c, d, and e, draw perpendiculars to X Y. Make $c'\,c'$ equal to $a\,b$, and bisect it by a horizontal line for the points a' and b'. Set off e E on each side of $a'\,b'$ for the points $e'\,e'$ and $d'\,d'$. Draw the curve of the ellipse through the points thus obtained.

3. The elevation will be the straight line $a'\,b'$ at an angle of 45° to X Y. For the plan describe a semicircle on $a'\,b'$, and divide it as in the preceding figure. Project from a', e', c', d', b'. Draw $a\,b$, and on each side of it set off the widths as in Fig. 2 and draw the ellipse.

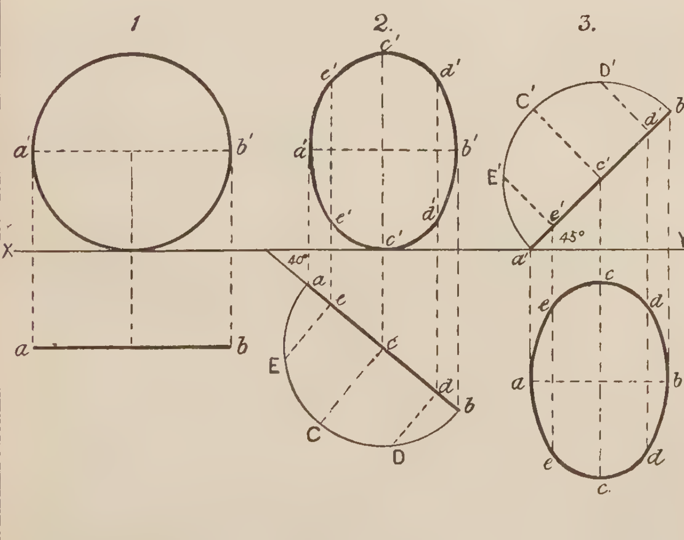
Notes.—1. It will be found convenient to divide the semicircle into 4 or 6 equal parts, thus obtaining either 8 or 12 points in the curve.

2. In large figures the French curve may be used to trace the ellipse.

PROB. 308.



PROB. 309.



PROBLEM 310.—To draw the plan and elevation of a circle of 3″ diameter when its plane is inclined at 60° to the horizontal plane, and a diameter is parallel to both planes. Also a second elevation, when the same diameter is inclined to the vertical plane at 45°.

First draw the elevation A B, when the plane of the circle is at right angles to the vertical plane and inclined to the horizontal plane at 60°.

Describe a semicircle on A B, and obtain the points D and E. From D, C, E, and B draw parallels to X Y. Draw a centre line *a′ b′*, and from this line set off on each side the distances C C, E E, &c., giving the points *c′ c′*, *e′ e′*, and *d′ d′*. The ellipse drawn through these points will be the elevation.

For the plan project vertically from each of the points *c′*, *d′*, *a′*. Then draw perpendiculars from D, C, E, and B, obtain these points on the projector from *c′*, and from each of the points draw parallels. The intersection of these lines with the vertical projectors will give the points *a*, *b*, *c*, *d*, and *e* of the plan. Through these points draw the ellipse.

For the **second elevation,** draw *x y*, making 45° with the diameter *c c*. Draw four lines parallel to *x y*, and the same distances above it as the lines from D, C, E, and B are above X Y. From *a* and *b* draw projectors to meet the lines A and B, giving the points *a″* and *b″*; from *d* and *d* draw projectors to D, giving the points *d″*, *d″*; from *c* and *c* draw projectors to C for the points *c″*, *c″*; and from *e* and *e* draw projectors to E for the points *e″*, *e″*. Through the points thus obtained draw the ellipse.

THE SPHERE

PROBLEM 311.—To draw the plan and elevation of two spheres of 3″ and 1″ diameters respectively, standing in contact with each other on the horizontal plane, the line joining their centres being parallel to the vertical plane.

As the line joining their centres is parallel to the vertical plane, the elevation will be two circles touching each other. Describe a circle of $\frac{1}{2}$″ radius on X Y. Produce its diameter and make *a′* B equal to the sum of the radii of the two circles, that is 2″. Draw a line parallel to X Y and $1\frac{1}{2}$″ distant from it. With centre *a′* and radius *a′* B intersect the parallel in *b′*. With *b′* as centre describe the larger circle.

For the plan, draw projectors from *a′* and *b′*, and with radii equal to $1\frac{1}{2}$″ and $\frac{1}{2}$″ describe two circles. Keep *a b* parallel to X Y.

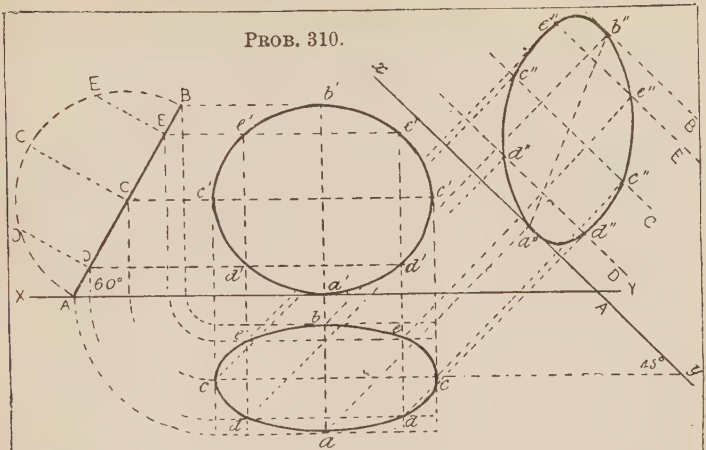
PROBLEM 312.—Four equal spheres of $\frac{3}{4}$″ radius, each touching the other three, rest on the ground plane; the lines joining the centres of the three spheres resting on the ground form an equilateral triangle, one side of which is at right angles to the vertical plane. Obtain the plan and elevation.

First draw the plan. Construct an equilateral triangle of $1\frac{1}{2}$″ sides, having its side *a c* perpendicular to X Y. With each angle of the triangle as centre, and a radius of $\frac{3}{4}$″, describe three circles. The centre *d* of the triangle will be the centre for the fourth sphere.

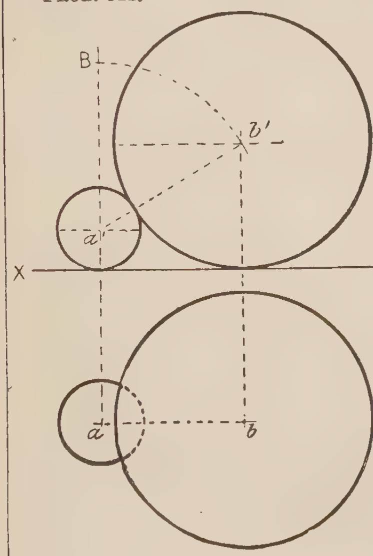
For the elevation project from *a* and *b*, and describe two circles from *a′* and *b′*. The circle described from *c* lies exactly behind the circle described from *a*. For the centre of the top circle draw a projector from *d*, and with centre *b′* and a radius of $1\frac{1}{2}$″ cut this projector in *d′*.

Note.—If *b d* had been inclined to the vertical plane then the height of *d′* above the line *a′ b′* would be found by obtaining the height of the tetrahedron formed by uniting the centres of the four circles. (*Problem* 307.)

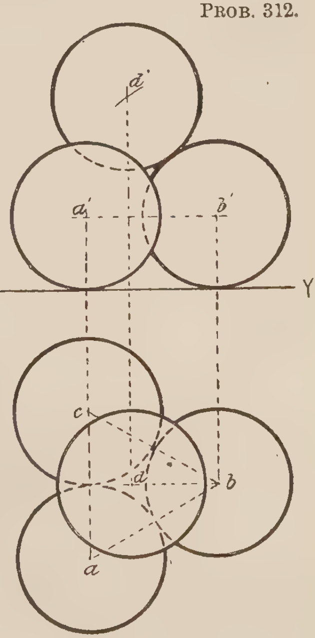
PROB. 310.



PROB. 311.



PROB. 312.



THE CYLINDER AND CONE

PROBLEM 313.—To draw the plan and elevation of a cylinder:—1. Standing with its end in the horizontal plane. 2. Lying with its axis at right angles to the vertical plane. 3. With its axis parallel to both planes.

The projections in this case explain themselves.

PROBLEM 314.—To draw the plan and elevation of a cone:—1. Standing on the horizontal plane. 2. With its axis horizontal and at right angles to the vertical plane. 3. With its axis parallel to both planes.

These projections, like the cylinder, require no comment.

PROBLEM 315.—To draw the plan and elevation of a cylinder, diameter 3″, length 4″, lying on the ground with its axis inclined to the vertical plane at 30°.

Draw *d c* making 30° with X Y, and on it construct a rectangle 4″ long and 3″ wide. This will be the plan of the cylinder.

On *b c* describe a semicircle. Divide it into 4 equal parts, and draw lines through each point of division parallel to *a b* and *c d*. From each point in *b c* and *a d* draw perpendiculars to X Y, and obtain the ellipses as in Problem 309, Fig. 2.

PROBLEM 316.—To draw the plan and elevation of a cylinder with its axis inclined to the horizontal plane at an angle of 30°, and parallel to the vertical plane. Also a side elevation of the cylinder when viewed from the left at right angles to its first elevation.

The elevation will be a rectangle with its side *a′ b′* inclined at 30° to the horizontal plane.

For the plan describe a semicircle on *a′ c′*, divide it, and draw parallels to *a′ b′* as in the previous problem. From each point in *a′ c′* and *b′ d′* draw perpendiculars. Draw *b c* parallel to X Y at any convenient distance, and on each side of it set off the ordinates as in the previous problems.

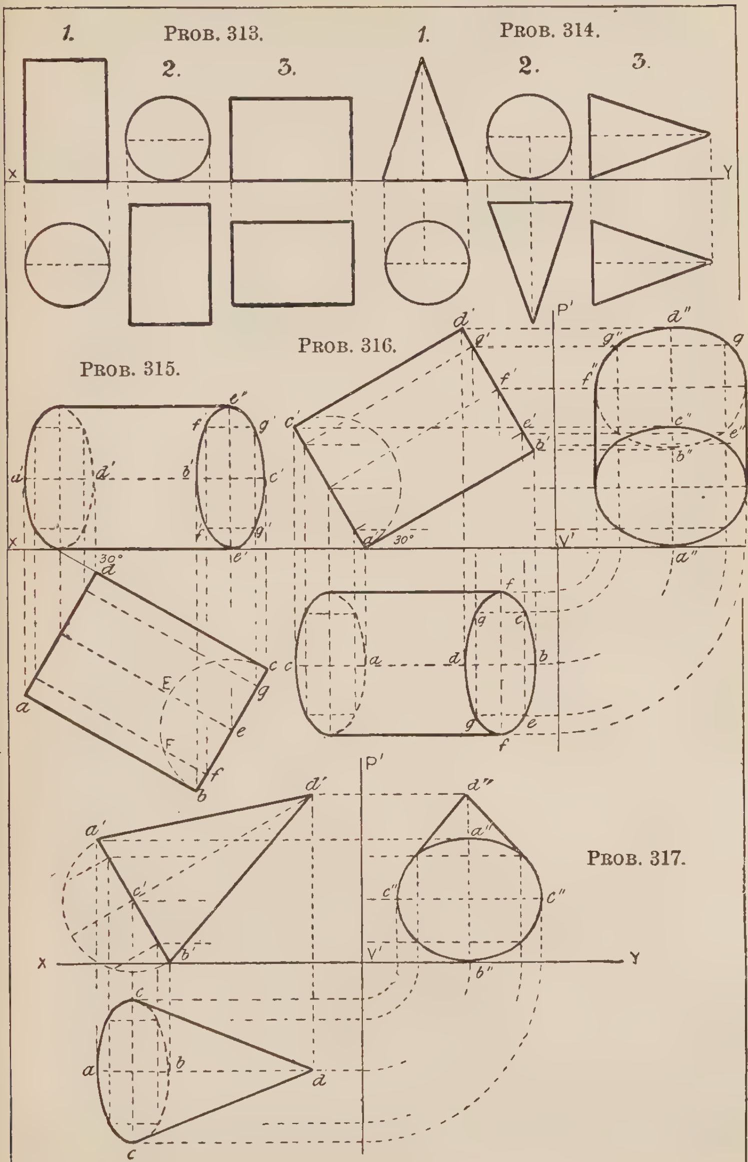
To obtain the **side elevation** first draw V′ P′, the side elevation of the vertical plane. Next draw the centre line *a″ d″*, and place it as far from V′ P′ as the centre line, *b c*, of the plan is from X Y. The cylinder will then be shown in elevation its correct distance from X Y; whatever distance *f* is from X Y in the plan, *f″* should be the same distance from V′ P′ in the side elevation. Project horizontally from each point of the elevation, and set off the ordinates on each side of the centre line as before; or turn the widths from the plan into X Y as shown.

PROBLEM 317.—To draw the plan and elevation of a cone, having its base inclined at 60° to the horizontal plane, and its axis parallel to the vertical plane. Also a side view at right angles to the first elevation.

Draw *a′ b′* making 60° with X Y. Bisect it and set up the height of the cone *c′ d′*. Join *a′* and *b′* with *d′*. This will be the elevation.

For the plan describe a semicircle on *a′ b′*, and divide it as in the previous problems. Draw a centre line *a d* parallel to X Y. Set off the ordinates, draw the ellipse and the sides of the cone.

For the **side elevation** draw V′ P′, and set off the centre line, *b″ d″*, as shown. The rest of the projection is similar to that of the cylinder. For the sides of the cone draw tangents to the ellipse from *d″*.


PROB. 313.
1.
2.
3.
PROB. 314.
1.
2.
3.
PROB. 315.
PROB. 316.
PROB. 317.

PROBLEM 318.—To draw the plan and elevation of a cone lying with its side in the horizontal plane and its axis parallel to the vertical plane.

Let the diameter of its base be 2½″, and its altitude 3″.

Draw the elevation **A** b' **F** when the cone has its axis vertical. Turn this elevation so that b' **F** will be horizontal, then $a'\ b'f'$ will be the required elevation.

For the plan describe a semicircle on $a'\ b'$, and draw perpendiculars from each point. Draw the centre line $b\ f$ parallel to X Y, and complete as in the previous problems.

Note.—This problem may be solved more readily by shifting the ground line as in 318*a*. Draw $x\ y$ along the side of the cone. If we now imagine the figure turned so that $x\ y$ is horizontal, the solution is exactly similar to that of 318, without the necessity of turning the elevation **A**b'**F**.

PROBLEM 319.—To draw the plan and elevation of a cone lying with its side in the horizontal plane, and with the plan of its axis inclined to the vertical plane at 45°.

Obtain the elevation and plan when the axis is parallel to the vertical plane as in Problem 318. This plan may now be placed so that the axis makes 45° with **X Y**, and a new elevation projected having the same height as $a'\ b'f'$. The better method, however, is to move the intersecting line to the required angle (45°). If the paper be now turned so that $x\ y$ is horizontal, then $b\ a\ f$ will represent the plan of the cone in the required position. In projecting the elevation, care must be taken to make each point in the new elevation the same distance above $x\ y$ as the corresponding point is above X Y. First project from b to $x\,y$, obtaining b'', next draw from a and make a'' as far above $x\ y$ as a' is above X Y. Then $a''\ b''$ is the elevation of $a\ b$. Now project $c\ c$, $d\ d$, $e\ e$, and f. It is only necessary to set off the height of c'', d'', and e'' once, as a parallel to $x\ y$ will give the opposite point. To complete the elevation, draw a tangent to the curve from f'' for the side of the cone.

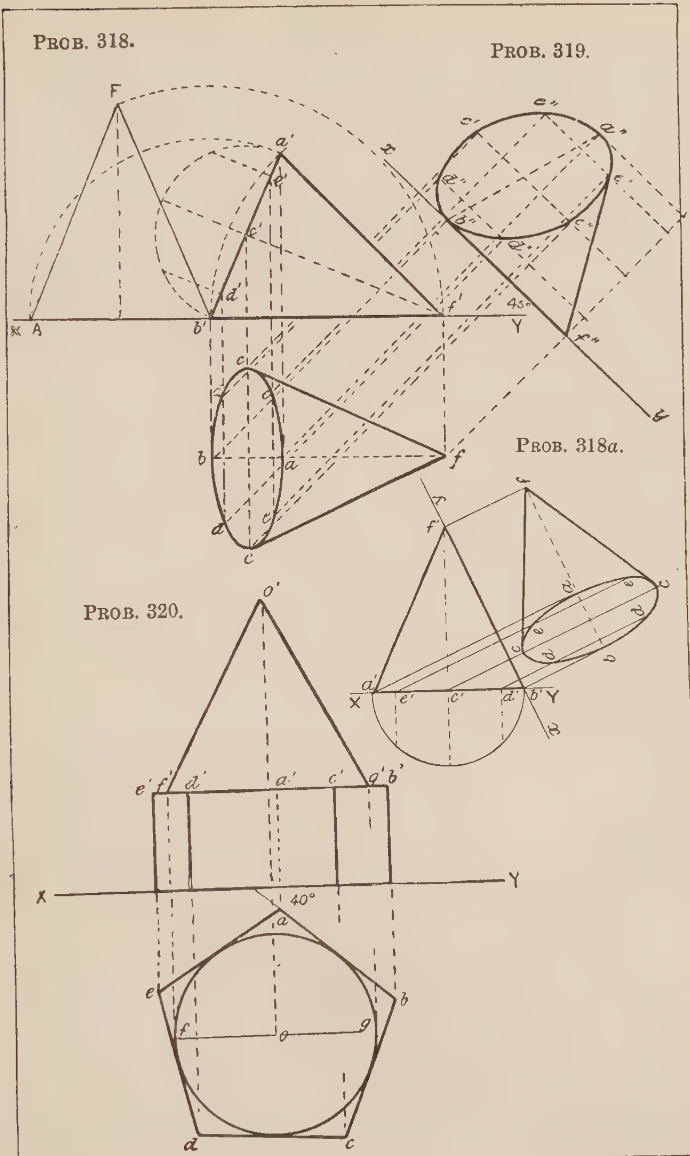
Notes.—1. A projection of the cone at any other angle may be obtained in a similar way, whether it be on its side, or as in Problem 317.

2. The projections of a cylinder having its axis inclined to the vertical plane may be obtained in the same manner.

PROBLEM 320.—A pentagonal right prism, side of base 1·5″, height 1″, rests on a horizontal plane. On it is placed a right cone, whose base circle touches the sides of the top of the prism. The height of the cone is 2″. Draw the plan and an elevation on a plane making 40° with one side of the base of the prism. (*Sc.*, 1889.)

Draw $a\ b$ making 40° with X Y, and on it construct a regular pentagon of 1·5″ sides. Find its centre and inscribe a circle. This will be the plan.

For the elevation, project from each point of the pentagon to a distance of 1″ above X Y, and draw $e'\ b'$ parallel to X Y, giving the elevation of the prism. Draw a diameter through o parallel to X Y. Project from each extremity of the diameter to $e'\ b'$, and from o to a distance of 2″ above $e'\ b'$. Join o' with f' and g'.


PROB. 318.
PROB. 319.
45°
PROB. 318a.
PROB. 320.
40°

SECTIONS

The plan and elevation of an object frequently do not furnish all the information that is necessary to construct it. In the construction of a building, for instance, not only is it necessary to show its height, and the space covered by it on the ground, but also its internal structure. To represent this we must imagine the building to be cut through by a vertical plane, and a drawing made of the cut or **section**. This drawing would show the position, arrangement, and thickness of the floors, &c.

The portion of the object that is cut through is usually indicated by drawing a series of equidistant parallel lines, making an angle of 45° with the intersecting line.

The difference should be noticed between the **sectional plan and elevation**, and the **true shape** of the section. The *sectional plan* is the appearance of the object when viewed vertically from above; the *sectional elevation* is its appearance when viewed horizontally forwards; the *true shape of the section* is seen when the section is looked at perpendicularly to the plane of section.

The sectional plan will only show the true shape of the section when the section is made by a *horizontal* plane passing through the object. The sectional elevation shows the true shape when the section is made by a plane at *right angles to the horizontal plane* and *parallel to the vertical plane.* In all other sections the true shape should be projected at right angles to the plane of section.

PROBLEM 321.—The elevation of a cube is given. Find the sectional plan and the true shape of the section made by a plane at right angles to the vertical plane, passing through $a' b'$.

Find the plan of the cube. From a' and b' draw perpendiculars to X Y. Then $a b$, $a b$ will represent the plan of the section. Draw the parallel section lines and complete.

Note.—The portion of the cube above $a' b'$ must be considered to be removed. The part of the plan in thick line would then be the only portion visible.

For the **true shape** project perpendicularly from a' and b', and set off the width $a a$ equal to $a a$ in the plan.

PROBLEM 322.—The plan of a cube cut by a vertical section through $a b$ is given. Obtain its elevation and the true shape of the section.

Project from c and d. Make $c' c'$, $d' d'$ equal to the side of the plan, and join. From a and b project to the elevation for the section.

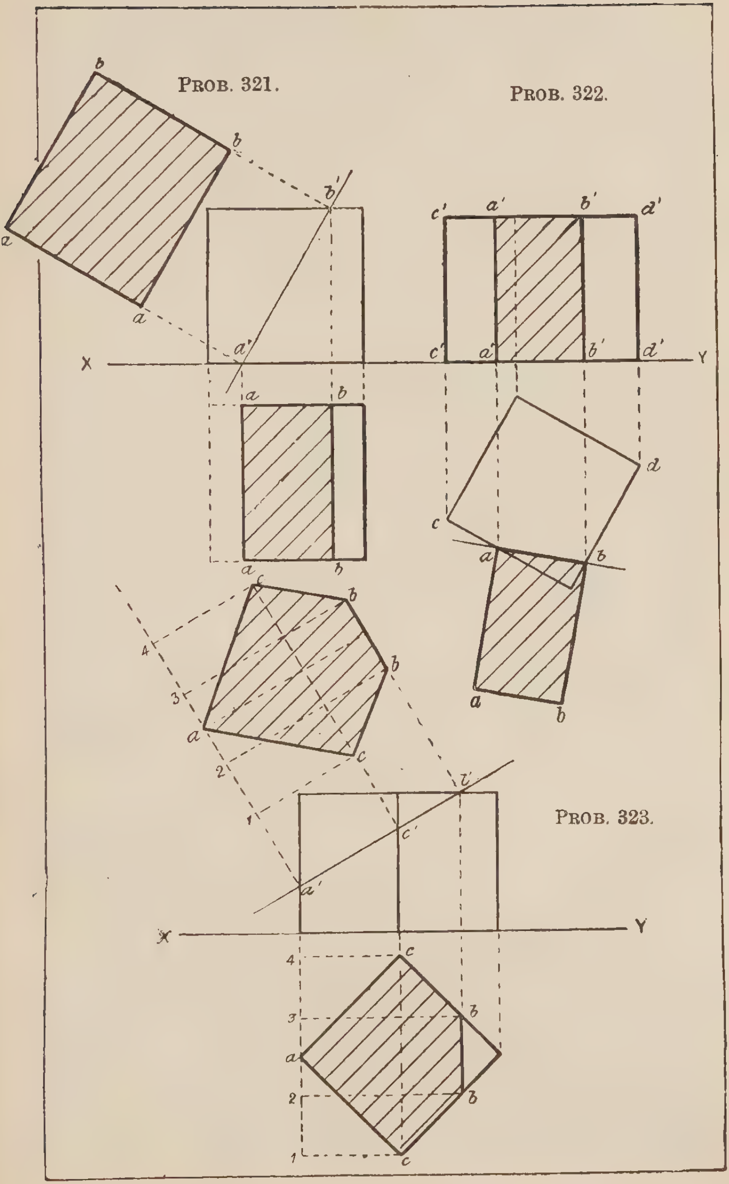
Note.—It is not necessary to project from the front corner, as the portion of the cube below $a b$ is supposed to be removed.

The **true shape** is the rectangle $aa\ bb$ having its side aa equal to $a' a'$.

PROBLEM 323.—A cube is cut by a plane passing through $a' b'$. Obtain the plan and the true shape of the section.

Draw the plan of the cube. From b' project to the plan. The whole of the top of the cube to the left of bb is cut away, and must be shaded with sectional lines.

For the **true shape**, project at right angles from each point where the section plane cuts the cube. Mark the point a on the projector from a', and draw a centre line perpendicular to $a' a$. Make $c c$ and bb equal to cc and bb in the plan.


PROB. 321.
PROB. 322.
PROB. 323.

PROBLEM 324.—A cube of 2″ sides stands on the horizontal plane, two of its vertical faces are inclined at 30° to the vertical plane. The cube is cut by a plane which is perpendicular to the vertical plane, makes an angle of 60° with the horizontal plane, and passes through the centre of the top face of the cube. Draw the plan, elevation, and section. (*Art*, 1887.)

Draw the plan of the cube with two of its sides inclined at 30° to X Y, and project the elevation. The position of the section plane must now be ascertained. Find the centre *o* of the plan. As the section plane is perpendicular to the vertical plane, the line *e e* will indicate where the plane cuts the top of the cube. Project this line to the elevation, and from *e′* draw *e′ f′* to meet X Y at an angle of 60°. From *f′* project to the plan. If we imagine the portion of the cube above *e′ f′* to be removed, then *f a b c f* will be the sectional plan.

To obtain the **true shape** of the section, we may project at right angles from each point where the section plane cuts the elevation, and transfer the widths of the section from the plan as in Problem 323, or we may project from the plan and transfer the distances *f′* 1 and *f′ e′* from the elevation. *Both methods* are shown in this problem. The student must be guided by the space at his disposal, and the general arrangement of the figure, as to which is the better course to adopt. From *e*, *e*, *f*, *f*, draw lines parallel to X Y. Set off *f′* 1 and *f′ e′* equal to *f′* 1 and *f′ e′* in the elevation. From *e′* draw a perpendicular to meet the parallels from *e*, *e*. From 1 draw a perpendicular to meet the parallel from *a*, and from *f′* draw a perpendicular to meet the parallels from *f*, *f*. Join the points thus obtained.

Note.—The student will notice that the *verticals* in the sectional plan and the true shape are equal.

PROBLEM 325.—The plan of a cube cut by a vertical section plane, 1 4, is given. Draw the elevation and the true shape of the section.

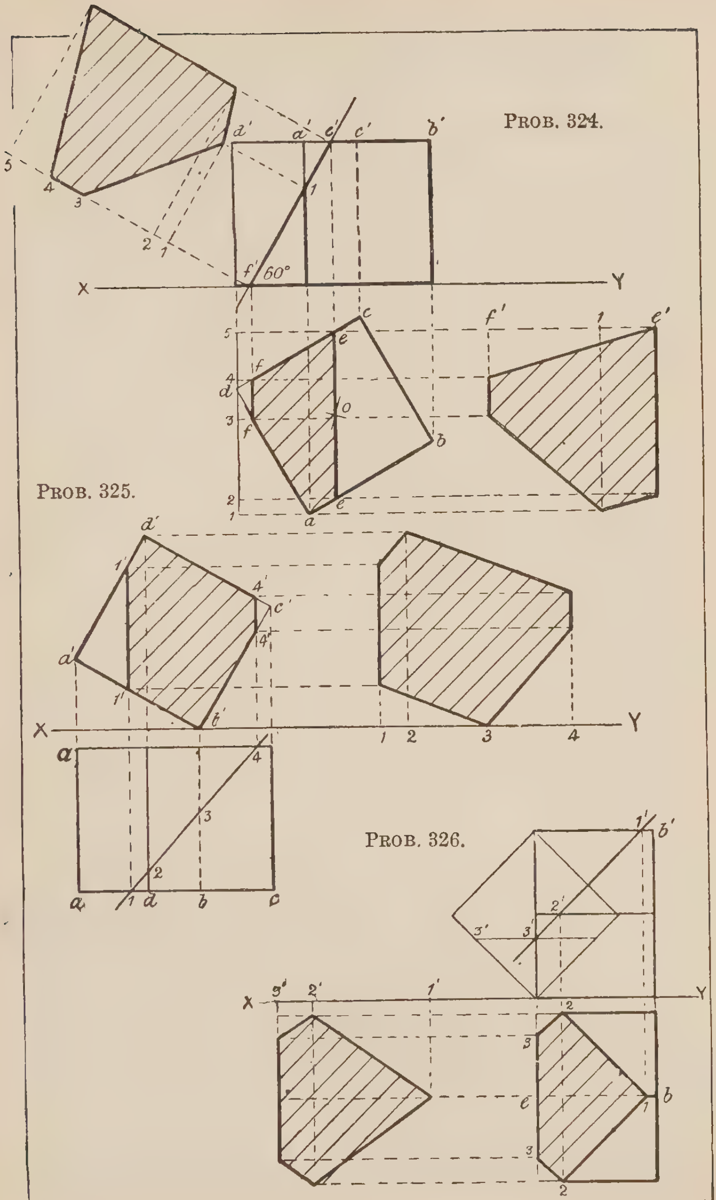
Draw perpendiculars from *a*, *b*, *c*, *d*. From *b′* with radius equal to *a a*, cut the projectors from *a* and *c* in *a′* and *c′* and complete the square. Project from 1 and 4 and complete the section.

For the **true shape** project from each point of the section parallel to X Y. On X Y set off 1, 2, 3, 4, equal to the same distances as in the plan. From 1 project to the parallels from 1′, 1′, from 2 to the top line, the point 3 is on X Y, and from 4 project to the parallels from 4′ 4′. Join the points thus obtained.

PROBLEM 326.—The elevation of a cube cut by a section plane, 1′ 3′, is given. Obtain its plan and the true shape of the section.

The plan will be the same shape as the elevation. From 1′ project to the corresponding line in the plan, from 2′ obtain 2 2. The point 3′ falls on the face of the square which is represented by a line in both elevation and plan; hence to obtain the width of the section draw the true shape of the end of the figure. The horizontal line drawn through 3′ will be the required width. Set off *e* 3 on each side of *e b* equal to 3′ 3′. Draw 32, 21.

For the **true shape** project as shown, setting off the distances 1′, 2′, 3′ on X Y equal to 1′, 2′, 3′, in the elevation.



PROB. 324.

PROB. 325.

PROB. 326.

PROBLEM 327.—The plan of a cube cut by a vertical plane, 1 4, is given. Obtain its sectional elevation.

The elevation of the cube will be of exactly the same shape as the plan. The points 2 and 3 of the section may be projected to the corresponding lines of the elevation, giving the two points 2′ and 3′. As the vertical faces of the cube are represented by one straight line in both plan and elevation, it will be necessary to draw the true shape of the vertical face as in the previous problem.

Draw parallels from each point of the elevation. Set off *b* 1, *b d*, *b f*, *b* 4, *b h* on X Y, and from *b*, *d*, *f*, and *h* draw perpendiculars to meet the parallels from the corresponding point of the elevation, giving the elevation of the cube when turned with its vertical face *b h* in front and parallel to the vertical plane. From 1 and 4 draw perpendiculars, giving the widths of the section on the vertical faces. From 1′ 1′ draw parallels to meet *e′ c′*, and from 4′ 4′ draw parallels to meet *f′ d′*. Join the points thus obtained and complete the section. Notice that the angles *d* and *f* are cut away, and are therefore not shown in the elevation.

PROBLEM 328.—The plan of a cube cut by a plane parallel to the vertical plane is given. Obtain its elevation.

Draw the elevation of the cube as in Problem 298. For the section find each point where the section plane cuts the lines of the plan, on the corresponding line of the elevation. Point 1 is on line *a e*, then project from 1 to *a′ e′*, point 2 is in *d h*, then 2′ will be in *d′ h′*, 3 is in *b f*, then 3′ will be in *b′ f′*, 4 cuts *b c* and *c d*, then its projections will be in *b′ c′* and *c′ d′*.

Join the five points and complete the section. The **true shape** will be the same as the section obtained, because the section plane is parallel to the vertical plane.

PROBLEM 329.—The elevation of a cube cut by an oblique plane at right angles to the vertical plane is given. Obtain its sectional plan, and true shape of the section.

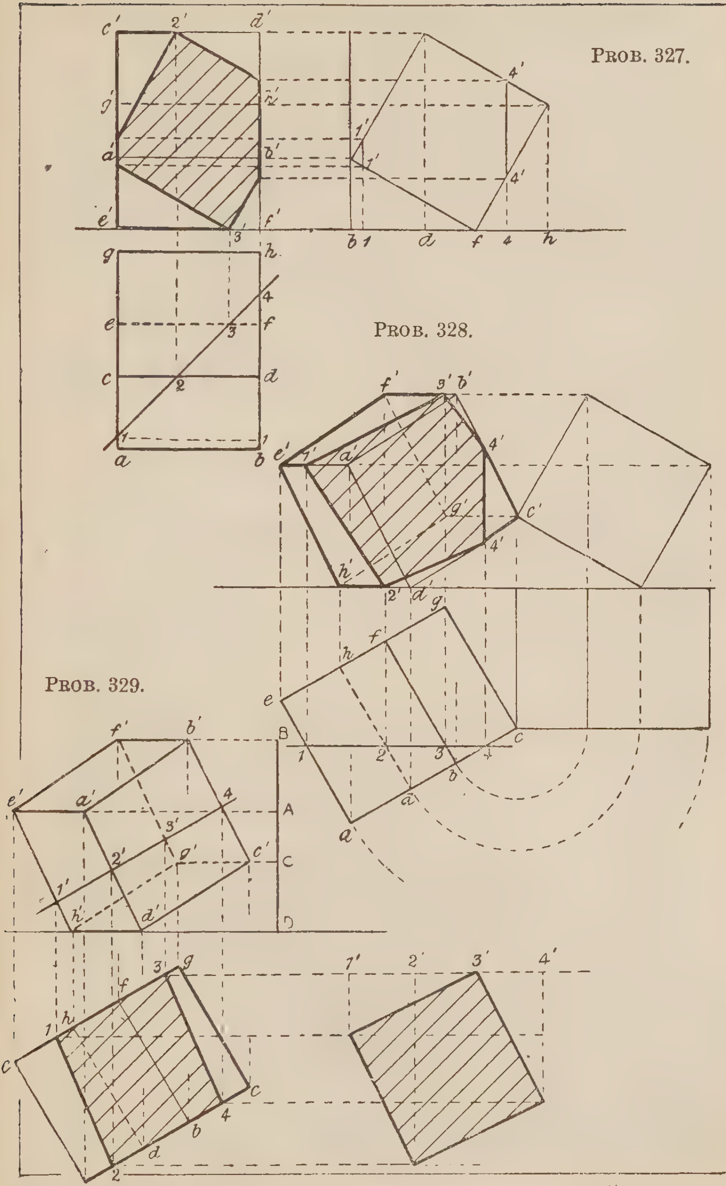
To obtain the plan, first draw the elevation D C A B of one face of the cube when at right angles to the vertical plane. The plan of this vertical face would be a line of exactly the same length. Project from each of the points *a′*, *b′*, *c′*, *d′*, and mark *c*. With *c* as centre and D B as radius, intersect the projector from *a′* giving *c a* the plan of the front vertical face. On *c a* mark off *c b* and *c d* equal to D C and D A.

Draw projectors from each point of the elevation of the back face, and from *a* draw a perpendicular *a e* to meet the projector from *e′*. Draw *e g* parallel to *a c*. Join *g c*, *b f*, and *d h*.

For the **section** project from 1′ to *e h*, 2′ to *a d*, 3′ to *f g*, and 4′ to *b c*. Join 1 and 2, 3 and 4, and complete the section.

The **true shape** of the section will be found as in the previous problems.

Note.—The cube has been treated in a variety of positions, and the student should apply similar methods of treatment to the square prism.



PROB. 327.

PROB. 328.

PROB. 329.

PROBLEM 330.—The plan of a square prism cut by a vertical plane is given. Find the elevation.

In drawing the elevation, $b' d'$ will equal $a\ c$. The section may be readily understood by following the lines.

PROBLEM 331.—The plan of an equilateral triangular prism is given. Obtain the sectional elevation and the true shape of the section when cut by a vertical plane passing through 13.

Project the elevation, obtaining its height by finding the height of the equilateral triangle described upon $a\ b$. Draw perpendiculars from 1, 2, and 3, to the corresponding lines of the elevation. Join the points thus obtained, and complete the elevation.

For the **true shape**, draw as shown, obtaining the heights from the elevation, or project from the elevation as in Problem 324.

PROBLEM 332.—The elevation of an hexagonal prism having one of the diagonals of its ends parallel to X Y is given. Draw the plan and the true shape of the section when cut by a plane, 1′ 4′, at right angles to the vertical plane.

Draw the plan, and project from 4′. For the **true shape** project horizontally from each point of the section. Transfer 1′, 2′, 3′, 4′, from the elevation, and complete as shown.

PROBLEM 333.—The elevation of an hexagonal prism is given. Obtain the sectional plan when cut at right angles to the vertical plane by the plane, 1′ 3′.

Draw the plan as in Problem 302. For the width of the section on the vertical face make 3 3 equal to $a\ b$. The problem may be easily followed from the construction lines.

PROBLEM 334.—The plan of an hexagonal prism with its axis inclined to the vertical plane is given. Obtain the sectional elevation made by a vertical plane passing through 1 4.

Project from each point of the plan. On $a\ d$ construct half a regular hexagon as shown, and set off twice $c\ c$ for the height of the elevation. Project from 1, 2, 3, and 4 for the section as shown.

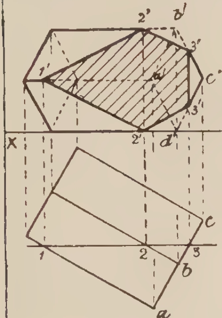
PROBLEM 335.—The elevation of an hexagonal prism with its axis inclined to the vertical plane is given. Obtain the sectional plan when cut by a plane, 1′ 4′, at right angles to the vertical plane.

First obtain the true shape of the end. Project horizontally from e' and f'. Take any point A and draw the diagonal A D at an angle of 60° with X Y. On it describe a circle and inscribe the hexagon in it. Project from each angle of the elevation. In the projector from f' take any convenient point f.

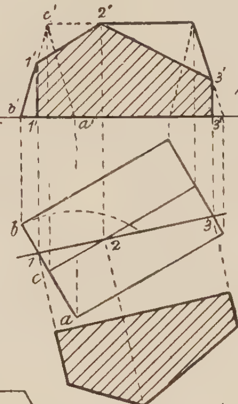
With centre f and radius F C, intersect the projector from c'. Draw $f\ c$ and mark e and d. Now draw perpendiculars to $f\ c$ from f, e, d, and c, meeting the projectors from the back face in n, m, l, and k. Join $n\ k$.

For the **section** draw perpendiculars from 1′, 2′, 3′, and 4′. Join the points thus obtained, and complete the figure.

PROB. 330.



PROB. 331.



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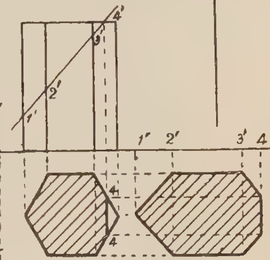

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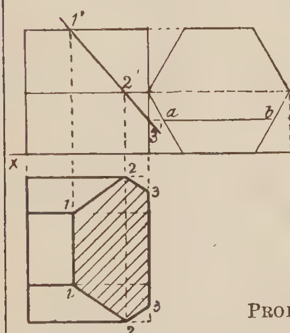

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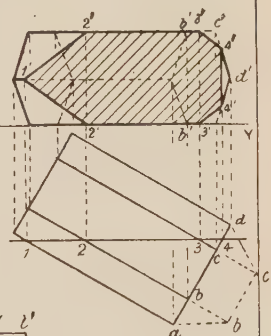
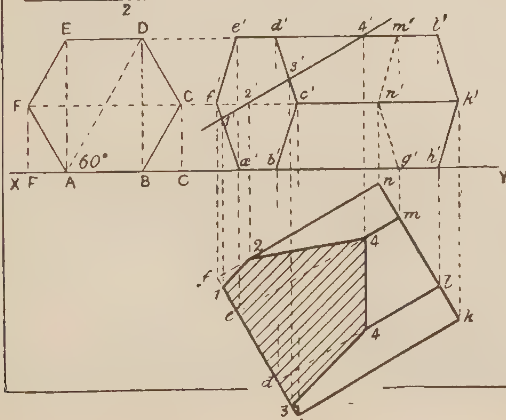


PROB. 335.



PROBLEM 336.—The elevation of a square pyramid cut by an oblique plane is given. Draw the plan, side elevation when viewed at right angles from the left, and true shape of the section.

Draw the plan. Project 1′ and 3′ to it. It is evident that 2′ cannot be projected to the corresponding line in the plan, because the line is vertical.

To obtain the plan of 2′ draw a horizontal line *a a* through 2′. Make 2 2 in the plan equal to *a a*, and complete the section.

For the **side elevation** draw V′ P′, project the angles of the plan on to it, transfer the points to X Y, and complete the second elevation. From 1′ 2′, and 3′ project horizontally to meet the edges of the pyramid. Join the points thus obtained, and complete the figure.

Note.—Finding the side elevation is the *general method* for ascertaining the width of the section of a pyramid when the point cannot be projected directly from the elevation to the plan. 2′ 2′ in the elevation is the required width for 2 2 in the plan. In a pyramid having the diagonals of its base at right angles, as in the square and octagon, a horizontal line drawn through the centre of the section as *a a* will give the width of the section at that point directly.

For the **true shape** project as shown, making 2 2 equal to 2 2 in the plan.

PROBLEM 337.—The elevation of an hexagonal pyramid cut by a plane, 1′ 3′, is given. Draw the sectional plan, and the true shape of the section.

Draw the plan of the pyramid. Project 1′ and 3′, giving 1 1 and 3 3.

As in the preceding problem, 2′ cannot be directly projected. Find the side elevation as in the last problem. 2′ 2′ will then represent the true width of the section. Transfer this to the plan, and complete the section.

The **true shape** may be easily followed from the lines.

PROBLEM 338.—The plan of an octagonal pyramid cut by a vertical plane is given. Draw the elevation and true shape of the section.

Draw the elevation, and project points 1, 2, 4, and 5. To obtain the elevation of 3, draw a side elevation of the pyramid, and project from 3 to V′ P′. Turn 3 into X Y, and erect a perpendicular to meet the outside edge of the pyramid at 3′, giving the highest point of the section.

From 3′ draw 3′ 3′ parallel to X Y, giving the required point.

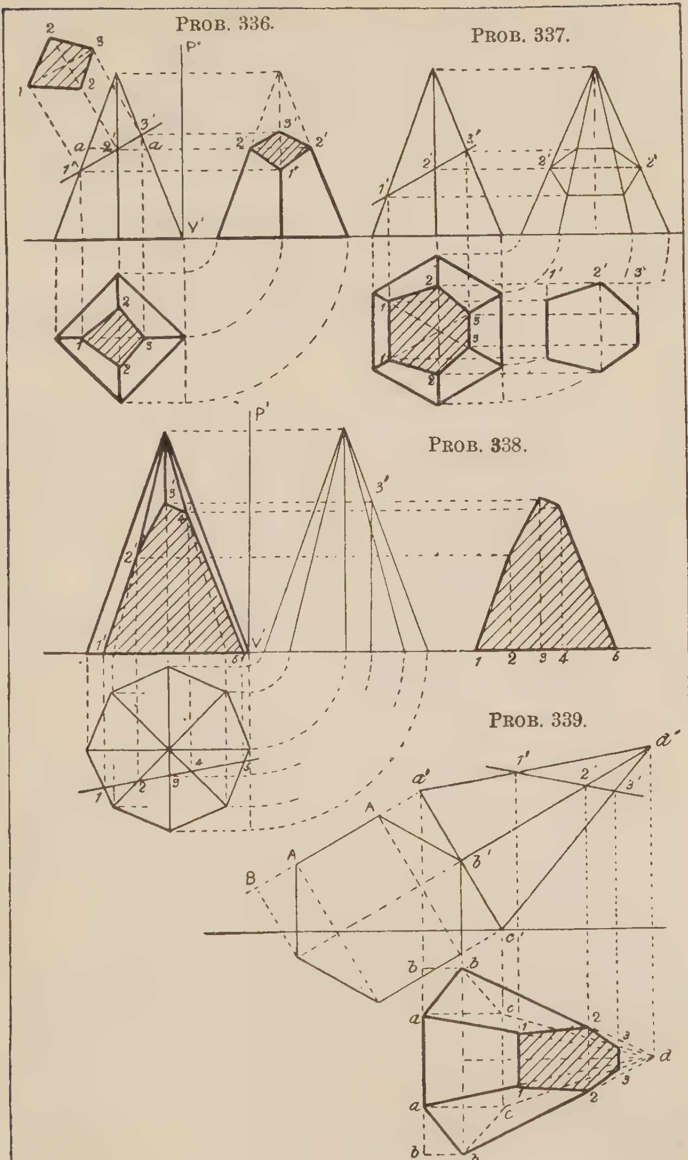
Join the points, and complete the section.

For the **true shape**, set off 1, 2, 3, 4, 5 on X Y and from 2′, 3′, 4′, project horizontally to meet the perpendiculars from 2, 3, and 4.

PROBLEM 339.—The elevation of an hexagonal pyramid cut by an oblique plane perpendicular to the vertical plane is given. Obtain its sectional plan.

First draw the true shape of the base of the pyramid.

Project from each point of the elevation. Make *b a*, *a a*, *a b* equal to B A, A A, A *a*′, and complete the plan. For the section, project from 1′ to *d a*, *d a*, from 2′ to *d b*, *d b*, and from 3′ to the remaining two edges. Join the points and complete the section.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PROBLEM 340.—The elevation of a hexagonal pyramid lying on one of its sides with its axis parallel to the vertical plane is given. Find the plan when cut by a plane passing through 1′ 3′.

Draw the true shape of the base and obtain the plan as in the previous problem. From 1′, 2′, and 3′ project to the corresponding lines of the plan, and complete the section. Notice that *b b* will not be seen in the plan, because *b*′ would be cut away by the section plane.

PROBLEM 341.—The plan of the same pyramid with its axis parallel to the vertical plane is given. Obtain its sectional elevation when cut by a vertical plane 1 5.

First draw a regular hexagon on *a b*. This will be the plan of the pyramid when standing on its base. Project to X Y and obtain the elevation. For the height, with centre E′ and radius *a g*, cut the projector from G in G′.

Turn this elevation so that A′ G′ lies on X Y.

For the **section** project from 3, 4, and 5 to the corresponding lines in the elevation. As the section plane cuts the lines *a b* and *d e* whose elevations are the points *a*′ and *e*′, these points will represent points of the section. Join the points and complete the section.

PROBLEM 342.—The plan of the same pyramid has its axis inclined to the vertical plane. Obtain the elevation when cut by a vertical plane passing through 15.

The elevation *a*′ *e*′ *g*′ must be first obtained as in the previous problem. Project from *a* and *b* to X Y, from *c* and *f* to meet the parallel from *f*′, and from *d* and *e* to meet the parallel from *e*′. Join the points thus obtained. Project *g* to X Y, and join *g*′ with the angles of the hexagon.

For the **section** project as shown from 1, 2, 3, 4, and 5.

PROBLEM 343.—The elevation of a sphere cut by a horizontal plane is given. Obtain its plan.

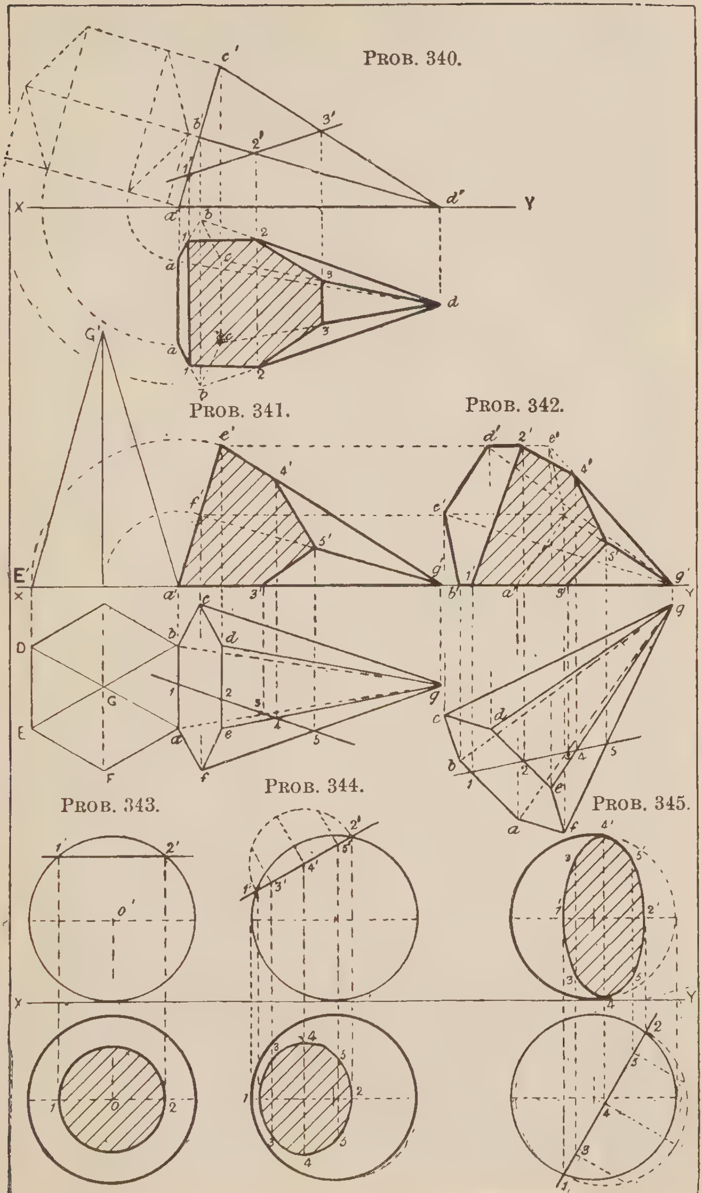
Project the centre *o*′ of the sphere, and describe a circle having the same radius as the given circle. From 1′ and 2′ project to the diameter of the plan, and describe the circle.

PROBLEM 344.—To draw the sectional plan of a sphere, when its elevation is cut by an oblique plane.

Project the circle and points 1′ and 2′ as before. We have now to obtain the plan of the circle of which 1′ 2′ is the elevation. Describe a semicircle on 1′ 2′, divide it, and draw perpendiculars. Project from 3′, 4′, and 5′, and on each side of the diameter set off distances equal to the perpendiculars at 3′, 4′, and 5′. Sketch in the curve, and complete the section.

PROBLEM 345.—The plan of a sphere cut by a vertical plane is given. Obtain its elevation.

This is solved in an exactly similar manner to the preceding problem.



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PROBLEM 346.—The elevation of a cylinder cut by a plane passing through 1′ 2′ is given. Find its plan, and a side elevation when viewed at right angles to its first position.

The plan will be a rectangle. Project from 1′ and 2′ as shown.

For the **side elevation**, turn the plan at right angles as shown, and project to meet the parallels from 1′ and 2′.

PROBLEM 347.—The elevation of a cylinder when its axis is parallel to both planes is given. Find the sectional plan made by a plane passing through 1′ 2′.

The plan of the cylinder will be a rectangle equal to the elevation. To obtain the width of the section draw half the true shape of the end. Then 2′ 2′ will be half the width of the section. Set off 2′ 2′ on each side of the centre line of the plan.

PROBLEM 348.—A cylinder standing on one end is cut by a plane, 1′ 2′. Draw its sectional plan, also the true shape of the section.

The plan of the cylinder will be a circle. Project from 2′ and complete the section.

The **true shape** of the section will be a segment of an ellipse. Project at right angles to 1′ and 2′, and draw 1 *a* at right angles to the projectors. Make 2 2 in the true shape equal to 2 2 in the sectional plan. To obtain other points in the curve, draw 3 3 through the centre of the plan, project to the section line 1′ 2′, from thence project at right angles, making 3 3 equal to 3 3 in the plan. Other points, as 4 4, may be obtained in a similar fashion. Through the points thus obtained draw the curve.

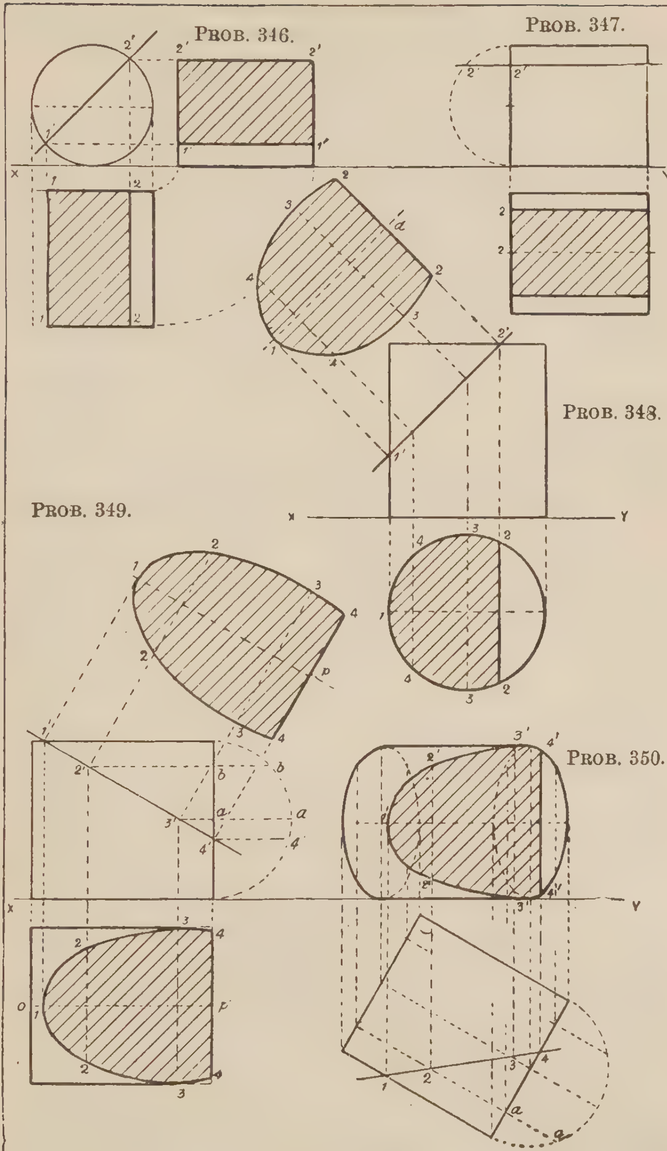
PROBLEM 349.—Given the elevation of a cylinder cut by a plane, 1′ 4′. Draw the sectional plan and true shape of section.

Draw the plan, which will be a rectangle, and mark the centre line *o p*. On the elevation describe a semicircle which will be half the true shape of the end. Project from 1′ to *o p*, then 1 will be the extremity of the axis of the ellipse. 4′ 4′ will be the width of the section on the end of the cylinder, then set off the distance 4′ 4′ on each side of *p*, giving the points 4 4. Through the centre *a* of the semicircle draw *a a*, and produce it to meet the section line at 3′. Project from 3′ to the plan, giving the points where the section is widest. Any number of points in the curve may be obtained. Take 2′ a point in the section line, and project horizontally and vertically. Set off the distance *b b* on each side of *o p* for the points 2 2. Through the points sketch the curve of the section.

For the **true shape** project at right angles as shown, and make the widths 2 2, 3 3, and 4 4 equal to the widths 2 2, &c. in the plan.

PROBLEM 350.—The plan of a cylinder cut by a vertical plane is given. Obtain its elevation.

First obtain the elevation of the cylinder as in Problem 315. Project from 1 to the centre line of the elevation for 1′, from 4 to the face giving 4′ 4′. From 3, the point where the section plane cuts the axis, project to the elevation for 3′ 3′. If the ordinate *a a* of the semicircle be produced to cut the section line in 2, two more points in the curve may be obtained. Project from 2, and set off the ordinate *a a*, on each side of the centre line for the points 2′ 2′. Through the points obtained sketch the curve and complete.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PROBLEM 351.—The elevation of a cylinder cut by a horizontal section plane 1′ 3′ is given. Obtain its plan. Also obtain an elevation when cut by a vertical plane, 3′ 5′, and viewed from the right.

First draw the plan of the cylinder as in Problem 316. Project from 1′ and 3′ for points 1 and 3 3. Produce *b′ b′* to cut the section line in 2′. Project from 2′ to the plan, and set off the distance *b′ b′* on each side of the centre line for the points 2 2, or draw parallels from *b b* in the plan as shown. Sketch in the curve and complete the section.

For the **second elevation** draw V′P′ the edge of the vertical plane, project the points to it from the plan, turn them into X Y, and erect perpendiculars to meet the parallels from the elevation as in Problem 316. Project horizontally from 3′ and 5′, thus obtaining the top and bottom of the section. From 4′ draw a parallel and set off *a′ a′* on each side of the centre line, thus obtaining the points 4′ 4′. Sketch in the curve and complete.

Note.—If another point *c′* between 1′ and 2′ be taken and projected to the plan, the ordinate *c′ c′*, set off on each side of the centre line, will give *c c*, two more points in the curve.

PROBLEM 352.—A cone stands with its base in the horizontal plane. Show:—1. A section made by a horizontal plane. 2. A section made by an oblique plane passing through the apex. 3. The true shape of a section made by a vertical plane passing through the apex.

These sections can be followed from the figures without difficulty.

PROBLEM 353.—The elevation of a cone cut by a plane 1′ 5′ is given. Draw its plan, the true shape of the section, and a side elevation viewed from the left. (See page 110 for the sections of the cone.)

There are two convenient methods for finding the sectional plan.

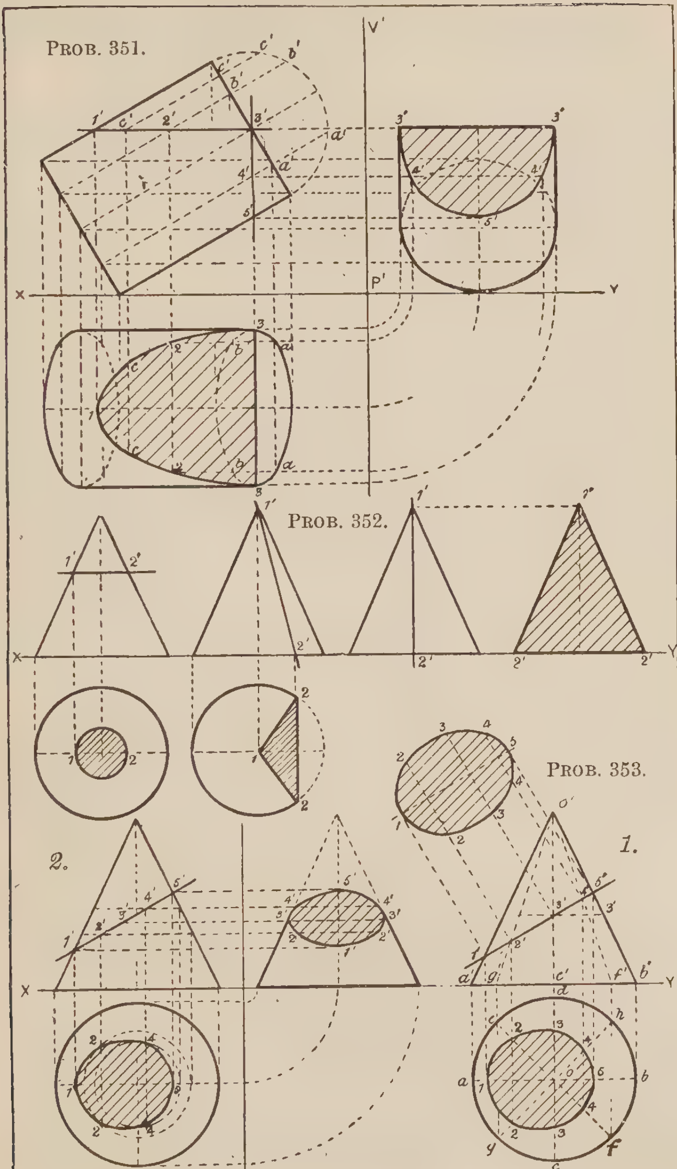
1. Draw the plan of the cone and divide it into any number of equal parts (in this case eight) by the lines *a b*, *c d*, *e f*, and *g h*. Obtain these lines on the elevation. The plane of section cuts these lines in the points 1′, 2′, 3′, &c.

Point 1′ is on line *a′ o′*, therefore its plan will be on *a o*, 2′ is on line *g′ o′*, and its plan will be on *g o*, and the corresponding point on the opposite side of the cone will be on *e o*. Project 4′ to *f o* and *h o*, and 5′ to *b o*. As 3′ cannot be projected vertically, set off 3′ 3′ on each side of line *a b*, as in Problem 336. Sketch in the curve and complete the section.

2. Draw the plan, and find points 1 and 5 as before. Take any number of points as 2′ and 4′, and imagine horizontal sections passing through these points. The plans of these sections will be circles. Draw two circles having diameters equal to the horizontal lines passing through 2′ and 4′. From 2′ and 4′, the points where the section plane cuts the circles, project to the plans of the corresponding circles. Draw the curve through the points obtained.

For the **true shape** of the section, project as shown, making 2 2, 3 3, and 4 4 equal to 2 2, 3 3, 4 4 in the plan.

For the **side elevation.** Turn the plan at right angles. From 1′, 2′, 4′, and 5′ project horizontally, and make the widths 2′2′ and 4′4′ equal to 2 2 and 4 4 in the plan.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PROBLEM 354.—The elevation of a cone cut by a section plane parallel to its side is given. Obtain the sectional plan and the true shape of the section.

Draw the plan of the cone, and find points 1 1 and 4 of the section. To find other points in the curve, take any number of points, as 2′ and 3′, and draw horizontal section lines through each point. Obtain the circles on the plan of which these lines are the elevations. Project from 2′ for points 2 2, and from 3′ for 3 3. Through the points obtained draw the curve.

For the **true shape**, project at right angles to the section line from 1′, 2′, 3′, and 4′ Draw a centre line, and make 1′ 1′, 2′ 2′, 3′ 3′ equal to 1 1, 2 2, 3 3 of the plan. The curve drawn through these points will be a *parabola*.

PROBLEM 355.—The plan of a cone cut by a vertical section plane is given. Find the elevation and true shape of the section.

Draw the elevation, and project points 1 and 7 of the section to X Y. Next obtain the height of the section. Draw *O* 4 perpendicular to the section line, and with centre *O* describe a circle touching the section line. From *a*, the point where the circle cuts *O d*, project to the elevation for *a′*. Through *a′* draw a horizontal line representing the elevation of the circle, and project from 4 to meet this line at 4′, giving the highest point of the section. To obtain other points in the curve, take 2 and 3, and describe circles through them from *O*. Project from *b* and *c*, where these circles cut *O d*, and draw the elevations at *b′* and *c′*. Projectors from 2, 3, 5, and 6 to meet these lines will give the required points of the curve. Sketch the curve, and complete.

For the **true shape**, make the perpendicular 4 4_1 equal to the height that 4′ is above X Y, and the perpendiculars at 2, 3, 5, and 6 equal to the heights of the corresponding points above X Y.

PROBLEM 356.—The elevation of a cone cut by a plane parallel to the axis is given. Draw the sectional plan.

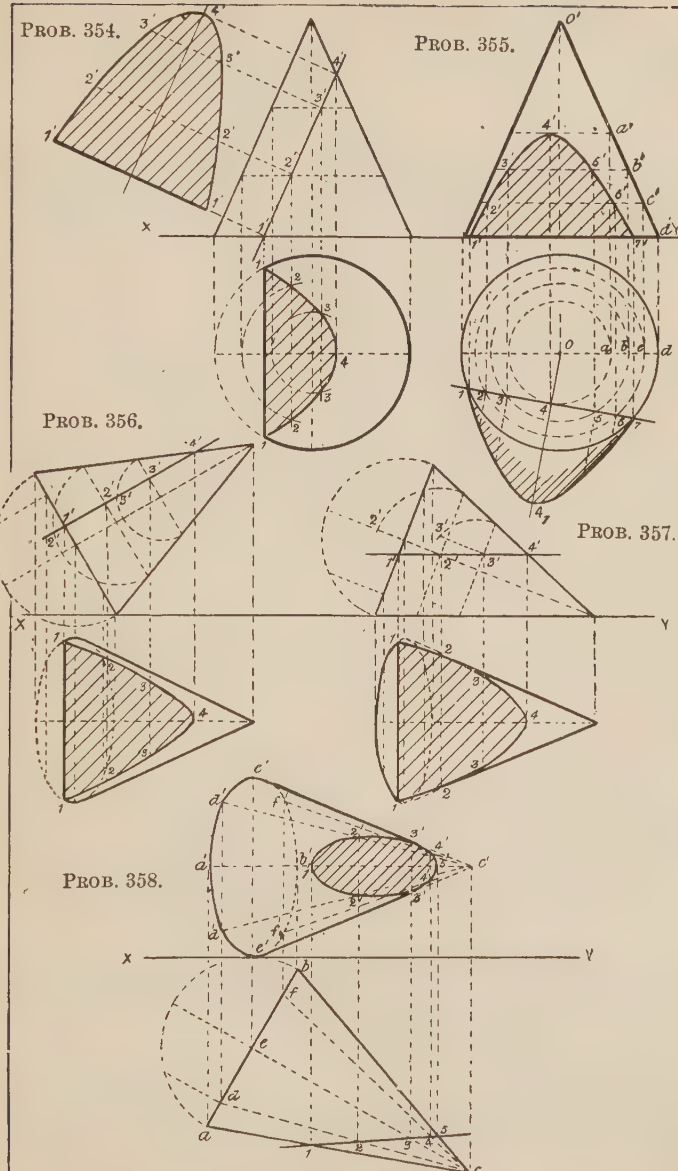
Obtain the plan of the cone by Problem 317. Project 1′ and 4′ for points 1 1 and 4. For other points, take 2′ and 3′, and draw sections through them parallel to the base. On each of these lines describe semicircles giving half the true shape of the sections at these points. Project from 2′ and 3′, and on each side of the centre line of the plan set off a distance equal to 2′ 2′ and 3′ 3′ for the points 2 2 and 3 3. Sketch in the curve through the points obtained.

PROBLEM 357.—A cone lying on its side is cut by a plane parallel to the ground-line. Draw the plan.

Draw the plan by Problem 318, and project for points 1 1 and 4. Assume other points in the section, as 2′ and 3′, and draw sections through them parallel to the base, as before. Draw half the true shape, and erect perpendiculars to the sections at 2′ and 3′. Project from 2′ and 3′, and set off distances equal to 2′ 2′ and 3′ 3′, on each side of the centre line of the plan, for points 2 2 and 3 3. Draw the curve through the points thus found.

PROBLEM 358.—The plan of a cone, its axis being horizontal, is given. Draw a sectional elevation when cut by a vertical plane, 1 5.

Project the elevation. The section may be obtained either as in the previous problem, or by assuming the cone to be a pyramid, as in Problem 353, Fig. 1. If treated as a pyramid, join *d*, *e*, and *f* with the vertex, *c*, and draw the elevations *d′ c′*, *f′ c′*, &c., of these lines. From the point 1 on *a c* project to *a′ c′*, from 2 on *d c* to *d′ c′*, from 3 on *e c* to *e′ c′*, from 4 on *f c* to *f′ c′*, and from 5 on *b c* to *b′ c′*. Through these points draw the curve of the section.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PROBLEM 359.—The elevation of three bricks is given. Draw the plan, making the width of each brick equal to half its length. Also draw a second elevation on a line, making 40° with the line of the wall.

Draw the plan as shown, making $a\,g$ = half $a\,b$. For the second elevation draw $x\,y$, making 40° with **X Y**. Project from a at right angles to $x\,y$. Set off a'' the same height above $x\,y$ that a' is above **X Y**, and draw a parallel to $x\,y$. Project from each of the corners of the bottom bricks, keeping the back lines dotted. Now project the top brick as shown, and complete the figure.

PROBLEM 360.—The elevation and plan of a square prism are given. The prism is cut by a vertical plane 1 4. Draw an elevation which shows the true shape of the section.

Suppose the portion of the prism on the right of 1 4 to be removed. If the true shape be required, the section must be viewed at right angles to 1 4. Draw $x\,y$ parallel to 1 4. Project the back end of the prism, making f'' the same height above $x\,y$ that b' is above **X Y**, and g'' and h'' corresponding to c' and d' respectively. From f'', g'', and h'' draw the long edges of the prism. The section plane cuts the edge $b\,f$ in 4, therefore project to meet the edge from f'', giving 4″. From 3 on $c\,g$ project to the edge from g'', and from 2 project for 2″. The edge $d\,h$ is not cut, and is projected as shown. The section plane cuts the square end in 1. Project from 1 and make 1″ 1″ the same height as 1′ 1′ on the given elevation. Join the points and complete the figure.

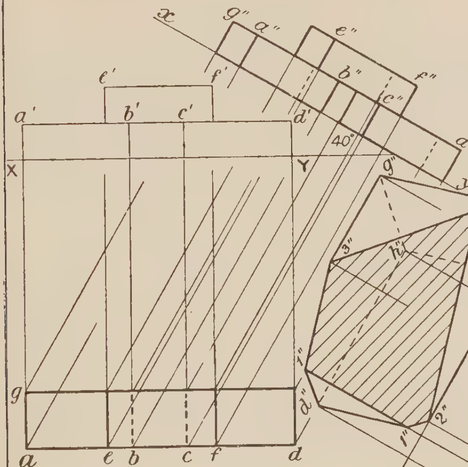
PROBLEM 361.—The plan and elevation are given of a cylinder pierced by a square opening. Draw a fresh elevation when the plane of the circular face is inclined at 45° to the V.P.

Draw $x\,y$ at 45° to the face of the cylinder. The solution may be readily followed from the figure.

PROBLEM 362.—Draw the plan of a square pyramid when one of its triangular faces lies in the horizontal plane, and when the plan of its axis is parallel to the vertical plane.

Draw the plan and elevation when standing with its base in the H.P. Now, instead of turning the elevation $a'\,b'\,c'$ so that $b'\,c'$ would rest on **X Y**, draw $x\,y$. Imagine $x\,y$ horizontal, then $a'\,b'\,c'$ becomes the elevation of the pyramid when lying on one of its triangular faces. Draw the plan as shown.

PROB. 359.



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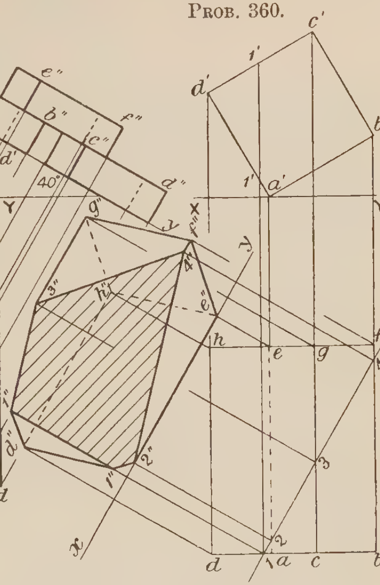

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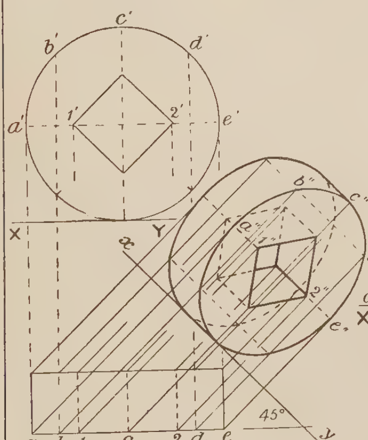

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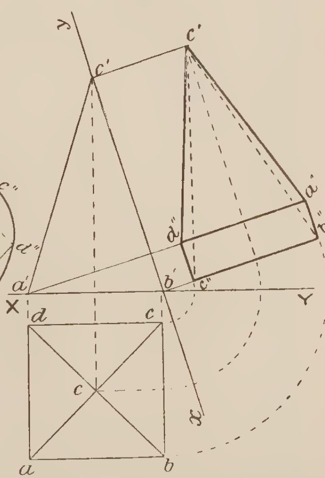


EXERCISES

CHAPTER XV

Notes.—1. Any Figures relating to these Exercises will be found on the opposite page. Copy the given Figures at least twice the given size.

1. The elevation of a right hexagonal pyramid is given. Determine its plan.

2. Draw the side elevation and plan of a pentagonal prism, 3″ long, of which the given figure is the end view.

3. The lines *a b*, *a c* are the elevations of two circles. Obtain their plans.

4. The elevation of a square pyramid cut by a plane *a b*, is given. Draw its plan.

5. The end elevation of a hexagonal prism, 1½″ long, cut by a plane *a b*, is given. Draw the sectional plan.

6. The plan of a sphere is given. Determine the sectional elevation made by a vertical plane passing through *a b*.

7. The elevation of a cylinder cut by a horizontal plane is given. Draw its plan.

8. The lines *a b*, *a c* are the plans of two unequal equilateral triangles. Draw the elevation.

9. The elevation of a square pyramid, lying on one of its triangular faces, with its axis parallel to the vertical plane, is given. Find the sectional plan made by a plane cutting the pyramid through *a b*.

10. The plan of a tetrahedron is given. Find an elevation on *a b*.

11. The elevation of a right square pyramid is given. Determine its sectional plan when cut by the plane *a b*.

12. Draw the sectional elevation of the square prism when cut by a vertical plane, *a b*.

13. The plan of a cube cut by a vertical plane is given. Find its elevation.

14. The plan of a cone is given. Find the elevation of the section made by the vertical plane, *a b*. (*Prob.* 355.)

15. The given figure represents the plan of a pentagonal prism, 1″ in height, surmounted by a cylinder 2″ in height. Draw the elevation when cut by a vertical plane through *a b*.

16. A pyramid having for its *base* a square of 2·5″ sides, and its axis 3·25″ long, rests with one *face* on the horizontal plane. Draw its plan, and a sectional elevation on a vertical plane, represented by a line bisecting the plan of the axis and making 60° with it. (*Sc.*, 1870.) (*Prob.* 305.)

17. Draw the plan of the above pyramid when its base is inclined at 47°, and one edge at 27°. (*Sc.*, 1870.)

18. Draw the plan of a right pyramid whose base is a hexagon of 1·25″ side, and its axis 3·25″, when it stands upright on a horizontal plane. Give the section made by a vertical plane which cuts off half one edge and a quarter of the next, measuring downwards from the vertex. (*Sc.*, 1871.)

19. Draw the plan of a cube of 3″ edge, in any position, as long as no edge is horizontal. (*Sc.*, 1872.) (*Prob.* 296.)

20. A pentagon, **A B C D E**, side 1·75″, revolves upon the line joining **A**, with the centre of the opposite side, till its plane becomes inclined to the vertical plane at 50°. Draw its plan. (*Prob.* 288.)

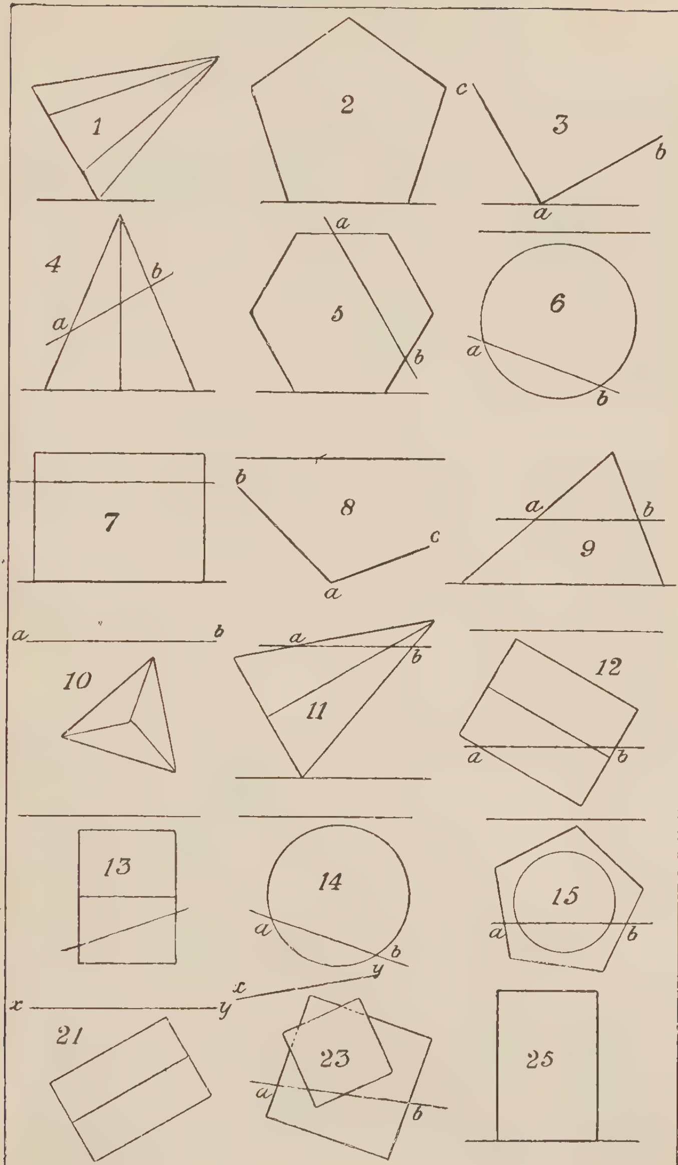
21. The plan of an equilateral triangular prism is given. Draw its elevation on **X Y**. (*Sc.*, 1876.)

22. A rectangle, length 2·5″, breadth 1·75″, is the plan of a square. Determine the inclination of the plane of the square. (*Sc.*, 1877.) (Converse of *Prob.* 286, *case* 3.)

23. The plans of two cubes are given, the smaller resting on the larger. Draw an elevation on *x y*, and a section on *a b*. (*Sc.*, 1878.)

24. A square prism, 3″ long and 1·5″ square, lies with one of its side edges horizontal, and one of its sides inclined at 30°. Make an elevation on a vertical plane which is at 50° to the axis. (*Sc.*, 1882.) (*Prob.* 295.)

25. The given rectangle represents the elevation of a square. Determine the inclination of its plane to the vertical plane. (*Sc.*, 1884.)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CHAPTER XVI

THE APPLICATION OF GEOMETRY TO THE SETTING OUT OF SCHEMES OF ORNAMENTAL PATTERNS, THE CONSTRUCTION OF UNITS OF PATTERNS, THE SPACING OF SURFACES FOR DECORATIVE PURPOSES, AND TO THE CONSTRUCTION OF ARCH FORMS, TRACERY, AND MOULDINGS

The importance of geometry as the groundwork of design can hardly be over-estimated. All patterns which require repetition must be arranged upon a geometrical foundation. The triangle, square, diamond, hexagon, octagon, and other polygons, with the circle and the ellipse, are the chief forms used in geometrical ornament, and when repeated form good patterns.

Natural forms, such as leaves and plants, however, give a much greater and more varied field of treatment. In using natural forms for ornament they must be arranged and adapted according to the nature of the material in which they are to be expressed, and also so as to properly cover the shape which is to be ornamented. For example, the treatment of a plant, used as the *motive* or principle for decorated ironwork, would be very different to its treatment when used for curtain or wall-paper patterns; in the first case the treatment would be more rigid and severe to be in keeping with the metal used, while in the latter it would be more flowing and less formal. This adaptation of natural forms is what is termed *conventional* treatment.

A knowledge of the application of geometry is essential in architecture and window tracery; in wall decoration; in mosaics, parquetry, tiles, floor-cloths, carpets, and rugs; in inlaid wood-work or marquetry; in pottery and coloured glass-work; in metal-work and jewellery; and in many other crafts.

The examples given are necessarily only suggestive of the many varieties of patterns which may be formed, and the student should not rest content with merely copying the given figures (*which should be done to a larger scale*), but should vary the treatment and invent fresh patterns of his own.

For further information the student should consult such works as Lewis Day's "Ornamental Design;" Jackson's books on "Design;" and Meyer's "Handbook on Ornament."

Most geometrical ornament may be arranged in one of the three following classes—

1. When the ornament is continuous, as in **bands** or borders, such as Figs. 1–19, 39, 43, 57, etc.

2. When the ornament is repeated in flat patterns over an unlimited surface as in **diapers**, such as Figs. 33, 59, 74, etc.

3. When the ornament fills an enclosed space as in **panels**, such as Figs. 112–155, etc.

EXPLANATION OF TERMS USED.

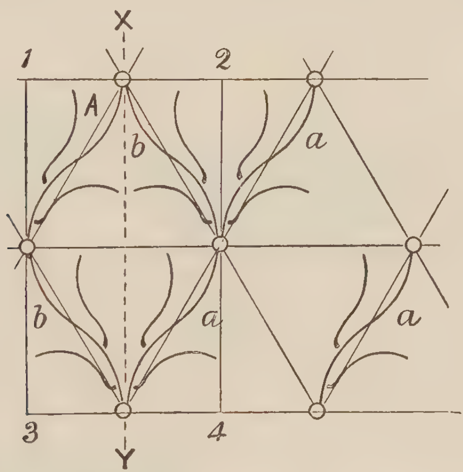
A few terms of frequent occurrence are explained here:—

Repetition means a succession of the same form. It is the basis of decorative art, but becomes monotonous when not varied; hence the reason for the introduction of the square panels in the fret bands, Figs. 13 and 15. In Fig. A the line **A** is *repeated* at *a, a, a*, while it is *reversed* at *b, b;* that is, the same form is repeated in an opposite direction, or *contrasted.*

The repetition of any form on its axis produces **symmetry**, one of the most important principles used in producing ornament; thus the repetition of the curved lines on the axis **X Y** produces the like-sided ogee form contained in the rectangle 1 2 3 4.

It is very important that the student should clearly understand what is meant by the **unit of design**, or the "unit of repetition," as it is often termed. In Fig. A the "unit of design" is the ornament

Fig. A.



included in the rectangle 1 2 3 4; this ornament when repeated forms an "**all-over**" **pattern**.

Note.—On many of the following patterns the extent of the unit is indicated by dotted lines.

The construction lines used in the setting out of geometrical patterns for parquetry (inlaid woodwork for flooring), mosaics (patterns composed of stone, glass, clay, etc.), floor-cloths, carpets, window-glazing, wall-decoration, ceilings, etc., are arranged to form a **network**, so as to secure accuracy both of construction and repetition. The nets most frequently used are those formed of squares and of equilateral triangles.

THE SQUARE NET.

The square network may be arranged in two positions:—First, as in Fig. B, where horizontal lines are crossed at right angles by vertical lines; or as in Fig. C, where the lines are drawn at an angle of 45°.

The best way to draw the first network is to rule horizontal lines at equal distances apart with the T-square, and cut these lines with a line at 45°. Through the points where this line cuts the parallels, rule the vertical lines with the set square. This secures both accuracy and quickness.

Fig. B.

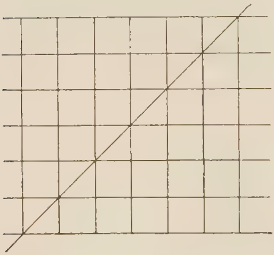
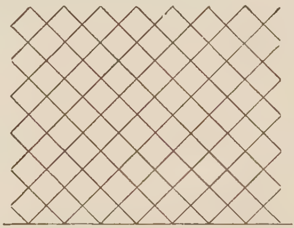


Fig. C.



For the second net, set off equal divisions on either a horizontal or a vertical line, and draw lines at 45° on each side of the points of division.

This second arrangement gives the **diamond** or **lozenge**. Upon these two nets an infinite variety of patterns may be formed.

Figs. 1–19 show a number of simple straight line **bands**. Figs. 5, 6, 11, 12, 13, 14, and 15 are Greek frets. Fig. 16 is a Chinese fret. Fig. 10 is a plait. Figs. 17 and 18 are Arabian or Moresque interlacement bands. Fig. 19 is the Gothic nail-head ornament. The monotony of Figs. 13 and 15 is broken by introducing square panels.

Some of the more common terms used in connection with the laying out of ornament can be simply illustrated by means of the network.

If each square be filled by a repetition of the same pattern, a **diaper** is formed, Fig. 20.

If the alternate squares only be filled the arrangement is called **chequering**, Fig. 21.

When larger intervals are left between the filled up spaces the method of setting out is called **spotting**. Sometimes smaller units are filled in between the spots; this is what is known as **powdering**. Fig. 22 *a a*, etc., indicate the latter process.

If the pattern be arranged in rows with spaces between, the method is called **striping** (Figs. 23, 24). Wide stripes are called **bands**, narrow ones **lines**.

The term **diaper** is so frequently used that a more detailed

description is given. Strictly speaking, the term was applied to those repeated patterns in which the unit of design was confined

FIG. 1.

FIG. 2.

FIG. 3.

FIG. 4.

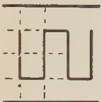


FIG. 5.

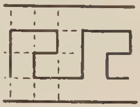


FIG. 6.

FIG. 7.

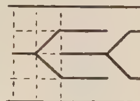


FIG. 8.

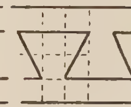


FIG. 9.

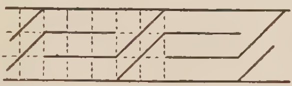


FIG. 10.

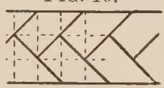


FIG. 11.

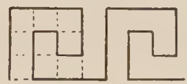


FIG. 12.

FIG. 13.

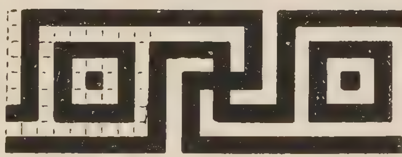


FIG. 14.

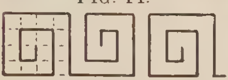


FIG. 15.

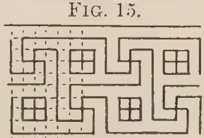


FIG. 16.

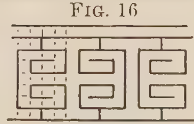


FIG. 17.

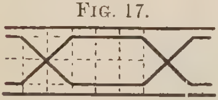


FIG. 18.

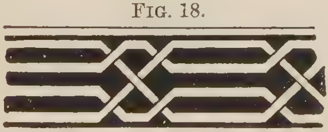
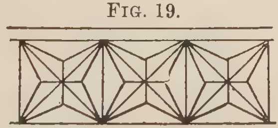
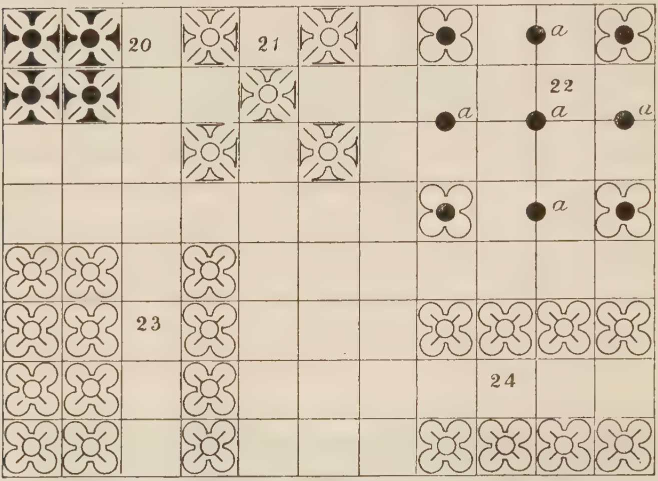


FIG. 19.



within a geometrical form, as in Fig. 20. Afterwards developments were made to relieve the monotony of the pattern, and portions of the unit were allowed to run out of one form into the other, as in Fig. 25; or the boundary lines of the geometrical form were ornamented, as in Fig. 26. These arrangements are known as "**all-over**" patterns.

Sometimes the formal geometrical shapes are entirely omitted, as in many wall-paper patterns, producing what are sometimes



called "free all-over" patterns. It must, however, still be remembered that the pattern is designed upon a geometrical foundation.

FIG. 25.

FIG. 26.

Figs. 27–34 are all formed on a square net; Figs. 27–31 by filling groups of squares. Fig. 32 is a plaid formed by "lining" in two directions. Figs. 33 and 34 are more difficult fret diapers, the pattern of which shows much clearer when filled in as shown. The *unit* of repeat is indicated by dotted lines. In drawing complicated frets the student should first mark in such parts as *a*, which recur at regular intervals; then such lines as *b*, which fall midway between.

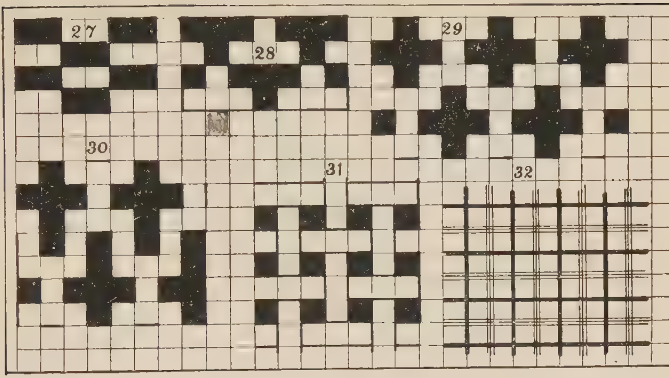


Fig. 33.

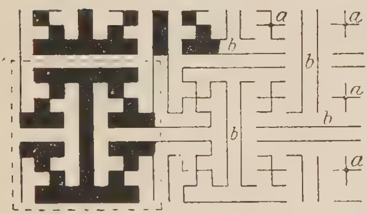
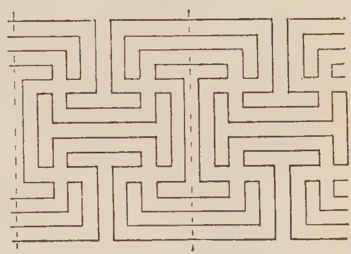
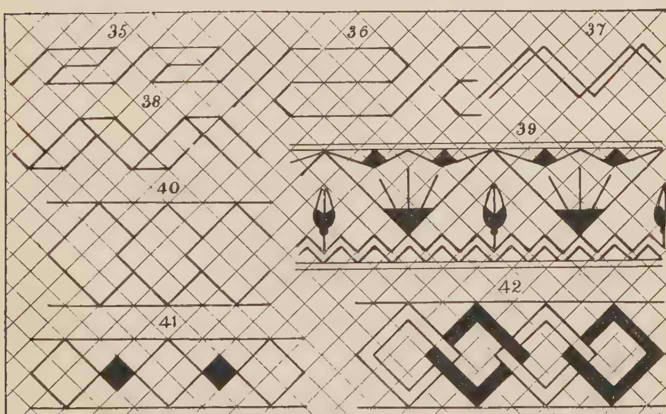


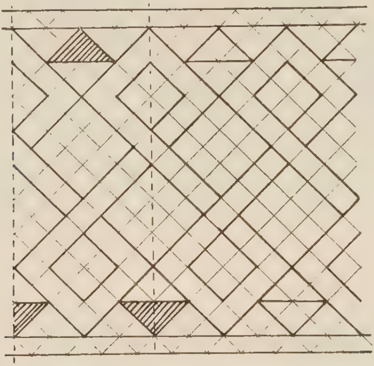
Fig. 34.





Figs. 35 and 36 are Arabian frets. Figs. 37 and 38 illustrate the **zigzag** or **chevron**. Fig. 39 is a variation on the zigzag similar to an Egyptian border ; the Egyptians used the zigzag to represent water. Figs. 40, 41, and 42 are borders of overlapping, alternating,

FIG. 43.



and interlacing squares. Fig. 43 is a more complicated interlacement band ; the unit of repetition is indicated by the dotted lines.

COMBINED SQUARE AND DIAMOND NETS.

A combination of the square and diamond nets gives octagonal forms which are not, however, equal sided. The method of inscribing a regular octagon in a square is shown in Figs. 52 and 53. In Fig. 52 the octagon has its alternate angles in the sides of the square ; in Fig. 53 its alternate sides coincide with the sides of the square. It will be noticed that the octagon will not repeat by itself when placed side by side, quadrangular spaces being left between.

FIG. 44.

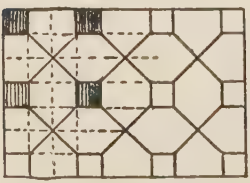
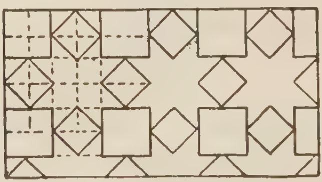


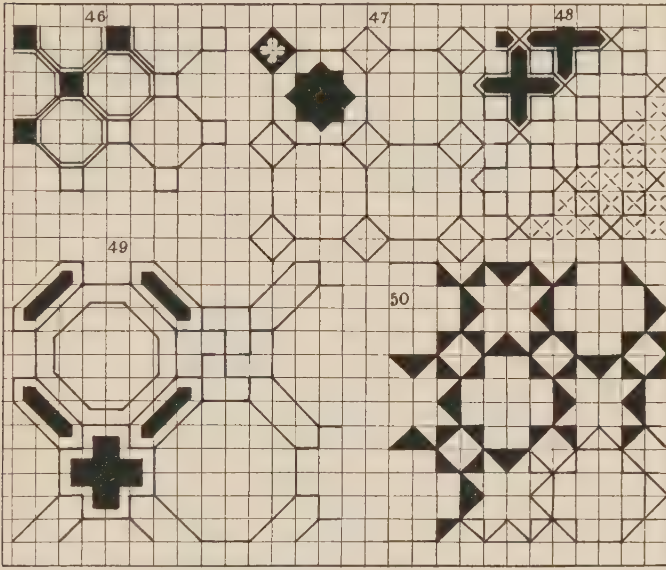
FIG. 45.



Figs. 44–51 are simple diapers formed on the net. Fig. 51 is an octagonal band.

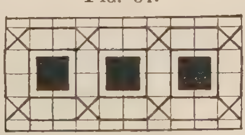
Fig. 58 is a diaper formed by intersecting octagons. Fig. 59 is a diaper of regular octagons and squares.

A network formed by producing the sides of intersecting squares is used for many Moorish patterns. First draw a square diamond



net, then describe a circle about one of the squares, and obtain the intersecting square as shown. Produce the sides of the second

FIG. 51.



square, giving the other lines for the net. Fig. 54 shows a diaper on this net. The spaces may be filled up in a variety of ways.

THE TRIANGULAR NET.

If we draw the lines of the net crossing at an angle of 60°, we obtain a rhombus. Cross this net by lines passing through the wider angles of the rhombus and we get the equilateral triangle. The net may be arranged in two positions, as in Figs. 55 and 56. It is the most useful net for the designer, as it is the readiest basis upon which "drop" patterns may be formed. The equilateral

FIG. 52.

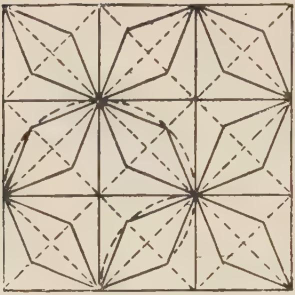


FIG. 53.

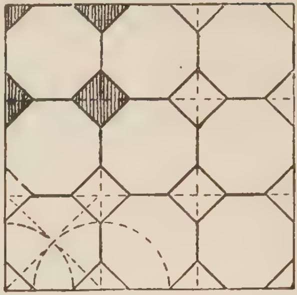


FIG. 54.

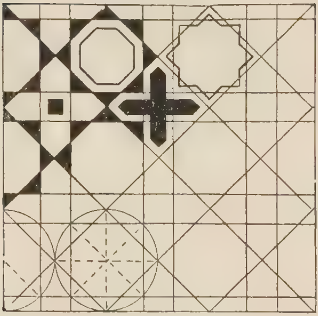


FIG. 55.

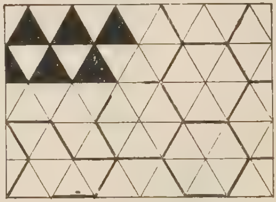


FIG. 56.

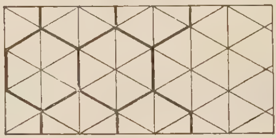


FIG. 57.

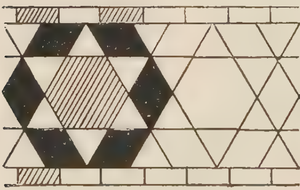


FIG. 58.

FIG. 59.

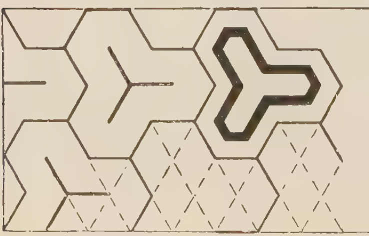
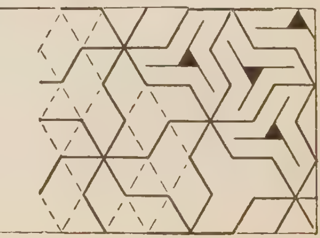


FIG. 60.



triangle, the rhombus, and the hexagon, and other shapes formed from them, all make perfectly fitting diapers upon the net.

Fig 61.

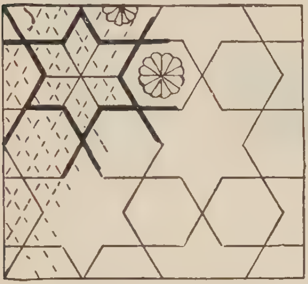


Fig. 62.

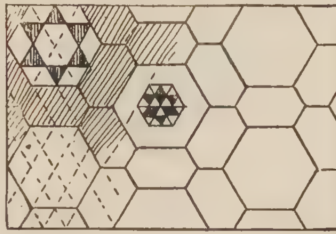
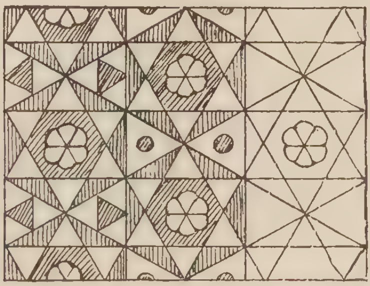


Fig. 63.



Figs. 57 and 58 are bands suitable for textile Mosaic borders.

Figs. 59–62 are diapers based on the hexagon. Fig. 63 is a marquetry pattern from an Indian box.

THE CIRCLE.

When curves are introduced the designer's field becomes still more fruitful. Figs. 64–73 are bands based on the circle; Figs.

Fig. 64.

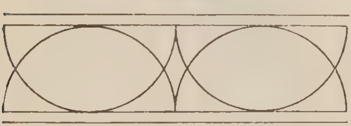
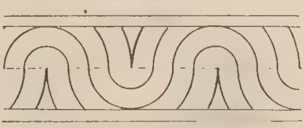


Fig. 65.



68–71 are various forms of the guilloche, a pattern largely used by the Assyrians and Greeks; Figs. 72 and 73 are "scale" bands. In Fig. 73 the treatment of a corner is shown. In designing a border,

set out the corner first ; the ornament should be arranged so as to tend to bind the borders together.

The simplest diaper formed from circles is when they are

FIG. 66.

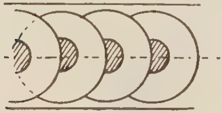


FIG. 67

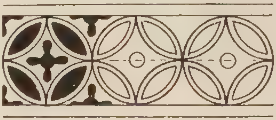


FIG. 68

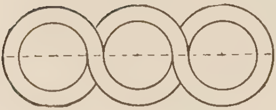


FIG. 69.

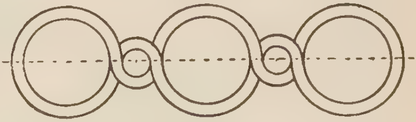


FIG. 70.

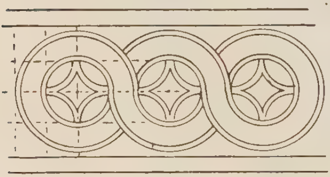


FIG. 71.

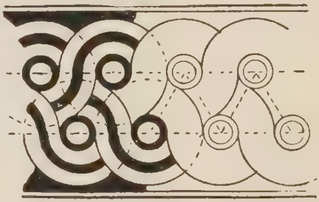


FIG. 72.

FIG. 73.

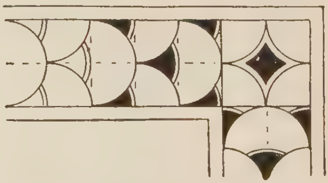


FIG. 74.

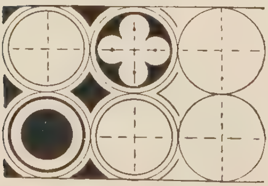
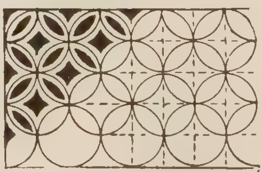


FIG. 75.



arranged on the square net, with their edges touching, as in Fig. 74. When the circles intersect, much more elaborate patterns are

FIG. 76.

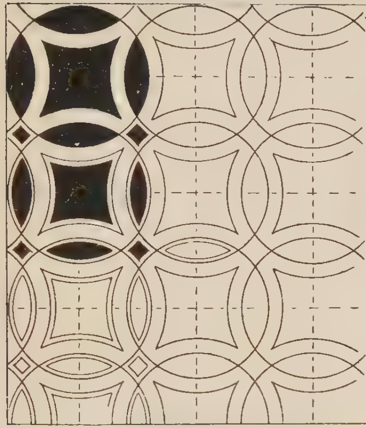


FIG. 77.

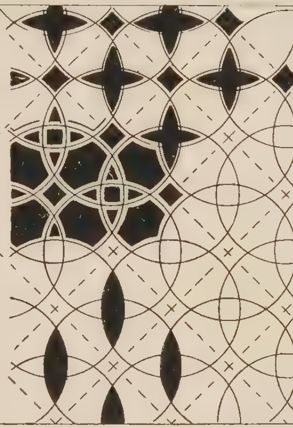


FIG. 78.

FIG. 79.

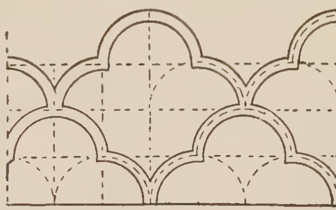


FIG. 80.

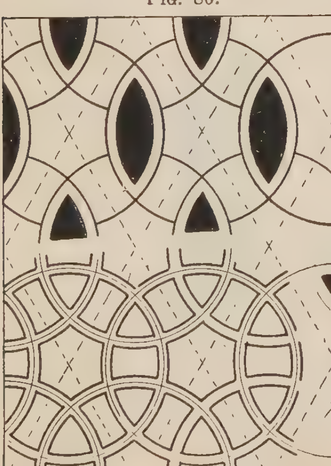


FIG. 81.

obtained. Figs. 75, 76, 77, show diapers formed by intersecting circles on square and diamond nets. These patterns may be enriched by the addition of ornament to the spaces. Figs. 78 and 79 are scale diapers.

Fig. 82.

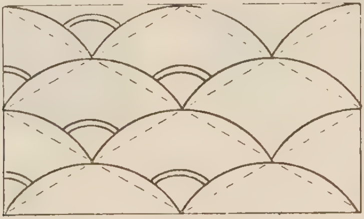


Fig. 83.

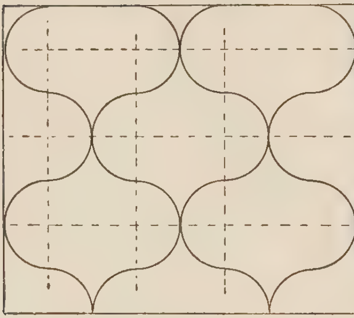


Fig. 84.

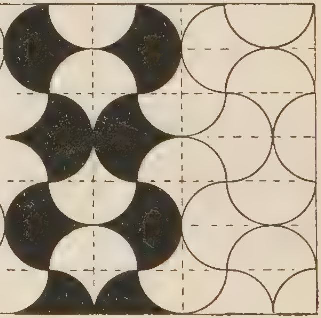


Fig. 85.

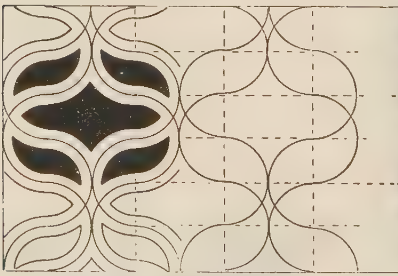
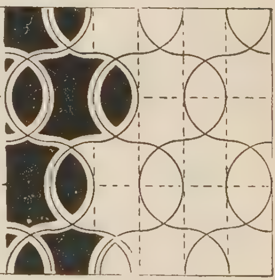


Fig. 86.



Figs. 80–82 are arranged on the diamond net. In Fig. 80 the alteration of the radii of the circle will produce a variety of patterns. Different methods of filling the spaces are shown. Figs. 81 and 82 are scale diapers.

Figs. 83–88 are diapers formed from ogee curves ; Fig. 84 is a double ogee ; Figs. 85 and 86 are interlacing ogees set out on an oblong foundation. Fig. 87 is based on circles arranged on a

FIG. 87.

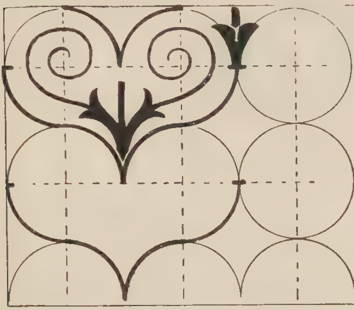
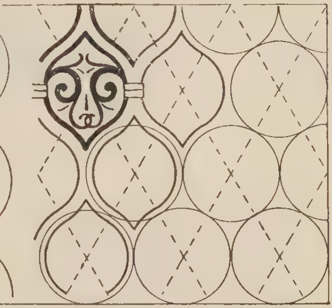


FIG. 88.



square net, while in Fig. 88 the circles are on a triangular net. The spaces may be enriched either by geometrical or by conventionalized natural forms.

COMBINATIONS OF THE STRAIGHT LINE AND CIRCLE.

Combinations of straight and curved lines afford a still greater variety of patterns. Figs. 89–96 show various band motives ; Figs.

FIG. 89.

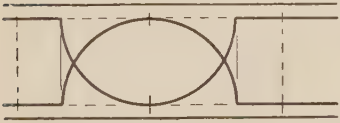


FIG. 90.

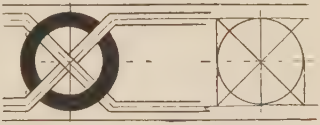


FIG. 91.

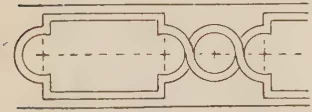
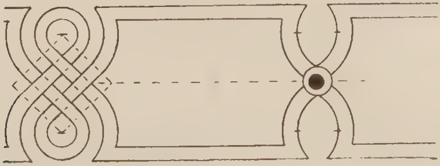


FIG. 92.

FIG. 93.



93 and 94 show combinations of tangential straight lines with curves ; Figs. 90, 91, 93, and 94 are interlacing bands ; Figs. 95 and

FIG. 94.

FIG. 95.

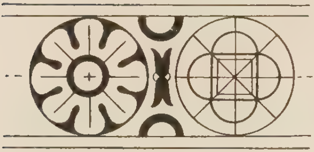


FIG. 96.

FIG. 97.

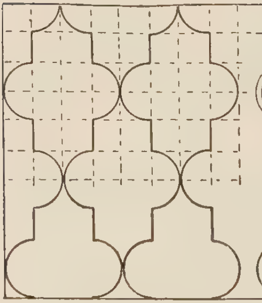


FIG. 98.

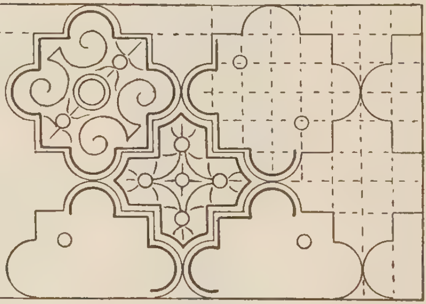


FIG. 99.

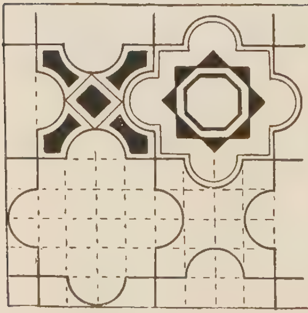
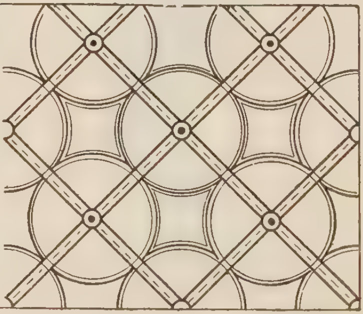


FIG. 100.



96 are rosette bands. Different methods of filling the rosette are suggested.

Figs. 97–100 show diapers composed of straight lines and circles. Fig. 100 is arranged upon a square diamond net; figs. 98 and 100 are suitable for the spacing of ceilings. The student must be accustomed to the analysis of the pattern as well as to the building of it up, that is, when a pattern is given, to be able to construct the geometrical foundation upon which it is based. This may be done by noticing the lines upon which any single feature recurs.

COMBINATIONS OF FREE CURVES WITH GEOMETRICAL FORMS.

In the following examples, free curves, chiefly plant forms, are used in combination with the more formal geometrical forms. It has already been pointed out that a much greater variety of more beautiful designs may be obtained by the use of free curves, such as those suggested by the growth and outlines of leaves and flowers.

In the repeating borders which follow, the lines of the setting-

FIG. 101.

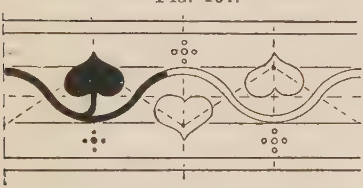


FIG. 102.

FIG. 103.

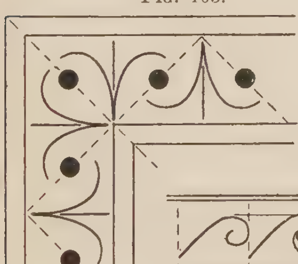


FIG. 104.

FIG. 105.

out are shown. The chief difficulty is the corner, which should be dealt with first. As a rule, it is better to ornament it more fully than the other part of the border, and to use lines which tend to hide the mitre line, and which add an appearance of strength to the joint.

Figs. 101–105 show various borders. The necessary construction

lines, which must be set out first, are clearly shown. The centres for the circular arcs, in Figs. 101 and 102, are shown by the small dots. In Figs. 103 and 104 two corners are shown. Where the lines are arranged, as in Fig. 105, it is necessary to reverse the direction in the centre of the border.

Figs. 106 and 107 show the setting-out of circular borders.

Fig. 106.

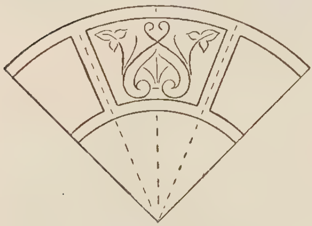


Fig. 107.

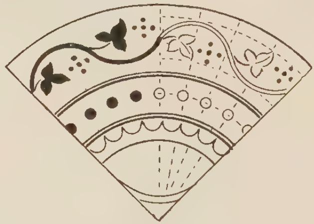


Fig. 108.

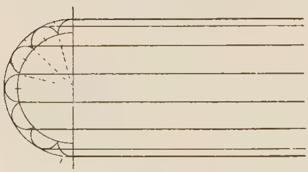


Fig. 109.

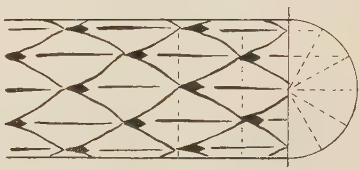


Fig. 110.

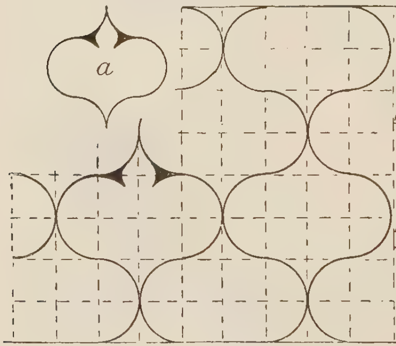
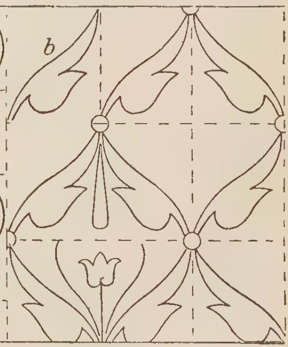


Fig. 111.



Large mouldings, like the torus moulding on the bases of pillars, are often ornamented in various ways. Figs. 108 and 109 show two simple examples and the constructions necessary to draw them.

Figs. 110 and 111 show how an "all-over" pattern may be formed.

In Fig. 110 *a* is the given unit. It is evident that the figure is formed of semicircles described from the angles of squares. Draw

the square net and construct the pattern. The spaces may be further enriched with ornament.

In Fig. 111 the given leaf-form, *b*, by repeating and reversing, forms a diaper. It will be seen that the figure can be enclosed in an oblong. First draw the rectangular net and form the pattern as in Fig. A, page 179.

Note.—These patterns should be drawn much larger, and tracing paper may be used for the repeats.

ENCLOSED ORNAMENT.

The preceding figures furnish examples of continuous ornament, as in *bands*, and of repeating ornament, as in "all-over" arrangements, such as *diapers*. The third division includes the spacing and ornamenting of enclosed figures, and may be termed **panel** ornament. The chief forms to consider are the square, the oblong, the triangle, the lozenge, and the polygons, of which the hexagon and the octagon are the most important.

THE SQUARE.

Figs. 112–123 indicate some of the ways in which the surface of the square may be spaced. The leading lines for the subdivisions

FIG. 112.

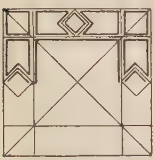


FIG. 113.

FIG. 114.

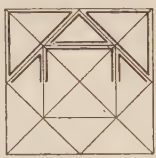


FIG. 115.

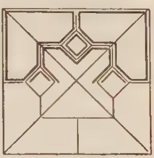


FIG. 116.

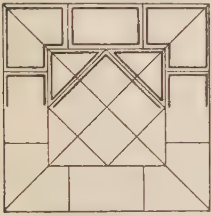


FIG. 117.

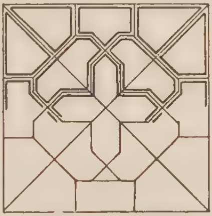


FIG. 118.

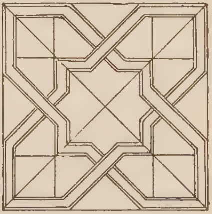


FIG. 119.

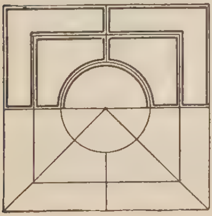


FIG. 120.

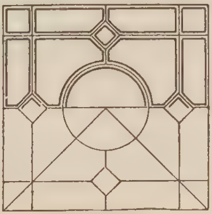
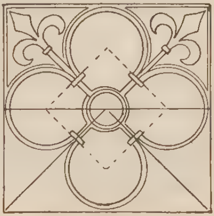


FIG. 121.



are the diagonals and the diameters. In Fig. 122 the square is subdivided into sixteen squares, and then lines are drawn from the middle points of the sides at an angle of 60°.

Fig. 122.

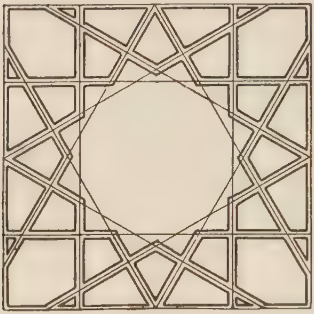
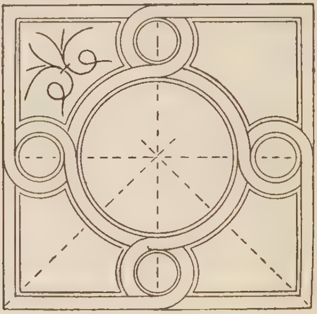


Fig. 123.



THE CIRCLE.

Under this heading will also be included sectors, stars, etc. The circle is spaced better, as a rule, by using curved lines, or a combination of curved and straight lines, particularly when applied

Fig. 124.

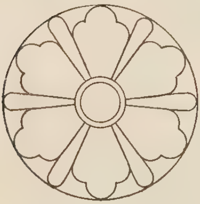


Fig. 125.

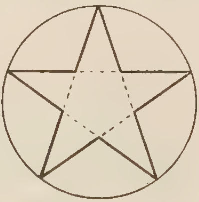


Fig. 126.

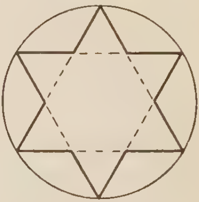


Fig. 127.

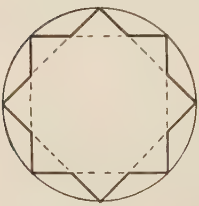


Fig. 128.

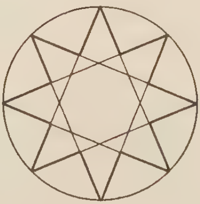
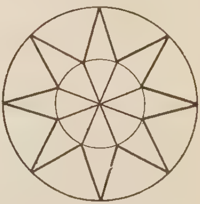


Fig. 129.



to tracery, as in Figs. 198–208. The problems dealing with the inscription of circles in Chapter XI. form the bases of the constructions in most cases.

By dividing the circumference of the circle into equal parts and drawing radii, sectors are formed, which are the foundation of rosettes, as in Figs. 95, 96, and 124. If the points of subdivision be joined the inscribed polygon is formed, as in Probs. 81–86, page 40. By joining every second or third point various stars are obtained, as in Figs. 125–128. Three forms of the eight-pointed star are shown ; other stars may be formed in a similar manner.

FIG. 130.

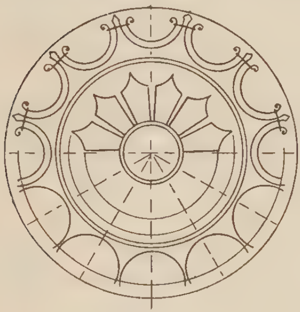


FIG. 131.

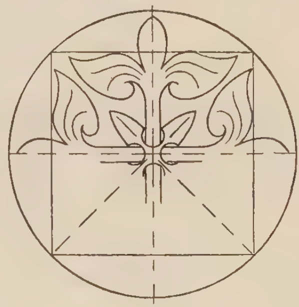


FIG. 132.

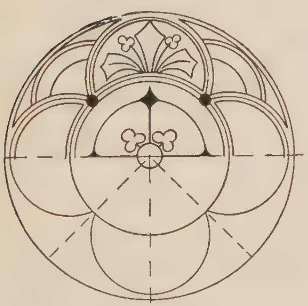


FIG. 133.

Circular panels may be ornamented in various ways :—by dividing into circular bands, as is often done with plates, salvers, etc., each band being separately ornamented. as in Figs. 106, 107, and 130 ; by inserting regular geometrical figures (squares, polygons, etc.), as in Fig. 131 ; by using arcs, as in Fig. 132 ; or they may be treated in a similar manner to the rosette, as in Fig. 133.

THE OBLONG.

This is, perhaps, the most common form of all. Ceilings, floors, walls, doors, book-covers, and many other objects take this shape. The relative proportions of the sides differ so much that the spacing

is necessarily very varied. The diagonal lines are not often used for dividing the surface, as the mitre line of the corner alone gives

Fig. 134.

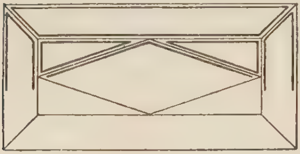


Fig. 135.

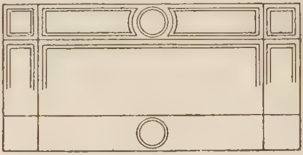


Fig. 136.

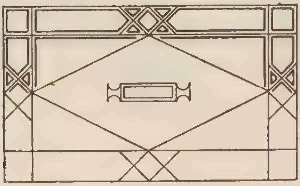


Fig. 137.

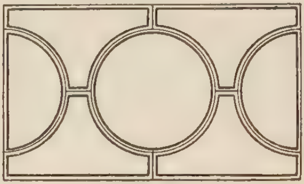
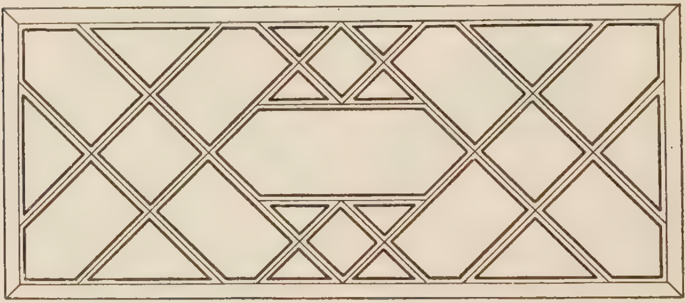


Fig. 138.



an equal width of border all round. Figs. 134-138 show simple subdivisions of the figure.

THE TRIANGLE, THE LOZENGE, THE SPANDREL, AND THE ELLIPSE.

The triangle and lozenge are not commonly used. Figs. 139-145 show subdivisions of these figures. When the rhombus (lozenge) is not subdivided, the ornament is grouped about the diagonals as in Fig. 146.

The spandrel is the irregular triangular space between the outside of an arch and the right angle enclosing it as in Fig. 147 ; or the space between two arches and the line touching them as in

Fig. 148. It often contains an inscribed circle, the centre of which is found by applying *Problem* 150, as shown.

FIG. 139.

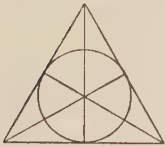


FIG. 140.

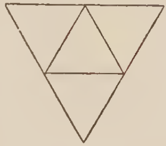


FIG. 141.

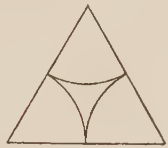


FIG. 142.

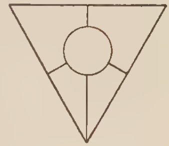


FIG. 143.

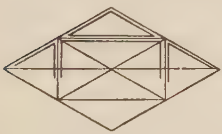


FIG. 144.

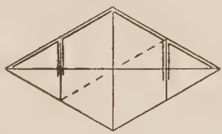


FIG. 145.

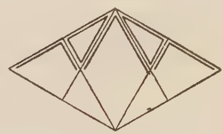


FIG. 146.

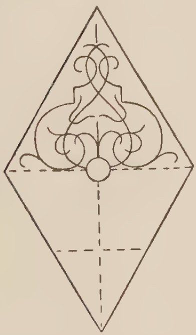


FIG. 148.

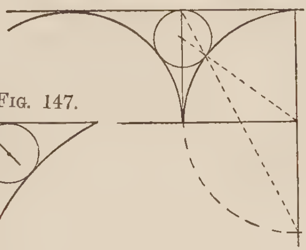


FIG. 147.

The surface of the ellipse may be divided by elliptical bands, similarly to Figs. 107 and 130, or circles may be inserted as in Fig. 123.

REGULAR POLYGONS.

The polygons most commonly used in ornament are the octagon and the hexagon. The diagonals and diameters intersecting give a great variety of subdivisions, of which Figs. 149-152 are examples. Star-shaped polygonal figures are shown among the examples on the square and circle. The hexagonal bands and diapers also suggest many suitable spacings of the hexagon. Figs. 153 and 154 show the simplest subdivisions obtained by using the diagonals and diameters respectively. Fig. 155 shows a division based on the six-pointed star.

FIG. 149.

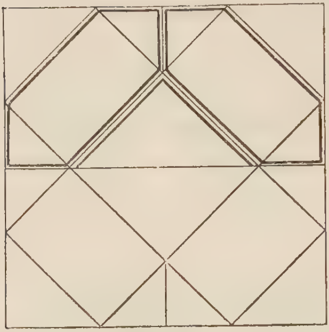


FIG. 150.

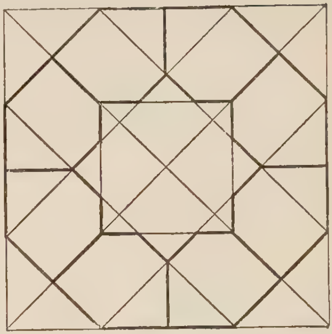


FIG. 151.

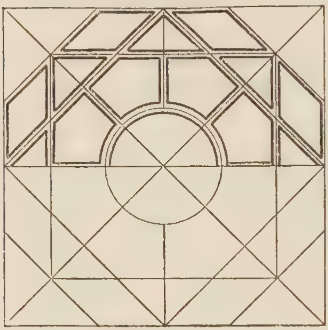


FIG. 152.

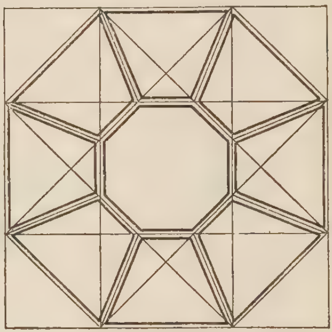


FIG. 153.

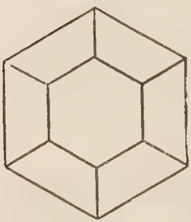


FIG. 154.

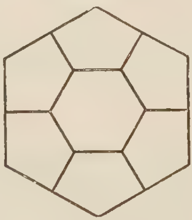
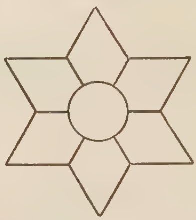


FIG. 155.

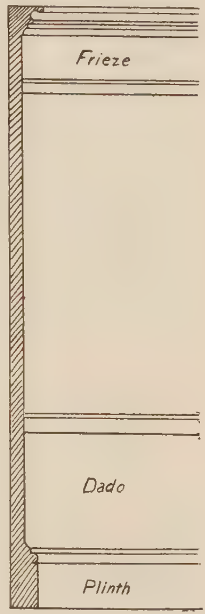


WALL SURFACE.

Wall surface is usually divided horizontally into five divisions, namely the plinth, dado, wall vail, frieze, and cornice, with borders and mouldings between, as in Fig. 156. Each of these divisions is ornamented in a different manner ; the dado being the

supporting part, as it were, has a more formal treatment, while the upper portion is usually decorated with a freer pattern. Sometimes vertical divisions, such as pilasters, are added. The spaces between are then usually decorated as enclosed ornament.

FIG. 156.



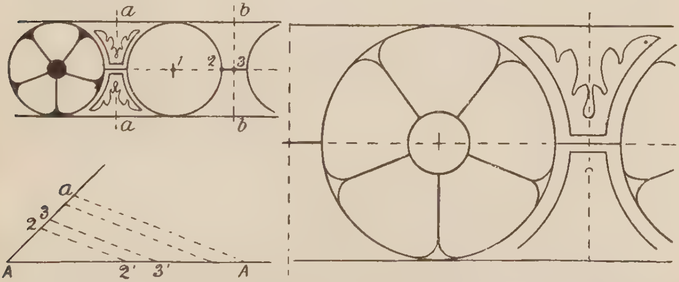
CONSTRUCTION OF PATTERNS TO A LARGER OR SMALLER SCALE.

This is easily done by applying *Problems* 117–119. Suppose, for example, it is required to make an enlarged copy of a unit of the given border, Fig. 157, making its height $1\frac{1}{4}''$.

The unit would be the portion of the pattern between the lines *a a* and *b b*. Draw the centre line, and set off the border line (half of $1\frac{1}{4}''$) on each side. Construct a proportional scale, making **A A** $= 1\frac{1}{4}''$ (the required height), and **A** $a = a\,a$ (the given height).

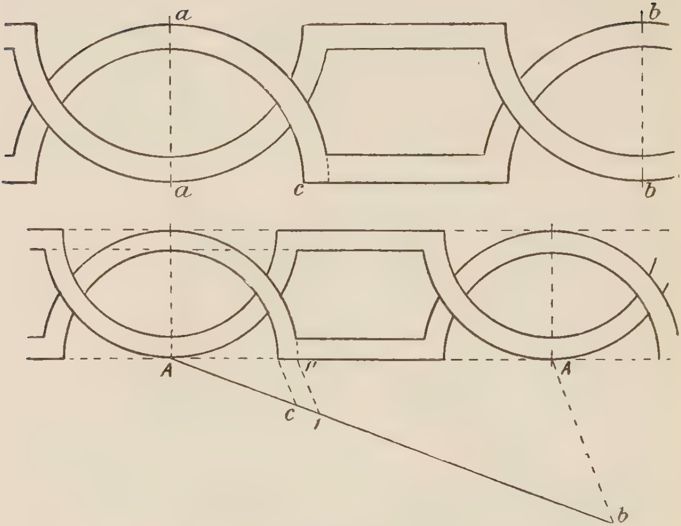
Set off **A** $2 = 12$ and **A** $3 = 13$. Draw parallels giving the required dimensions for the enlarged unit. With radius **A** $2'$ describe the circle, and with radius **A** $3'$ mark the boundary lines of the unit. Any other necessary dimensions may be found on the scale in a similar manner. The circle may be divided into five sectors by the help of the dividers, or by *Problems* 86, 81, or 108.

FIG. 157.



In Fig. 158 the border is reduced so that the unit *a a* to *b b* is reduced to $2''$. Make **A A** $= 2''$, and draw **A** $b = a\,b$. For the required height set off **A** $1 = a\,a$, and obtain **A** $1'$. Set off **A** $c = a\,c$, and obtain the radius for the smaller semicircle as shown. The rest of the construction may be easily followed.

FIG. 158.



ARCHES.

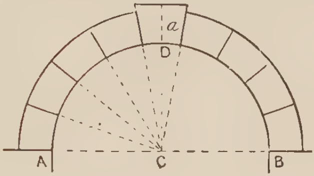
The curves of arches may be produced in such a number of ways, that it is only possible to show typical cases. The alteration of the position of the foci from which the curves are struck provides a great variety of forms. A few problems are worked out from given data; in the case of the other figures the position of the centres (**C**), the lengths of the radii, and the dotted construction lines, sufficiently indicate the method to be followed.

The distance from **A** to **B** (Fig. 159) is the **span** of the arch. **A** and **B** are termed the **springing points.** **D** is the **crown** and **C D** the **height** of the arch. The separate stones are called **voussoirs,** and the central stone (*a*) is the **key-stone.** The outside of the voussoirs is the **extrados,** or **back,** and the inside is the **intrados,** or **soffit.**

Some of the most typical forms of arches are—

The **semicircular** arch. Fig. 159.
The **stilted** arch, which stands upon upright lines. Fig. 160.
The **segmental** arch described from one centre. Fig. 161.
The **segmental** arch described from two centres. Fig. 162.
The **horse-shoe** or **Moorish** arch. Fig. 163.
The **pointed horse-shoe** or **Moorish** arch. Fig. 164.
The **equilateral** arch. Fig. 165.
The **lancet** arch. Fig. 166. The centres lie without the span.
The **obtuse** arch. Fig. 167. The centres lie within the span.
The **three-centred** arch. Fig. 168.

159



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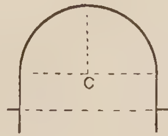

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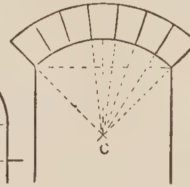

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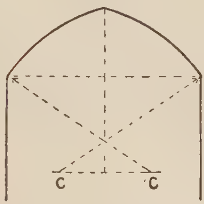

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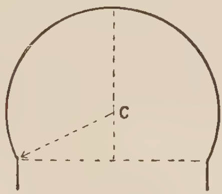

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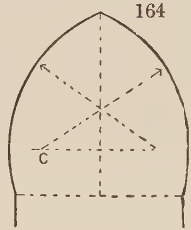

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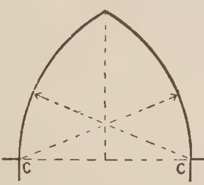

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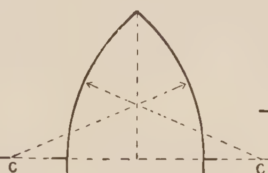

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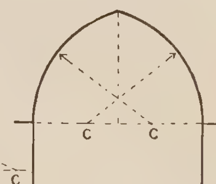

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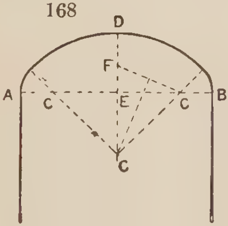

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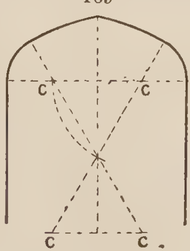

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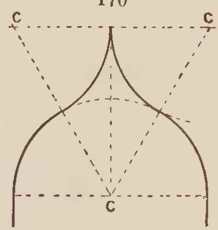

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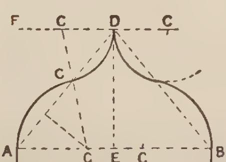

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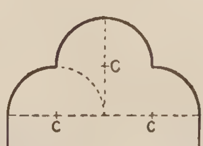
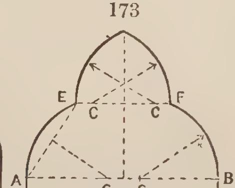


173



To construct this arch, the span **A B** and the height of **E D** being given. Make **A C**, **B C**, and **D F** all equal. Join **F** and **C**, and bisect. Where the bisecting line meets **D E** produced will be the third centre. From the points **C**, **C**, **C** describe arcs.

The **depressed Tudor** arch described from four centres. Fig. 169.
The **ogee** arch described from three centres. Fig. 170.
The **ogee** arch described from four centres. Fig. 171.

Given span **A B**, and height **E D**. Draw **D F** parallel to **A B**. Join **D A** and **D B**. Mark any point **G** where the arcs unite. Bisect **A G** for centres on span. Join **C G** and produce to meet **F D** for the other centres.

The **round-headed trefoil**. Fig. 172.
The **pointed trefoil**. Fig. 173. Obtain **A E** and **E F**, and bisect **A E** for the centres on **A B**.
The **elliptic** arch. Fig. 178. The joints between the voussoirs are obtained by drawing normals to the curve as in *Problem* 261. For the outer line make the normals equal, and sketch in the curve.

The parabolic curve in Problem 265 may also be used as an arch form.

MOULDINGS.

Mouldings are very important architectural features. They are used to improve the appearance of the angles, projections, arches, doors, windows, etc., of buildings. The profile of mouldings varies according to the style of architecture. The Greeks and Romans

Fig. 174.

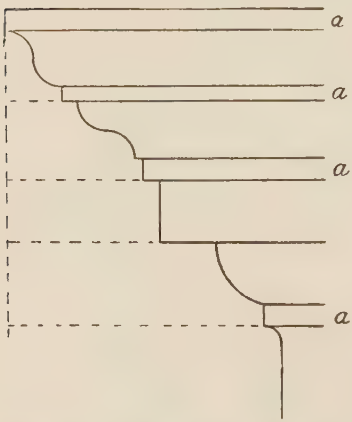
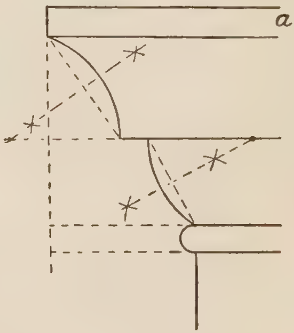


Fig. 175.



used eight forms, namely the **fillet**, the **cyma recta**, the **cyma reversa**, the **cavetto**, the **ovolo**, the **bead**, the **scotia**, and the **torus**. These they frequently enriched with either carving or colour. Figs. 174 and 176 show the cornice and base respectively of the Roman

Ionic order with the mouldings in position; those marked *a* are fillets. Fig. 175 shows another cornice with the cavetto, ovolo, and bead. As these curves are not quadrants like Figs. 183 and

FIG. 176.

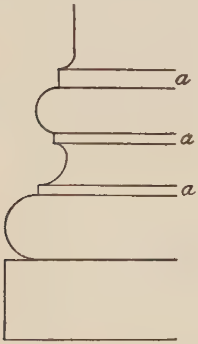
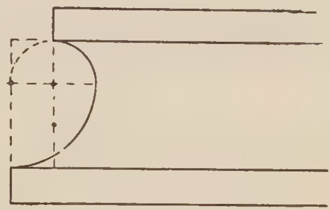


FIG. 177.



184, the method of obtaining the centres is shown. Fig. 177 is an enlarged drawing of the scotia in Fig. 176, showing how the centres of the arcs are obtained.

I. **Roman mouldings** are formed from arcs of circles.

Fig. 179 is a **fillet** or **band**, a narrow flat member generally separating other mouldings.

Fig. 180 is an **astragal,** or **bead.**

Fig. 181 is the **torus** moulding, used in the bases of columns, and frequently ornamented with the guilloche.

Fig. 182 is the **scotia,** a sunken moulding in the base of a column.

The centres for the quadrants must be in one line.

Fig. 183 is the **ovolo,** or **quarter-round,** often enriched with the egg-and-dart ornament.

Fig. 184 is the **cavetto,** the reverse of the ovolo.

Figs. 185–187 show three forms of the **cyma recta.**

In Fig. 185, $ab = ac$; and in Fig. 186, ab is greater than ac. In both figures the centres lie in the parallel drawn through the middle point of bc. In Fig. 187, the point d is not the centre of bc, and the centres are obtained as shown.

Figs. 188–190 show three forms of the **cyma reversa.**

In Fig. 188, $ab = ac$; in Fig. 189, ab is less than ac. The centres of both figures lie in the parallel to ab through the middle point d. In Fig. 190, the point d is not in the centre of bc. Figs. 185–190 are also known as ogee mouldings.

II. **Grecian mouldings** are usually formed from conic sections—the ellipse, parabola, and hyperbola—and are therefore much more varied and graceful in contour.

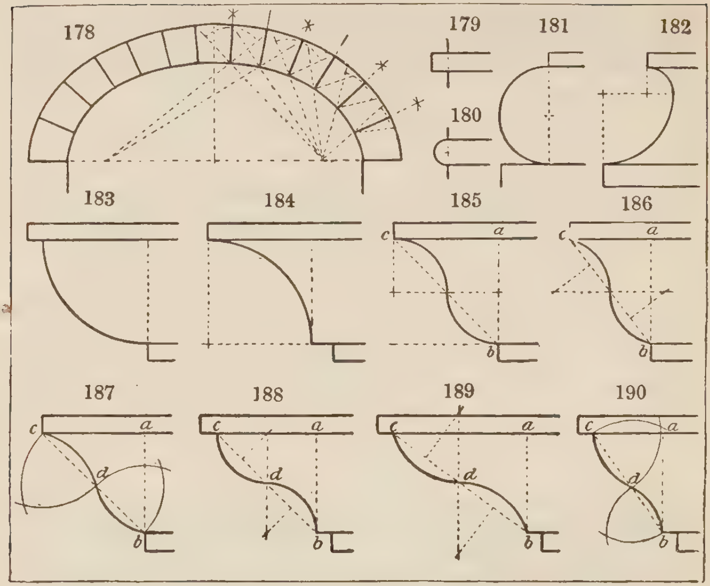


Fig. 191 is the **echinus, or ovolo.**

Given $a\,b$ and $b\,c$, also d, the point where the moulding turns. This turn is called a **quirk.** Divide $a\,e$ and $e\,d$ each into any number of equal parts, and draw the parabolic curve as in Problem 265.

Fig. 192 is another **echinus.** Here the hyperbola is used as in *Problem* 271.

Given $a\,b$ and $b\,c$ as before. Make $d\,f$ any convenient length, the alteration of the position of f changes the character of the curve.

Fig. 193 shows the **cavetto** formed by the quarter of an ellipse.

Fig. 194 is the **scotia** formed from a semi-ellipse.

Given $f\,e$, the depth of the curve. Draw $e\,h$ parallel to $c\,d$, and construct the semi-ellipse in the parallelogram $c\,h$.

Figs. 195 and 196 show the **cyma recta.**

In Fig. 195, $a\,b$ is greater than $b\,c$. The construction can easily be followed.

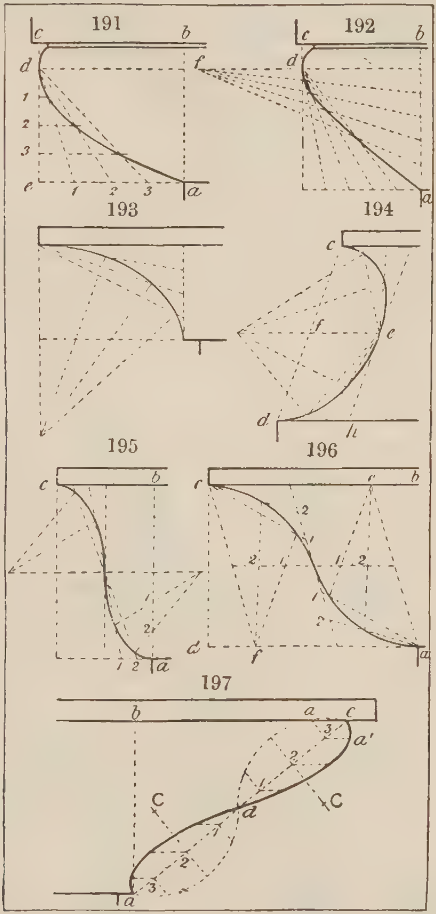
In Fig. 196, $a\,b$ is less than $b\,c$. Make $c\,e = c\,d$. Join e and a. Draw $c\,f$ parallel to $a\,e$, and proceed as in the last figure. The curve lies within the parallelogram $a\,e\,c\,f$.

The **cyma reversa** is similarly constructed to the cyma recta.

If Figs. 195 and 196 be turned at right angles, they will show the cyma reversa.

Fig. 197 shows Nicholson's method of obtaining any ogee curve.

Join a and c. On dc and ad describe arcs, from any convenient centres as **C C**. Take any number of points, 1, 2, 3, etc. From these points draw perpendiculars to the arcs. From the same points draw parallels to bc. Set off on these parallels, distances = the perpendiculars at the same point; thus $3a' = 3a$, etc.



GEOMETRICAL TRACERY, WINDOWS.

In these examples the light centre lines form the foundation. The main construction lines are shown where necessary. For further information, the

student is referred to books on architecture. *Draw all the examples at least twice as large.*

Fig. 198 is based upon Problem 135, Fig. 199 on Problem 166, Fig. 200 on Problems 127 and 143. In Fig. 200, first inscribe a circle in the equilateral

FIG. 198.

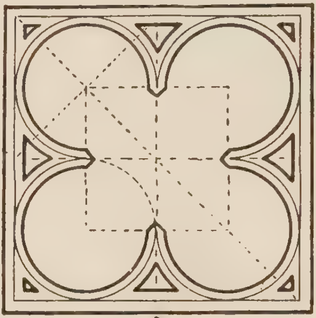


FIG. 199.

FIG. 200.

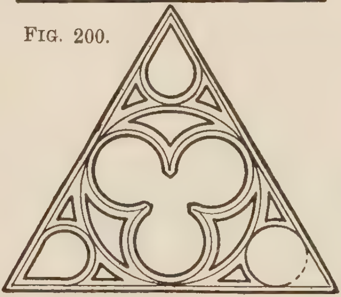


FIG. 201.

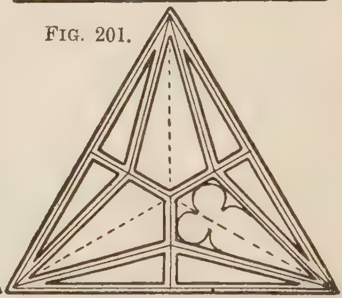


FIG. 202.

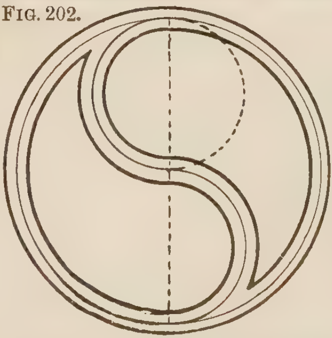
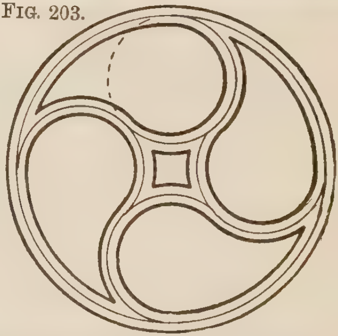


FIG. 203.



triangle, next three circles in the circle, and then a circle in each corner. In Fig. 201, a slight sketch indicates how the spaces may be further ornamented. Figs. 202 and 203 are based upon the inscription of two and four circles respectively in a circle.

Fig. 204 is based upon Problem 168: six semicircles in a circle, and the same process repeated within the inner circle. Fig. 205 depends upon the same

Fig. 204.

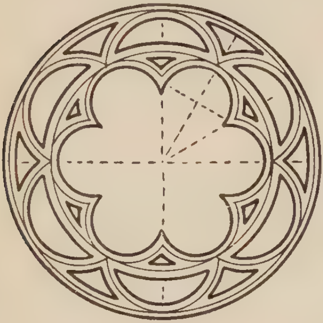


Fig. 205.

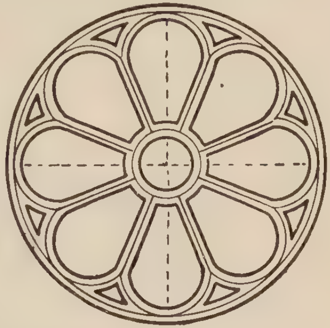
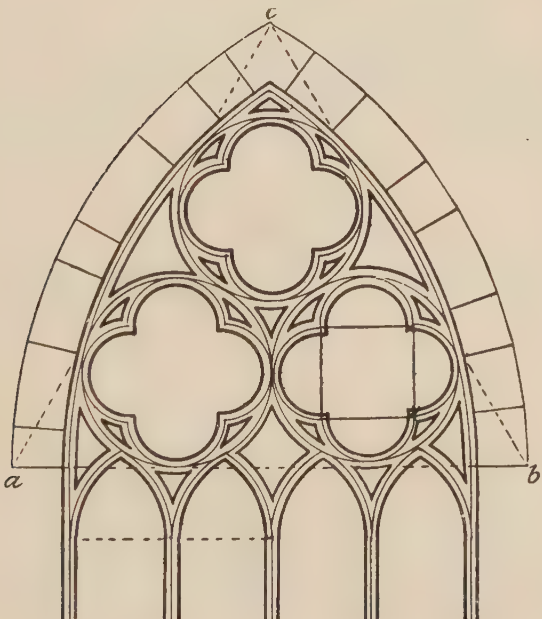


Fig. 206.



principle. In Fig. 206, first describe the equilateral triangle *a b c*, and inscribe within it three circles; then, from the centres *a* and *b*, describe the inner

lines of the arch. Within each circle inscribe four semicircles, and fill in the tracery as shown.

Fig. 207 shows the construction of a more elaborate example of tracery. First inscribe three circles in the circle. The centres for the inner foils are shown by the dots. Point 1 lies on the line used to find the centre of the

Fig. 207.

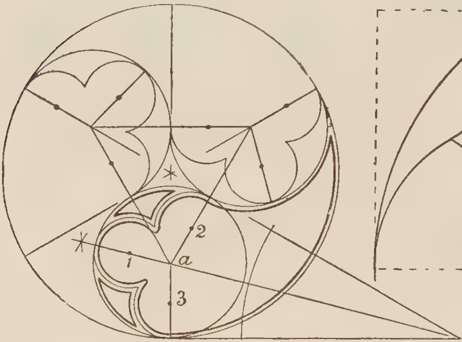
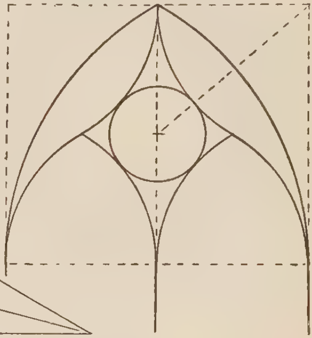


Fig. 208.



inscribed circle, 2 and 3 are equidistant from the centre *a*. The foils in this case are not tangential to one another, as tangential arcs would not combine so agreeably with the curves of the inscribed and the enclosing circles.

Fig. 208 shows simple leading lines for the window tracery of an equilateral arch.

SCIENCE AND ART DEPARTMENT.

MAY, 1897.

GEOMETRICAL DRAWING.

Only eight *questions are to be attempted. Questions marked (*) have accompanying diagrams.*

PLANE GEOMETRY.

1. Construct a scale to show yards and feet, on which $3\frac{1}{2}''$ represents 8 yards. Make the scale long enough to measure 10 yards, and finish and figure it properly. (12.)

***2.** Draw the figure shown, according to the given figured dimensions. (10.)

No marks will be given for merely reproducing the diagram the same size.

3. Construct a regular octagon of $1''$ side, and a second octagon having its angles at the middle points of the sides of the first. (12.)

4. About a square of $1''$ side describe a triangle having one of its angles 60°, and another 70°. (8.)

5. Within an equilateral triangle of $3''$ side inscribe three equal circles, each touching the two others and *two* sides of the triangle. (10.)

***6.** Draw the figure shown, adhering strictly to the figured dimensions. (12.)

No marks will be given for merely reproducing the diagram the same size.

***7.** Construct a figure *similar* to that given in the diagram, but having the side corresponding to **AB** $2''$ long. (10.)

***8.** Reduce the given figure to a rectangle of equal area, having **AB** for one of its sides. (10.)

***9.** Draw the "Scotia" moulding shown. The curve is made up of two quarter-circles of $1''$ and $\frac{1}{2}''$ radius respectively. (8.)

No marks will be given for merely reproducing the diagram the same size.

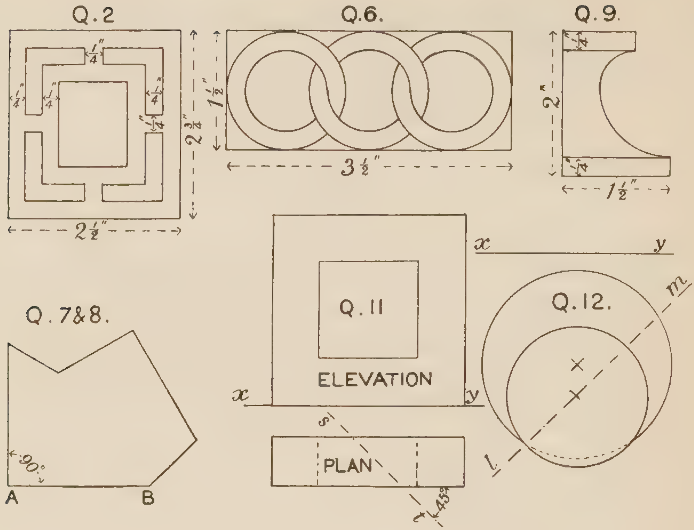
10. Describe the involute of a circle of $1''$ diameter, *i.e.* the spiral curve traced by the end of a string which is kept stretched as it is unwound from the circumference of the circle. (12.)

SOLID GEOMETRY.

***11.** Plan and elevation are given of a square frame. Make a new elevation on a plane parallel to the line *st*, and show on it the section made by the vertical plane of which *st* is the plan. The part in section must be clearly indicated by lightly shading it. (14.)

***12.** The plan is given of a cylindrical slab $\frac{5}{8}''$ thick, and of a sphere resting on the slab. Draw an elevation of the solids on the line xy, and also the elevation of the section made by the vertical plane represented in plan by the line lm. The part in section must be clearly indicated by lightly shading it. (16.)

13. A right prism, $2''$ long, and having for base a scalene triangle, sides $1''$, $1\frac{1}{4}''$, and $1\frac{1}{2}''$, lies with its smallest rectangular face on the ground. Draw the plan of the prism. (12.)



June, 1897 (Day Examination).

GEOMETRICAL DRAWING.

Only eight *questions are to be attempted. Questions marked* (*) *have accompanying diagrams.*

Plane Geometry.

1. Construct a scale of $7\frac{1}{2}'$ to $1''$, by which single feet may be measured up to $30'$. The scale must be neatly finished and correctly figured. (12.)

*2. Draw the "rosette" shown, according to the figured dimensions. (10.)

No marks will be given for merely reproducing the diagram the same size.

3. Construct a square of $1\frac{1}{2}''$ sides, and within it inscribe a rectangle having one of its angles on each side of the square and one of its sides $1''$ long. (8.)

4. Construct a triangle, sides $1\frac{1}{2}''$, $1\frac{7}{8}''$, and $2\frac{1}{8}''$ long, and a *similar* triangle having its longest side $2\frac{1}{2}''$ long. Measure and write down the number of degrees in each of the angles. (10.)

*5. Draw a figure similar to the one shown, the points of the star being at the angles of a regular heptagon inscribed within a circle of $1\frac{1}{4}''$ radius. (10.)

No marks will be given for merely reproducing the diagram the same size.

6. Draw a line **AB**, $2''$ long. Describe a circle of $\frac{3}{4}''$ radius touching **AB** at **A**, and another of $1''$ radius touching **AB** at **B**. Draw a second line which shall touch both circles, showing clearly the points of contact. (8.)

*7. Draw the figure shown, according to the figured dimensions. (12.)

No marks will be given for merely reproducing the diagram the same size.

8. Within the rhombus, sides $2\frac{1}{2}''$ one angle 60°, inscribe an ellipse touching the sides of the rhombus at their middle points. (12.)

9. Construct a square equal in area to an equilateral triangle of $2''$ side. (10.)

*10. Draw the "four centred" arch shown, adhering to the figured dimensions. (12.)

No marks will be given for merely reproducing the diagram the same size.

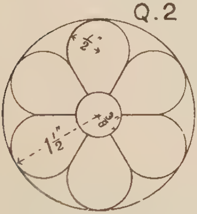
Solid Geometry.

*11. Plan and elevation are given of a solid letter **H**. Draw an elevation when the horizontal edge *ab* makes an angle of 60° with the vertical plane of projection. (14.)

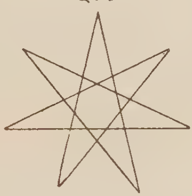
*12. Plan and sectional elevation are given of a short length of moulding which has been cut across for "mitring" by a vertical plane shown in plan at *lm*. Determine the true form of the section. (16.)

13. A sphere of $1''$ radius has a portion cut off by a *horizontal* plane $\frac{5}{8}''$ above its centre, and another portion by a *vertical* plane passing $\frac{1}{2}''$ from the centre. Draw a plan of what remains of the sphere. The section must be clearly indicated by lightly shading it. (12.)

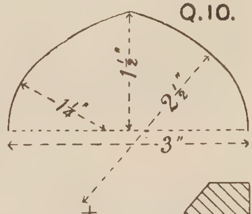
Q.2



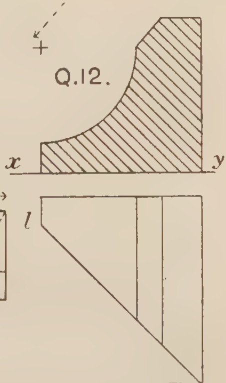
Q.5.



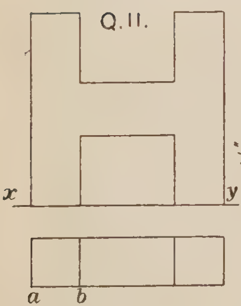
Q.10.



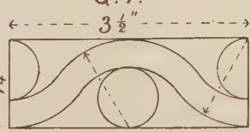
Q.12.



Q.11.



Q.7.



APRIL, 1898.

GEOMETRICAL DRAWING.

Only eight *questions are to be attempted. Questions marked (*) have accompanying diagrams.*

PLANE GEOMETRY.

*1. The diagram represents an incomplete scale of feet. Complete the scale so that distances of 2′ may be measured by it up to 50′. The scale must be properly finished and figured. (12.)

*2. Draw the figure shown, using the figured dimensions. (10.)

3. Construct an equilateral triangle of $2\frac{3}{4}''$ side. Bisect all three of its sides and join the points of bisection. Within each of the four equilateral triangles thus formed inscribe a circle. (10.)

4. Describe two circles, each of $\frac{1}{2}''$ radius, touching each other at a point **A**. Find a point **B** on one of the circles, $\frac{7}{8}''$ from **A**. Describe a third circle, touching both the others and passing through **B**. (8.)

5. Within a circle of $1\frac{1}{2}''$ radius inscribe a regular nonagon. Within the nonagon inscribe a rectangle having all its angles in the sides of the nonagon, and one of its sides $1\frac{1}{2}''$ long. (12.)

6. Construct a rectangle $2\frac{1}{2}'' \times 1\frac{3}{4}''$. Within it inscribe two other rectangles, each *similar* to the first, concentric with it, and having their longer sides $1\frac{1}{2}''$ and $1''$ long respectively. (10.)

*7. Copy the diagram, enlarging it to the dimensions figured. (12.)

8. Construct a quadrilateral **ABCD** from the following data:—

Sides—$\mathbf{AB} = 1\frac{1}{2}''$, $\mathbf{BC} = 1\frac{1}{2}''$
Angles—$\mathbf{ABC} = 105°$, $\mathbf{BAD} = 75°$

The four angles of the figure all lie in the circumference of a circle. (10.)

*9. Draw the moulding shown, adhering strictly to the figured dimensions. The arc of $\frac{1}{2}''$ radius is a quadrant. (10.)

10. An arch in the form of a semi-ellipse is 6′ wide and 2′ high. Describe the curve, and draw two lines perpendicular to it from two points on the curve, each 2′ from the top point of the arch.

Scale (which need not be drawn) $\frac{1}{2}''$ to 1′. (12.)

SOLID GEOMETRY.

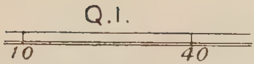
*11. The plan is shown of three bricks, each $9'' \times 4\frac{1}{2}'' \times 3''$, one resting upon the other two. Draw an elevation upon the given line *xy*.

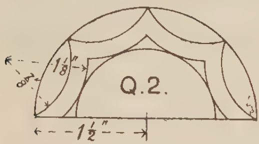
Scale (which need not be drawn) 2″ to 1′, or $\frac{1}{6}$ of full size. (14.)

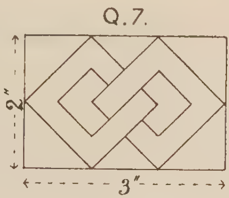
*12. Plan and elevation are given of a rectangular block with a semi-cylindrical hollow in it. Draw a new elevation upon a vertical plane which makes an angle of 30° with the horizontal edge **AB**. (16.)

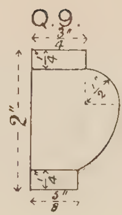
*13. The diagram shows the end elevation of a right prism $1\frac{3}{4}''$ long with square base, and a horizontal plane *lm* cutting the prism. Draw a plan of the portion below *lm*.

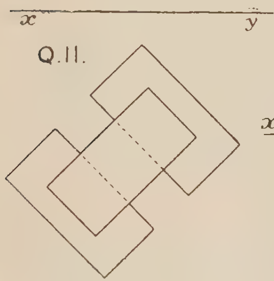
(12.)

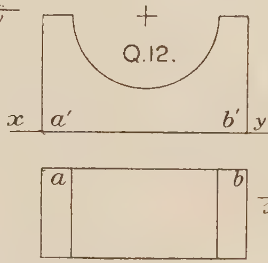


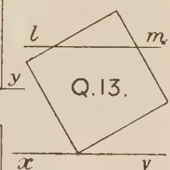












June, 1898 (Day Examination).

GEOMETRICAL DRAWING.

Only eight *questions are to be attempted. Questions marked* (*) *have accompanying diagrams.*

Plane Geometry.

1. Construct a scale one-tenth of full size, to measure feet and inches up to 5 feet. The scale must be properly finished and figured. (12.)

***2.** Make an enlarged copy of the given diagram, using the figured dimensions. (10.)

3. Construct a regular pentagon of $1\frac{1}{2}''$ side. Describe five circles of $\frac{3}{4}''$ radius, having their centres at the five angles of the pentagon. (10.)

4. Construct a triangle, sides $1\frac{1}{2}''$, $2''$, and $2\frac{1}{2}''$, and within it inscribe an equilateral triangle having its three angles in the three sides of the first triangle. (8.)

5. Within a circle of $2\frac{3}{4}''$ diameter, inscribe four equal circles each touching the given circle and two of the others. (10.)

***6.** Copy the diagram according to the given dimensions. Show clearly how the centre of the small circle is determined. (10.)

***7.** Draw the figure shown in the diagram, making the side of the outer hexagon $1\frac{1}{4}''$ long. (12.)

8. Make an irregular pentagon **ABCDE** from the following data :—

Sides : $\mathbf{AB} = 2\frac{1}{8}''$, $\mathbf{BC} = 1\frac{1}{4}''$, $\mathbf{CD} = 2\frac{3}{8}''$, $\mathbf{DE} = 1''$.
Angles: $\mathbf{ABC} = 105°$, $\mathbf{ABE} = 30°$, $\mathbf{BAE} = 105°$.

Then make a *similar* figure in which the side corresponding to **AB** is $1\frac{5}{8}''$ long. (12.)

***9.** Draw the "cyma recta" moulding shown in the diagram, using the figured dimensions. The curve is composed of two equal tangential arcs each of $\frac{3}{4}''$ radius. (10.)

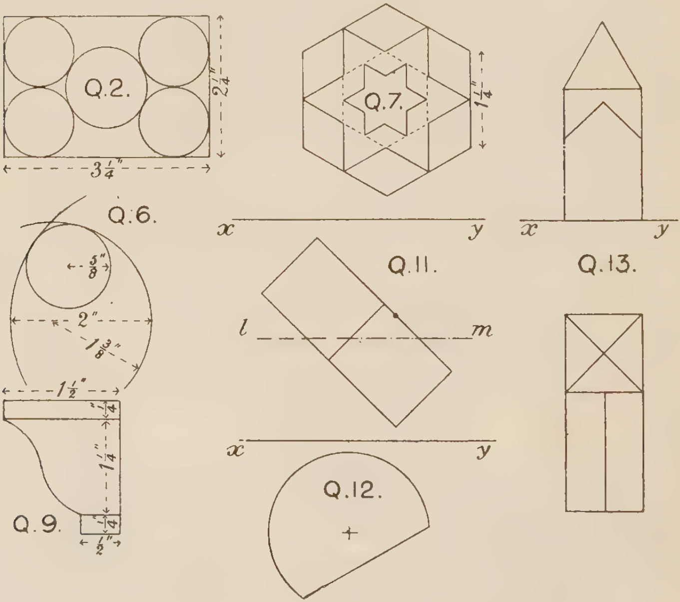
10. A circle of $1''$ radius rolls along a straight line. Draw the curve (cycloid) traced by a point on the circumference of the circle during half a revolution. (12.)

Solid Geometry.

*11. The plan is given of a right prism having equilateral triangles for its bases. These bases are vertical. Draw an elevation of the prism on the line xy. Show the form of the section of the prism made by the vertical plane lm. The part in section should be indicated by lightly shading it. (16.)

*12. The diagram shows the plan of a portion of a sphere. Draw an elevation upon the given line xy. (14.)

*13. The "block" plan and end elevation are given of a building having a square tower with pyramidal roof. A side elevation of the building is required. (12).



April, 1899.

GEOMETRICAL DRAWING.

Only eight *questions are to be attempted. Questions marked* (*) *have accompanying diagrams.*

Plane Geometry.

1. Construct a scale of feet and inches one-ninth ($\frac{1}{9}$) of full size, long enough to measure 4 feet. The scale must be properly finished and figured, and should not be "fully divided" throughout; *i.e.* only one distance representing 1 ft. should be divided to show inches. (12.)

*2. Draw the corner ornament shown, using the figured dimensions. (10.)

3. Construct a rhombus having its sides 2″ long, and one of its angles 75°. Within it inscribe two equal circles touching each other, and each touching two sides of the rhombus. (10.)

4. Within a square of $1\frac{3}{4}''$ side inscribe a regular octagon having all its angles in the sides of the square. (10.)

5. Within a circle of $1\frac{3}{8}''$ radius inscribe a regular heptagon. Draw a second similar heptagon, of which the longest diagonals are 2″ long. (12.)

*6. Draw the given figure, making the side of the square $2\frac{1}{4}''$ and the radii of all the arcs $\frac{3}{8}''$. (12.)

*7. Reduce the given figure to a rectangle of equal area, having one side equal to **AB**. (10.)

8. Two conjugate diameters of an ellipse are $3\frac{1}{2}''$ and $2\frac{1}{2}''$ long respectively, and cross one another at an angle of 60°. Draw the curve. (10.)

9. Make a scale of chords of 2″ radius, to read to 10° up to 90°. The scale must be finished and figured. At the ends of a line $2\frac{1}{2}''$ long construct, from the scale, angles of 20° and 70° respectively. (10.)

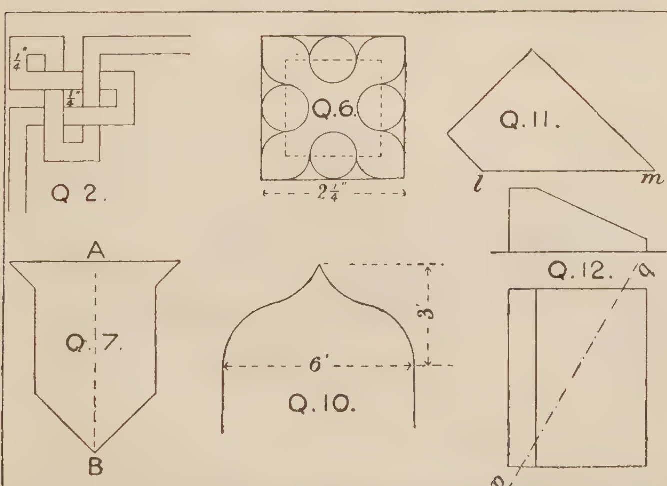
*10. Draw the "ogee" arch shown, to the scale of 2′ to 1″. The arcs are all of 2′ radius. The methods of finding the centres and points of contact must be clearly shown. (12.)

Solid Geometry.

*11. The plan is given of a piece of cylindrical rod, cut by a vertical plane shown at *lm*. Draw an elevation of the solid upon an *xy* parallel to *lm*. (16.)

*12. Plan and elevation are given of a sloping desk. Draw an elevation upon a vertical plane parallel to the line *pq*. Show upon this elevation the outline of the section made by the vertical plane represented by *pq*. (14.)

13. Show in plan and elevation a shallow circular metal bath. Diameter at top 2′, 6″, at bottom 2′, height 6″. The thickness of metal may be neglected. Scale (which need not be drawn) 1′ to 1″. (12.)



June, 1899 (Day Examination).

GEOMETRICAL DRAWING.

Only eight *questions are to be attempted. Questions marked* (*) *have accompanying diagrams.*

Plane Geometry.

*1. The line **AB** represents a distance of $2\frac{1}{2}$ feet. Make a scale by which feet and tenths of a foot may be measured, up to 4 feet. The scale must be properly finished and figured, and should not be "fully divided" throughout, *i.e.* only one distance representing 1 foot should be divided to show tenths. (12.)

*2. Draw the given diagram of window tracery, using the figured dimensions. The arch is "equilateral." (10.)

3. Within a circle of $1\frac{1}{2}''$ radius inscribe a regular hexagon. Within the hexagon inscribe *three* equal circles touching each other and each touching two sides of the hexagon. (10.)

*4. Construct a figure *similar* to the given figure, but having the distance corresponding to **CD** $1''$ long. (10.)

5. Construct a regular heptagon of $1''$ side, and within it inscribe an equilateral triangle. (12.)

*6. Describe a circle touching the given circle at **T**, and passing through the point **P**. (10.)

*7. Draw the figure shown, using the figured dimensions. The spaces are $\frac{1}{4}''$ wide throughout. (10.)

*8. Draw the arch opening shown, using the figured dimensions. The curve is a semi-ellipse, of which **PQ** is a diameter, and **RS** is half its conjugate character. (12.)

*9. Make a square equal in area to the given figure. (10.)

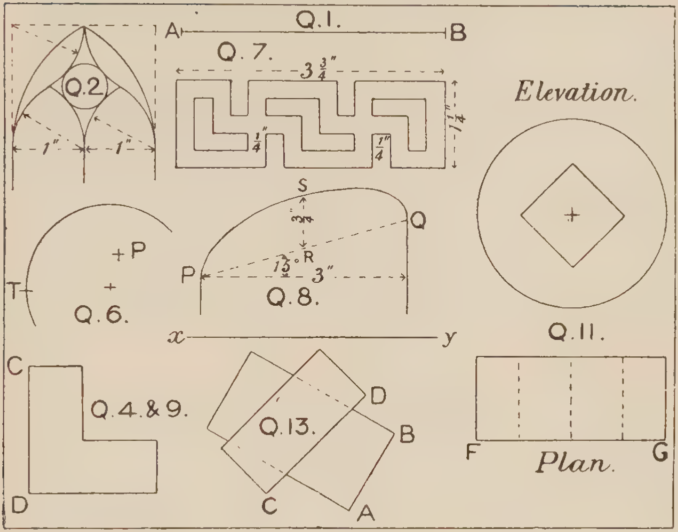
10. Within a circle of $2''$ radius inscribe a "Spiral of Archimedes" of one revolution. (12.)

Solid Geometry.

*11. Plan and elevation are given of a cylinder through which a square opening has been cut. Draw a fresh plan and elevation of the solid, the plane of the circular base **FG** being inclined at 45° to the vertical plane of projection. (16.)

12. A right square pyramid, edge of base $1\frac{1}{2}''$, height $2'$, is cut by a plane which contains one edge of the base, and is inclined at 45° to the plane of the base. Draw the plan of the section, and, if you can, its true form. (14.)

*13. The diagram shows the plan of two square prisms, one resting upon the other. Draw their elevation upon the given xy. The lines **AB** and **CD** are plans of square surfaces. (12.)



APRIL, 1900.

GEOMETRICAL DRAWING.

Only eight *questions are to be attempted. Questions marked* (*) *have accompanying diagrams.*

PLANE GEOMETRY.

*1. The given line **AB** represents a distance of 35 feet. Construct a scale by which single feet may be measured up to 40 feet. The scale is not to be "fully divided," *i.e.* single feet are not to be shown throughout the whole length, and it must be properly finished and figured. (12.)

*2. Draw the given outline of window tracery, using the figured dimensions. The arch is "equilateral," and all the arcs are of equal radius. (10.)

3. The sides of a triangle are 1″, $1\frac{1}{4}$″, and $1\frac{1}{2}$″ long. About this triangle describe a circle, and about the circle describe a triangle of the same shape as the given one. The points of contact must be found and clearly shown. (10.)

*4. Draw the figure, using the figured dimensions. (10.)

5. Construct a regular pentagon of $1\frac{1}{2}$″ side, and a rectangle of the same area and altitude as the pentagon. (12.)

*6. Describe a circle touching both the given circles, and passing through the point **P** on the circumference of the larger one. (12.)

*7. Draw the given frame, using the figured dimensions. The border is $\frac{1}{4}$″ wide throughout. (12.)

*8. Make a figure *similar* to the given one, but having the length corresponding to, **CD** 2″ long. (10.)

9. About a circle of 1″ diameter describe six equal circles, each touching the given circle and two of the others. Then describe a circle touching and enclosing all six of the outer ones. (10.)

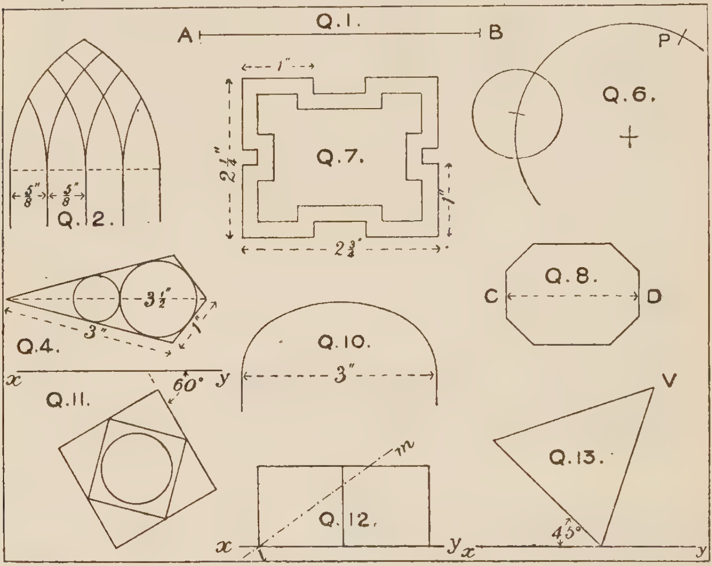
*10. Draw the "three-centred" arch shown. The two lower arcs are of 1″ radius, and the upper arc is of $2\frac{1}{4}$″ radius. (10.)

Solid Geometry.

*11. The diagram shows the plan of two cubes, one resting upon the other, with a sphere resting on the upper cube. Draw an elevation on the given *xy*. (12.)

*12. An elevation is given of a regular hexagonal prism, with its bases horizontal. Draw its plan. Also show the true form of the section made by the inclined plane shown at *lm*. (16.)

*13. The diagram shows the elevation of a right cone having its vertex at V. Draw the plan. (14.)



June, 1900 (Day Examination).

GEOMETRICAL DRAWING.

Only eight *questions are to be attempted. Questions marked* (*) *have accompanying diagrams.*

Plane Geometry.

1. A drawing is made to a scale of $1\frac{1}{2}$″ to 1′, and another drawing is required on which the dimensions shall be three-quarters of those on the first drawing. Make a scale for the second drawing to show feet and inches up to 5′. The scale is not to be "fully divided" (*i.e.* only one length of 1′ is to be divided to show inches), and it must be properly finished and figured. (12.)

2. Construct a square of 2″ side. In the centre of the square place a circle of $\frac{1}{2}$″ radius. Then describe four other circles, each touching the first circle and two sides of the square. (12.)

*3. Draw the given figure, using the figured dimensions.

(This problem is intended as an exercise in the use of T and set squares.) (10.)

4. Within a circle of $2\frac{3}{4}''$ diameter inscribe a regular pentagon. Draw also a second regular pentagon concentric with the first one, its sides being parallel to those of the first and $1\frac{1}{4}''$ long. (12.)

*5. Describe the four given circles, using the figured radii. The necessary construction lines and points of contact must be clearly shown. (10.)

6. Draw two lines **AB, AC**, containing an angle of 75°. Find a point **P**, $\frac{3}{4}''$ from **AB**, and $\frac{1}{2}''$ from **AC**. Complete the isosceles triangle, of which **BAC** is the vertical angle, and the base passes through **P** (10.)

*7. Copy the given figure, using the figured dimensions. (12.)

8. The foci of an ellipse are $2\frac{1}{2}''$ apart, and its minor axis is $2''$ long. Draw the curve, and draw also a tangent from a point on the curve $1''$ from one of the foci. (10.)

9. Construct a square equal in area to a regular hexagon of $1''$ side. (10.)

10. Draw a straight line **AB**, $3''$ long, Bisect **AB** in **F**. At **F** draw **FV** $\frac{3}{4}''$ long at right angles to **AB**. **F** is the focus, and **V** the vertex of a parabola, **A** and **B** being points on the curve. Draw the curve from **A** to **B**, showing the construction for at least 4 points. (10.)

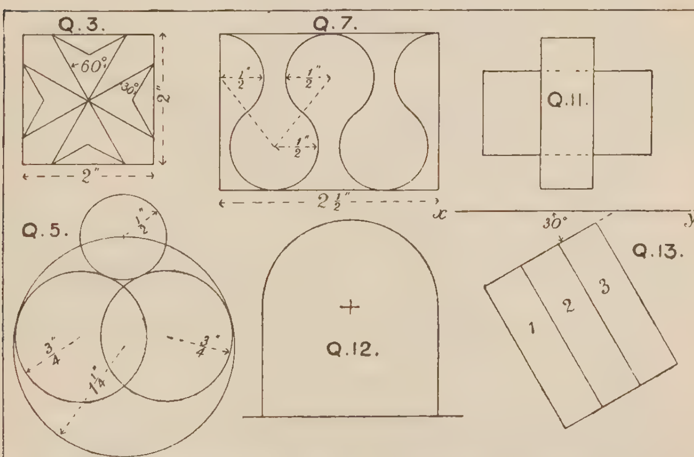
Solid Geometry.

*11. The diagram shows a side elevation of a square prism, pierced through its centre by a cylinder. Draw a front elevation of the solids. (12.)

*12. An elevation is given of an archway with semi-circular head, in a wall 1′ 6″ thick. Draw a second elevation upon a vertical plane which makes an angle of 45° with the face of the wall.

(Scale, which need not be drawn, $\frac{1}{2}''$ to 1′.) (16.)

*13. The plan is given of a flight of three steps each $\frac{3}{4}''$ high, of which 3 is the uppermost Draw an elevation on the given xy. (14.)



APRIL, 1901.

6 to 7.30 p.m.

GEOMETRICAL DRAWING.

GENERAL INSTRUCTIONS.

You may not attempt more than five questions, of which three only may be chosen from Section A, and two only from Section B. But no award will be made to a candidate unless he qualifies in both sections.

All your drawings must be made on the single sheet of drawing paper supplied, for no second sheet will be allowed. You may use both sides of the paper.

None of the drawings need be inked in.

Put the number of the question close to your workings of problems, in large distinct figures.

The number of marks assigned to each question is stated in brackets.

A single accent (′) signifies *feet*; a double accent (″) *inches*.

Questions marked (*) have accompanying diagrams.

SECTION A.

Read the general instructions.

YOU MAY ATTEMPT THREE QUESTIONS ONLY, WHICH MUST BE SOLVED ACCURATELY, USING INSTRUMENTS.

The constructions must be strictly geometrical, and not the result of calculation or trial.

All lines used in the constructions must be clearly shown.

Set squares may be used wherever convenient. Lines may be bisected by trial.

1. Construct a scale $\frac{1}{8}$ of full size, by which feet and inches may be measured up to 2 feet. Show also distances of $\frac{1}{8}''$ by the diagonal method. Finish and figure the scale properly, and show by two small marks on it how you would take off a distance of 1′ 7½″. (20.)

2. Within a regular hexagon of 1¼″ side, inscribe a square, having all its angles in the sides of the hexagon. Within the square inscribe four equal circles, each touching two of the others and two sides of the squares. (16.)

*3. With each of the points **PQR**, as centre, describe a circle of $\frac{5}{8}''$ radius. Then describe two other circles each touching the first three. Show clearly how you would determine all the points of contact. (14.)

*4. Draw the given figure, making the radius of the outer circle 1¼″. (16.)

5. The base of a triangle is 1·65″ long, and the angles at the base are 88° and 53°. Construct the triangle, and find a fourth proportional less to the three sides. Measure and write down the length of this line.

(The angles should be found from the protractor or scale of chords.) (14.)

*6. A plan and elevation are given of a buttress projecting from a wall. Draw a fresh elevation on a vertical plane which makes an angle of 45° with the plane of the wall. (16.)

*7. The diagram shows an elevation of a square slab, **AB** being one side of a square face. Draw the plan of the slab, and show a cylindrical hole of 2″ diameter pierced through its centre. (22.)

SECTION B.

Read the General Instructions.

YOU MAY ATTEMPT TWO QUESTIONS ONLY.

In the solution of these accuracy of measurement and the use of instruments are not demanded so long as the required results be clearly and precisely indicated.

*8. Draw clearly, with instruments or freehand, the geometrical basis on which the given "diaper" pattern is constructed. Alter the scale of your drawing from that of the diagram so as to show two "repeats" of the pattern in a width of 3½″.

Show 5 or 6 repeats in all.

(Only sufficient of the ornamental detail need be sketched to indicate its position.) (18.)

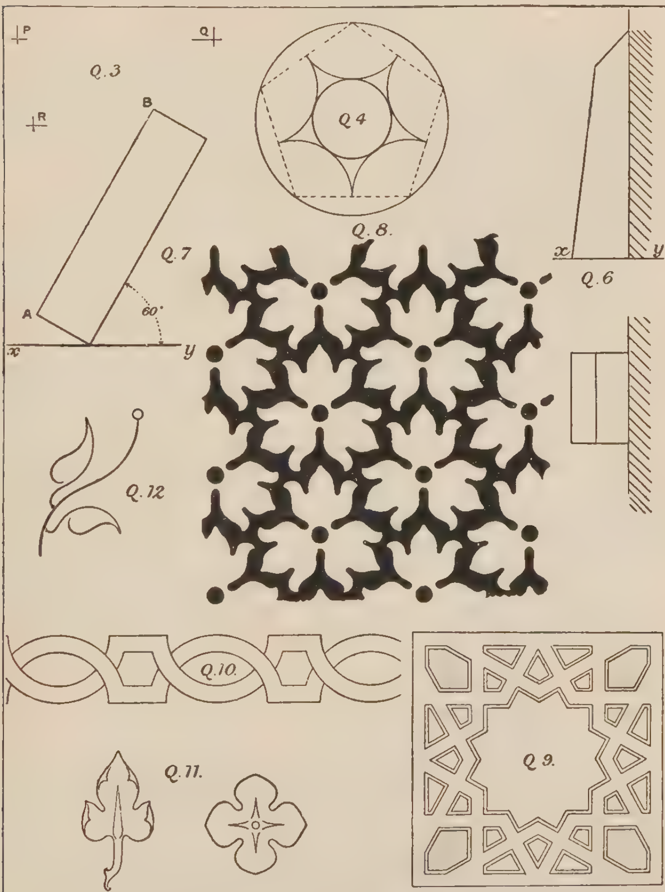
*9. Show the construction lines upon which the ornament in this square panel has been designed. (18.)

*10. Make a drawing of one unit of the given border, increasing the length of the the "unit" to 2¼″, and the other dimensions in proportion. Indicate the method by which the figure should be constructed (22.)

*11. Draw, with instruments or freehand, two different arrangements by which the two given ornaments may be used, alternating with one another, so as to form a "diaper" pattern.

(The ornaments may be roughly sketched, simply to indicate their position.) (20.)

*12. Show how, by repeating and reversing the given lines, an "all-over" pattern may be obtained. Indicate the lines of construction. (16.)



JUNE, 1901.

10 to 11.30 a.m.

GEOMETRICAL DRAWING.

SECTION A.

Read the General Instructions. (p. 222).

YOU MAY ATTEMPT THREE QUESTIONS ONLY, WHICH MUST BE SOLVED ACCURATELY, USING INSTRUMENTS.

The constructions must be strictly geometrical and not the result of calculation or trial.

All lines used in the constructions must be clearly shown.

Set squares may be used wherever convenient. Lines may be bisected by trial.

1. A distance of 4 8″ represents 40 yards. Make a scale of yards, showing feet by the diagonal method. Finish and figure the scale properly. (20.)

*2. Show how you would find the centre of the circles inscribed in the given spandrel form. (18.)

3. Construct a regular pentagon of 2″ side, and a similar pentagon of 2″ diagonal. The two figures should have the same centre.

[N.B.—The protractor may not be used for obtaining the angle of the pentagon.] (20.)

4. Within a square of $2\frac{1}{2}$″ side inscribe four equal semicircles, each touching one side of the square and having their diameters adjacent. Within the quatrefoil so constructed describe, with the same centres as the semicircles, another quatrefoil, but of tangential arcs. Show clearly how the points of contact are found. (18.)

5. Construct an isosceles right-angled triangle having its hypotenuse (or side opposite the right angle) $2\frac{3}{8}$″ long. Within it inscribe a square having one of its sides in the hypotenuse of the triangle. Measure and state, as accurately as you can, the length of one side of the square. (16.)

*6. The *end* elevation is given of a small coffer or caddy, the *length* of which is to be 3″. The lid has four sloping faces, which all make the same angle (30°) with the horizontal. Draw the plan of the lid. (18.)

*7. The plan is given of a square prism of which **AB** represents a square face. Determine the elevation of the prism on the given xy, and add the elevation of a circular hole of 2″ diameter piercing the centre of the prism. (20.)

SECTION B.

Read the General Instructions. (p. 222).

YOU MAY ATTEMPT TWO QUESTIONS ONLY.

In the solution of these, accuracy of measurement and the use of instruments are not demanded so long as the required results be clearly and precisely indicated.

*8. Sketch, with instruments or freehand, *one unit* of the given diaper; showing clearly your method of setting out its details. Make your drawing about three times as large as one of the units in the diagram. (18.)

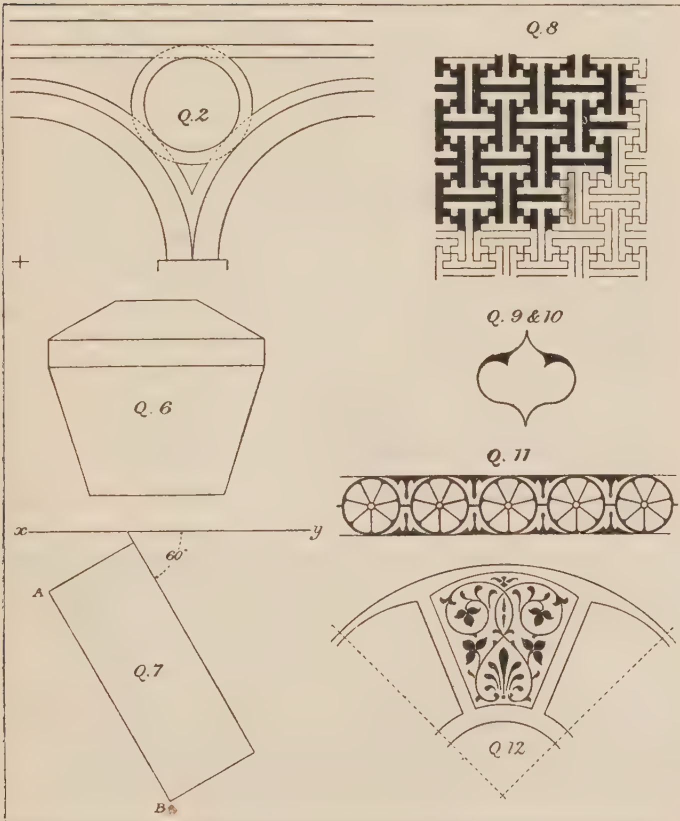
*9. Sketch, with instruments or freehand, an "all-over" diaper pattern formed by repeating the given unit. Show 9 repeats of the unit, with the leading lines of the construction. Make each unit about the same size as that in the diagram.

[The height and width of the figure are equal.] (20.)

*10. The outline of the diaper pattern formed by placing repeats of the given figure in contact with one another is made up of semicircles. Draw at least four repeats of the outline so as to show clearly where centres and points of contact of the semicircles occur. Make each unit about twice the size of that in the diagram. (16.)

*11. Make an enlarged copy of *one unit* of repeat of the given border, the height of your drawing being increased to $1\frac{1}{2}''$ and the length in proportion. Show a construction for obtaining the divisions of the circle. (18.)

*12. The figure shows one quarter of the decoration of a circular *plaque*. Complete the circle, and set out the panels. The position only of the freehand ornament need be indicated. (20.)



April 19, 1902.

6 to 7.30 p.m.

GEOMETRICAL DRAWING.

Section A.

Read the General Instructions (p. 222).

You may attempt three questions only, which must be solved accurately, using instruments.

The constructions must be strictly geometrical, and not the result of calculation or trial.

All lines used in the constructions must be clearly shown.

Set squares may be used wherever convenient. Lines may be bisected by trial.

*1. Make a diagonal scale of feet, to show single feet up to 400 feet, for a plan on which the given line AB represents 320 feet. Figure the scale properly, and show by two small marks on it how you would take off a distance of 468 feet. (22.)

2. About a circle of ·75″ radius describe an equilateral triangle. Describe three equal circles touching the given circle, and having their centres at the angles of the triangle. Determine the points of contact. (14.)

*3. Draw the given figure, making the side of the square $2\frac{1}{4}''$ long. Show clearly how all the points of contact are determined. (18.)

*4. Copy the cornice given, increasing the total height to $2\frac{1}{4}''$, and the other measurements in proportion. You may draw the "cyma" moulding by any geometrical construction that seems to you suitable. (20.)

*5. The given figure is to be drawn according to the following data :—

The point P is to be 1″ from the line CD.
Each of the two small circles is to be of ·75″ radius.
The large arc is to be of 1·75″ radius. (18.)

*6. The plan is given of an octagonal tray or dish, the height of which is $\frac{1}{2}''$. Make an elevation on the given xy.

(Only the visible lines need be shown in the elevation, and the thickness of the material is to be neglected.) (18.)

7. A right cone, height $2\frac{1}{2}''$, radius of base 1″, stands with its base on the horizontal plane. A sphere of 1″ radius rests on the horizontal plane and *touches* the cone. Draw a plan and elevation of the two solids, showing clearly your construction for finding the centre of the sphere. Show also the true form of the section of each solid by a horizontal plane $\frac{1}{2}''$ above the centre of the sphere. (22.)

Section B.

Read the General Instructions (p. 222).

You may attempt two questions only.

In the solution of these, accuracy of measurement and the use of instruments are not demanded so long as the required results be clearly and precisely indicated.

Short notes to fully explain your meaning should be written where you think they are necessary.

*8. Draw, freehand or with instruments, a system of construction lines on which the given repeating pattern of quatrefoils can be built up. Show how you would determine the centres of the arcs, and the points of contact. Three or four repeats of the unit should be indicated, about the same size as in the diagram. (18.)

*9. Show clearly any geometrical construction you think would be useful in setting

out the given "Tudor rose" about twice the dimensions of the diagram. You need only sketch so much of the flower as is needed to illustrate your construction. (16.)

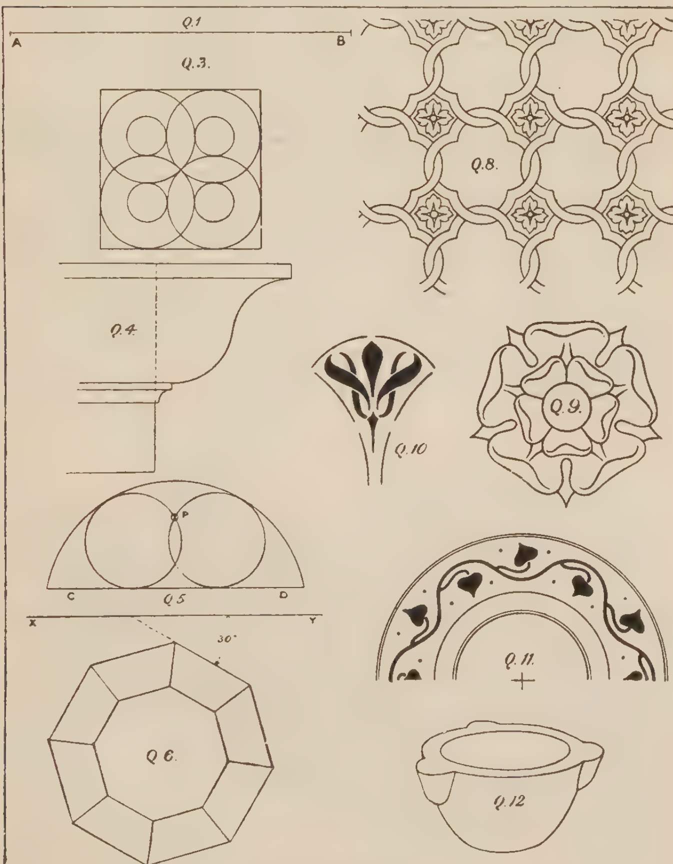
***10.** The diagram represents a stencilled ornament which it is desired to repeat so as to form a "diaper" pattern. Sketch two ways in which this can be done, the repeats of the unit being placed adjacent to one another. Show four repeats in each case, about the same size as the diagram. (16.)

***11** The diagram shows half a circular plate. Indicate a method by which the leading wave line of the ornament could be made up of arcs of circles of equal radius.

(Your drawing may be the same size as the diagram.) (18.)

***12.** Make an approximate sketch plan, and also a sectional elevation, of the mortar of which a perspective sketch is given, assuming that the inside form is a hemisphere. Show clearly any construction you would suggest.

(The size of your drawing should not be less than that of the diagram.) (18.)



JUNE 18, 1902.

10 to 11.30 a.m.

GEOMETRICAL DRAWING.

SECTION A.

Read the General Instructions (p. 222).

YOU MAY ATTEMPT THREE QUESTIONS ONLY, WHICH MUST BE SOLVED ACCURATELY, USING INSTRUMENTS.

The constructions must be strictly geometrical, and not the result of calculation or trial.

All lines used in the constructions must be clearly shown.

Set squares may be used wherever convenient. Lines may be bisected by trial.

1. Make a diagonal scale one hundredth ($\frac{1}{100}$) of full size, showing feet and inches up to 50 feet. Figure the scale properly, and show by two small marks on it a distance of 18′ 5″. (22.)

2. About a circle of ·8″ radius describe a rhombus having one of its angles 54°. Each side of the rhombus touches the circle. Determine the four points of contact. (14.)

***3.** The given figure is made up of a rectangle and *semicircles.* Make a copy of it, using the figured dimensions. (18.)

4. Draw a line AB 1·5″ long. With A as centre describe a circle of 1″ radius. Describe a second circle touching the first, and also *touching* the line AB at B. Show clearly the point at which the two circles touch one another. (16.)

***5.** Copy the diagram, using the figured dimensions. (20.)

***6.** The diagram represents a doorway in a wall, the door being shown opened at an angle of 45° with the surface of the wall. Draw an elevation of the door when closed; *i.e.* showing the true form of the panels. *Only* the door need be drawn, not the surrounding mouldings. Your construction *must* be shown. (20.)

7. A right cylinder of 2″ diameter is cut by a plane making an angle of 30° with the axis of the cylinder. Show the true form of the section. (22.)

SECTION B.

Read the General Instructions (p. 222).

YOU MAY ATTEMPT TWO QUESTIONS ONLY.

In the solution of these, accuracy of measurement and the use of instruments are not demanded so long as the required results be clearly and precisely indicated.

Short notes to fully explain your meaning should be written when you think they are necessary.

***8.** Draw, with instruments or freehand, the system of construction lines on which you would build up the given repeating pattern. Show also one complete unit of the repeat. (18.)

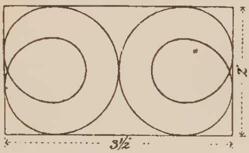
***9.** Show how you would proceed to modify the given figure so as to make the central panel a *regular* octagon, the width of the four side panels remaining unchanged. (16.)

***10.** Indicate clearly a geometrical basis for the given repeating pattern, and show what you consider to be the unit. (The freehand ornament should only be shown once.) (16.)

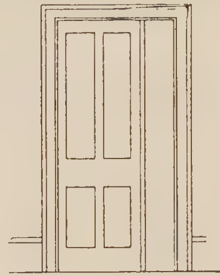
***11.** It is desired to rest re the complete circular ornament of which a fragment is shown. How would you do this? The restorations of the dark portions should be disregarded altogether in your drawing. (18.)

***12.** The diagram shows a perspective sketch of part of a buttress. Make an approximate sketch plan and a side elevation. (18.)

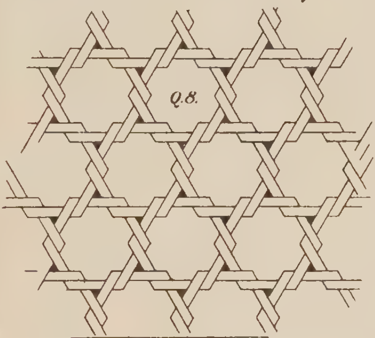
Q. 3.



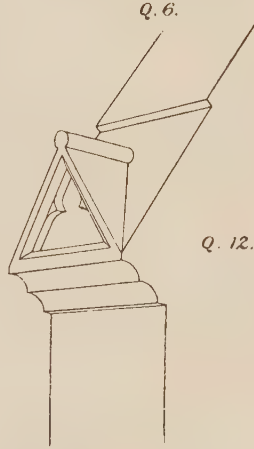
Q. 5.



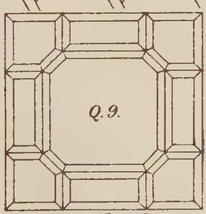
Q. 6.



Q. 8.



Q. 12.



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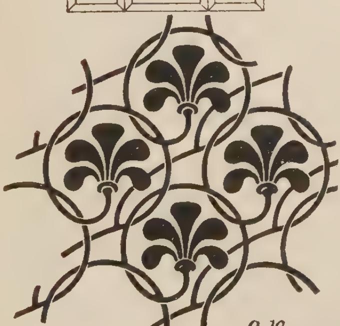
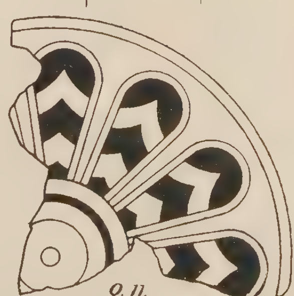


Q. 10.



Q. 11.

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